Lesson 1: Introduction on Filters and Backgrounds on Distributed Circuits

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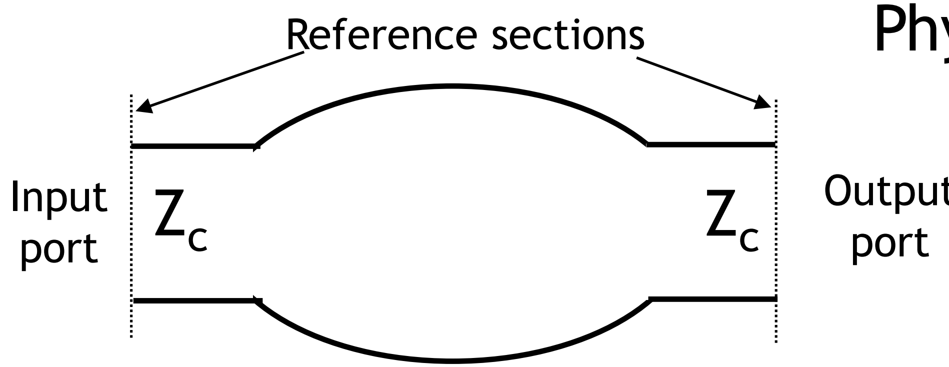
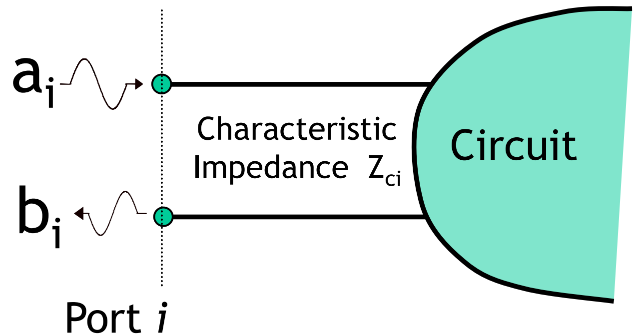
Representation of a physical structure

Physical Structure



Given a physical structure, the *Scattering Parameters* (**S** matrix) allow a representation of its behavior in terms of the waves propagating on the transmission lines that connect the structure with the external world.
Each line is characterized by its characteristic impedance (Z_c), the phase velocity (v) and a reference section (defined *port*) at which the structure is observed.
The physical structure must be linear to allow the matrix characterization.

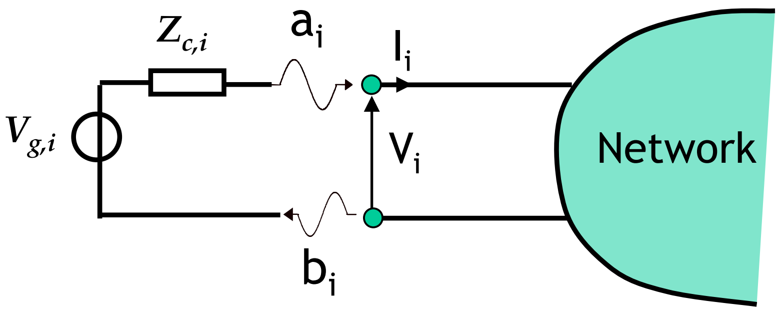
Power Wave definition



(1/2)$|a_i|^2$: Incident power wave = Available power from a source with impedance Z_{ci}

(1/2)$|b_i|^2$: Reflected power wave = Difference between the available power and the power absorbed by the port (i.e. flowing into the port)

Definition of conventional *V* and *I*



$$V_{g,i} = Z_{c,i} I_i + V_i$$

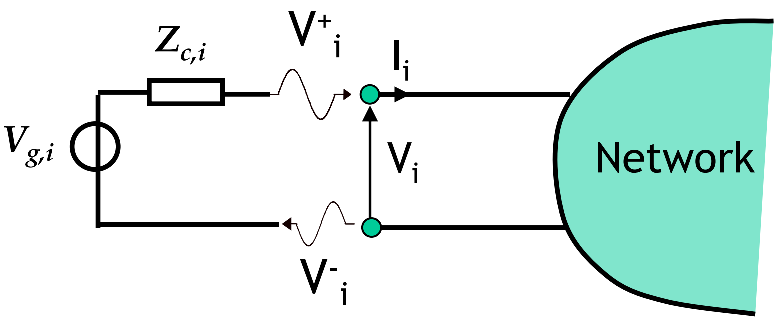
$$P_{IN,i} = \frac{1}{2}\mathrm{Re}\left\{V_i \cdot I_i^*\right\}$$

$$P_{av,i} = \frac{\left|V_{g,i}\right|^2}{8 \cdot \mathrm{Re}\left\{Z_{c,i}\right\}} = \frac{\left|V_i + Z_{c,i} \cdot I_i\right|^2}{8 \cdot \mathrm{Re}\left\{Z_{c,i}\right\}} = \frac{1}{2}\left|a_i\right|^2 \quad \Rightarrow \quad a_i = \frac{V_i + Z_{c,i} \cdot I_i}{2\sqrt{\mathrm{Re}\left\{Z_{c,i}\right\}}} = I_i \frac{Z_i + Z_{c,i}}{2\sqrt{\mathrm{Re}\left\{Z_{c,i}\right\}}}$$

$$P_{IN,i} = \frac{1}{2}\mathrm{Re}\left\{V_i \cdot I_i^*\right\} = \frac{1}{2}\left(\left|a_i\right|^2 - \left|b_i\right|^2\right) \quad \Rightarrow \quad b_i = \frac{V_i - Z_{c,i}^* \cdot I_i}{2\sqrt{\mathrm{Re}\left\{Z_{c,i}\right\}}} = I_i \frac{Z_i - Z_{c,i}^*}{2\sqrt{\mathrm{Re}\left\{Z_{c,i}\right\}}}$$

$$\left(a_i = \frac{I_i + Y_{c,i} \cdot V_i}{2\sqrt{\mathrm{Re}\left\{Y_{c,i}\right\}}}, \quad b_i = \frac{-I_i + Y_{c,i}^* \cdot V_i}{2\sqrt{\mathrm{Re}\left\{Y_{c,i}\right\}}} \right)$$

Power and Voltages waves



$$V_i = V_i^+ + V_i^-, \quad I_i = I_i^+ + I_i^-$$

$$V_i^+ = Z_{c,i} I_i^+, \quad V_i^- = -Z_{c,i} I_i^-$$

Power waves:

$$a_i = \frac{V_i + Z_{c,i} \cdot I_i}{2\sqrt{\mathrm{Re}\{Z_{c,i}\}}}, \quad b_i = \frac{V_i - Z_{c,i}^* \cdot I_i}{2\sqrt{\mathrm{Re}\{Z_{c,i}\}}}$$

Voltage waves:

$$V_i^+ = \frac{V_i + Z_{c,i} \cdot I_i}{2}, \quad V_i^- = \frac{V_i - Z_{c,i} \cdot I_i}{2}$$

Absence of reflected wave:

Conjugate matching

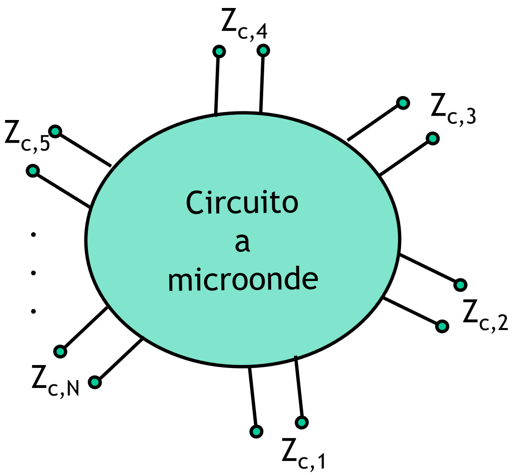
$$Z_i = Z_{c,i}^*$$

Matching

$$Z_i = Z_{c,i}$$

Coincide for $Z_{c,i}$ real

Generalized Scattering Matrix



For a linear circuit, incident and reflected waves are linearly related:

$$b_1 = s_{11}a_1 + s_{12}a_2 + \dots + s_{1N}a_N$$
$$b_2 = s_{21}a_1 + s_{22}a_2 + \dots + s_{2N}a_N$$
$$\dots\dots\dots\dots\dots\dots$$
$$b_N = s_{N1}a_1 + s_{N2}a_2 + \dots + s_{NN}a_N$$

In matrix form:

$$\underline{\mathbf{b}} = \underline{\underline{\mathbf{S}}} \cdot \underline{\mathbf{a}} \qquad \underline{\underline{\mathbf{S}}} = \begin{pmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{N1} & \cdots & s_{NN} \end{pmatrix}$$

Meaning of S parameters

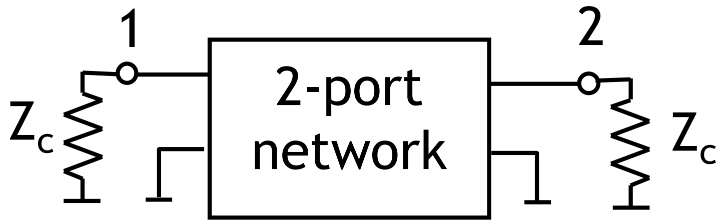
$$s_{ii} = \left.\frac{b_i}{a_i}\right|_{a_{k \neq i}=0}$$

➡ Reflection coefficient at port i when the other ports are connected to their reference impedances Z_{cj} (*matched*)

$$s_{ij} = \left.\frac{b_i}{a_j}\right|_{a_{k \neq j}=0}$$

➡ Transmission coefficient from port j to port i with the other ports matched. Note that $|s_{ij}|^2$ rappresents the transducer power gain between the two ports

S matrix of 2-port passive network



Equivalent Representation

S_{11}, S_{22} = Reflection at the ports
S_{21}, S_{12}=Transmission between the ports

Passive structures: $|S_{i,j}|<1$
Reciprocity: $S_{12}=S_{21}$
Lossless structures: **S** unitary ($|S_{11}|^2+|S_{12}|^2=1$, …)

In filters application the transmission parameter (S_{21}) is replaced by its inverse (>1). In decibel it has:

$$\text{Attenuation (dB)} = -20 \log_{10}(|S_{21}|)$$

A very general definition of RF Filters

"A RF filter is a 2-port junction exhibiting a selective frequency behavior in the transmission from the input port to the output port"

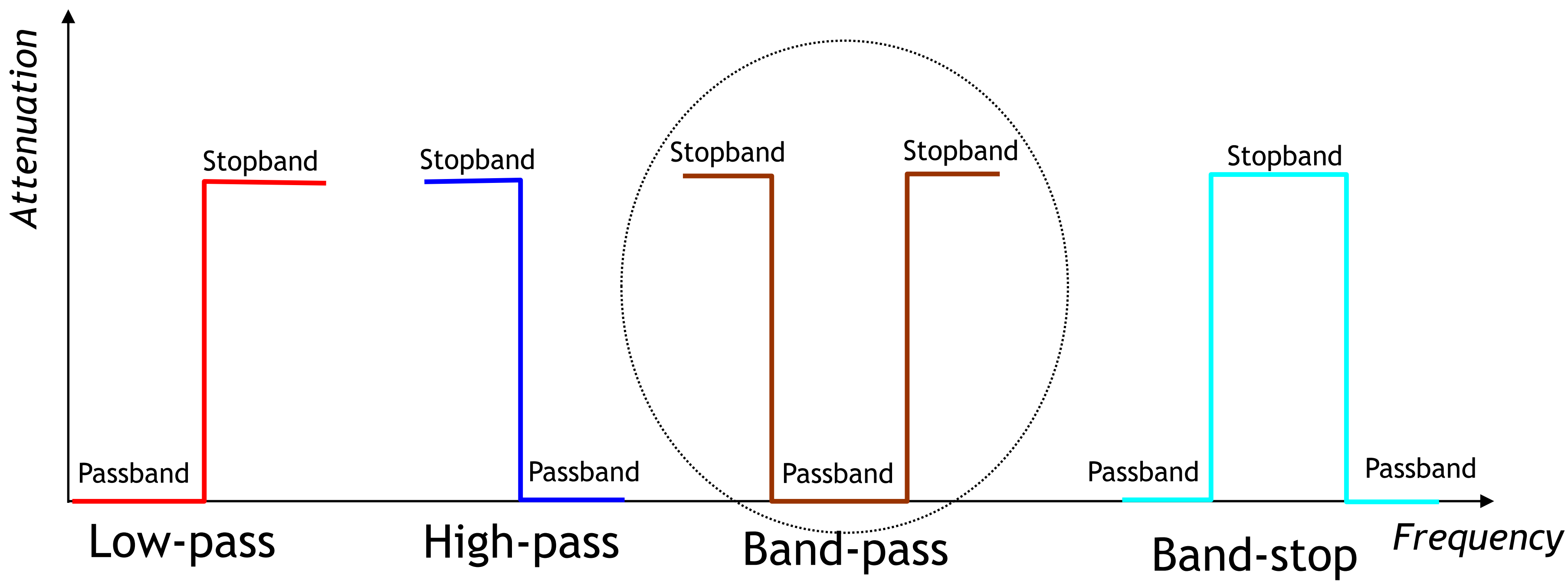
Passband(s): Frequency band(s) which is transferred from input to output without attenuation (ideally)

Stopband(s): Frequency band(s) where the transmission is blocked. In general, the signal rejection is obtained by reflection

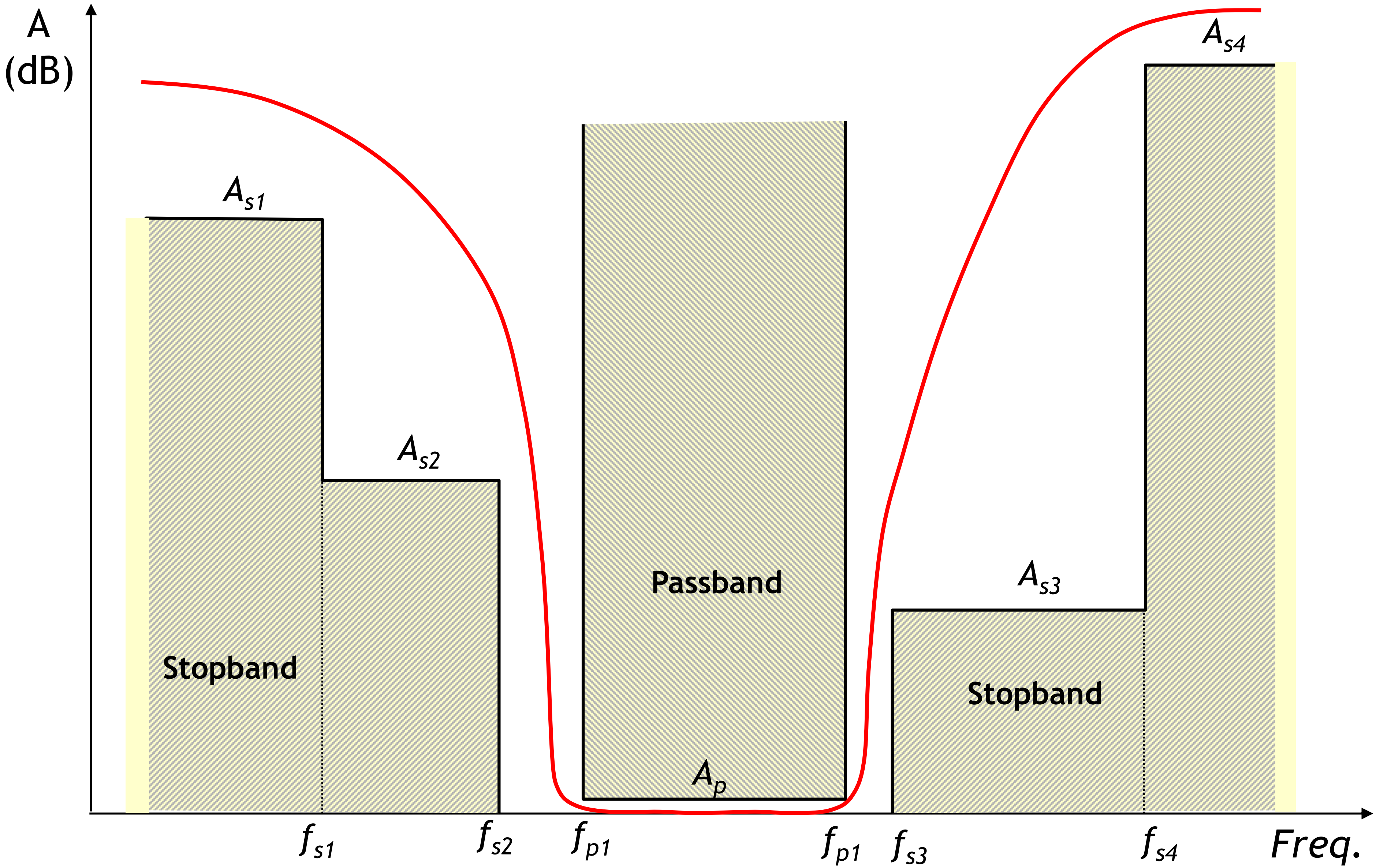
Basic Classification

Filter are classified according to the number and location of passbands and stopbands. The most basic classification considers 4 filters classes:

Low-pass, High-pass, Band-pass, Band-stop



Specification of filter requirements: the Attenuation Mask



Attenuation and matching

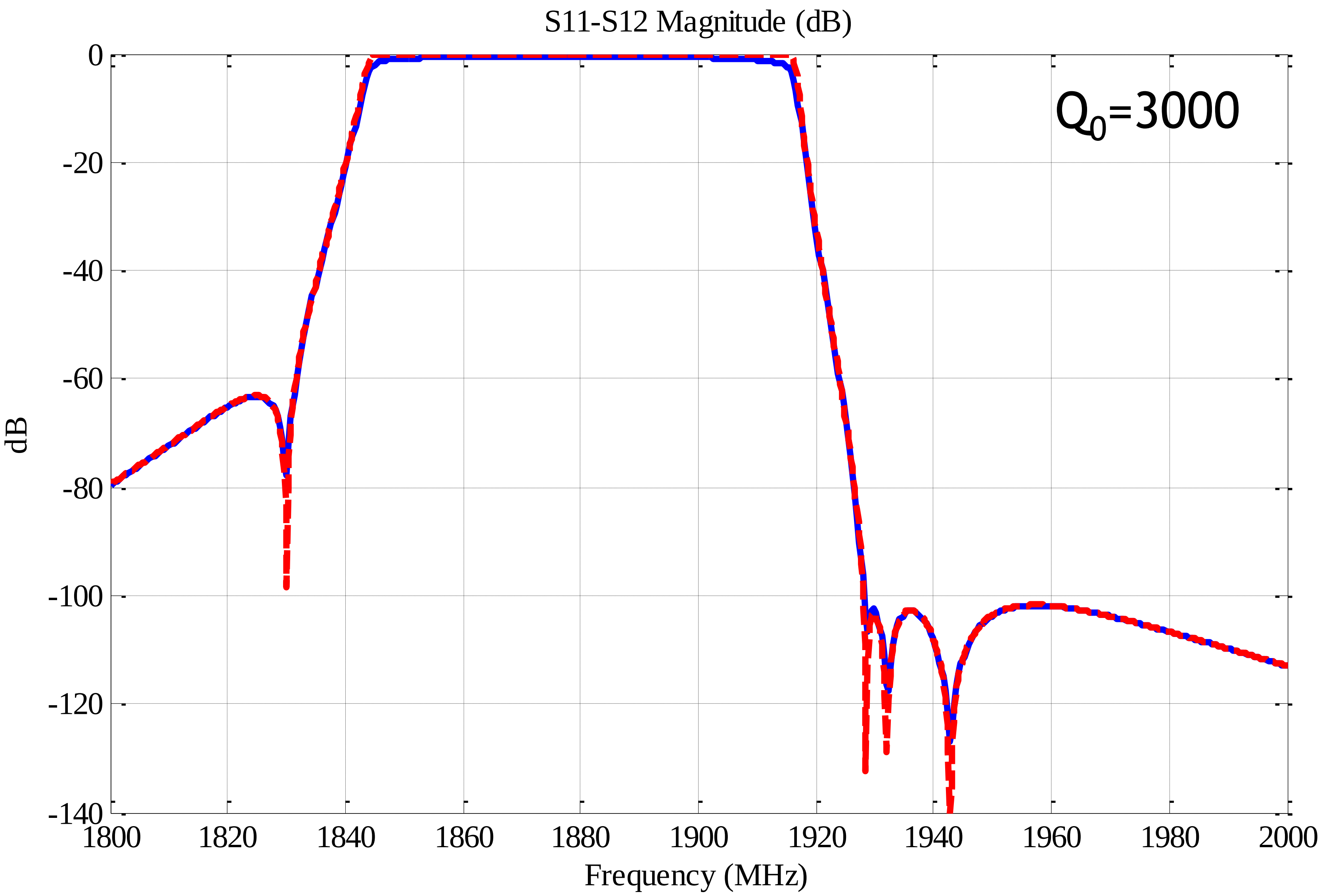
Ideally, a filter is a lossless network. In this case the attenuation in the passband is produced only by the not perfect match at input/output ports.

In the real world the filter components introduce **dissipation**, which becomes the main contribution to passband attenuation.

In general, passband losses have relatively little effect on port matching

In addition to the attenuation mask (which specifies the attenuation requirements), the matching requirements at the ports must also be given

Effect produced by losses on attenuation shape



Requirements on transmission phase

- To have a distortion-less two port, the phase of transmission parameter should be linear in the passband
- Often, the requirement is given on the *group delay*, which is defined as the derivative of transmission phase with respect the radian frequency. The group delay should be ideally constant in the passband
- Phase linearity is generally assessed a posteriori, after the attenuation requirements are satisfied.
- If the requirements on phase linearity is not verified, a *phase equalizer* may be required.
- In alternative, complex transmission zeros can be introduced in the response for phase equalization

Approaches to the design of microwave filters

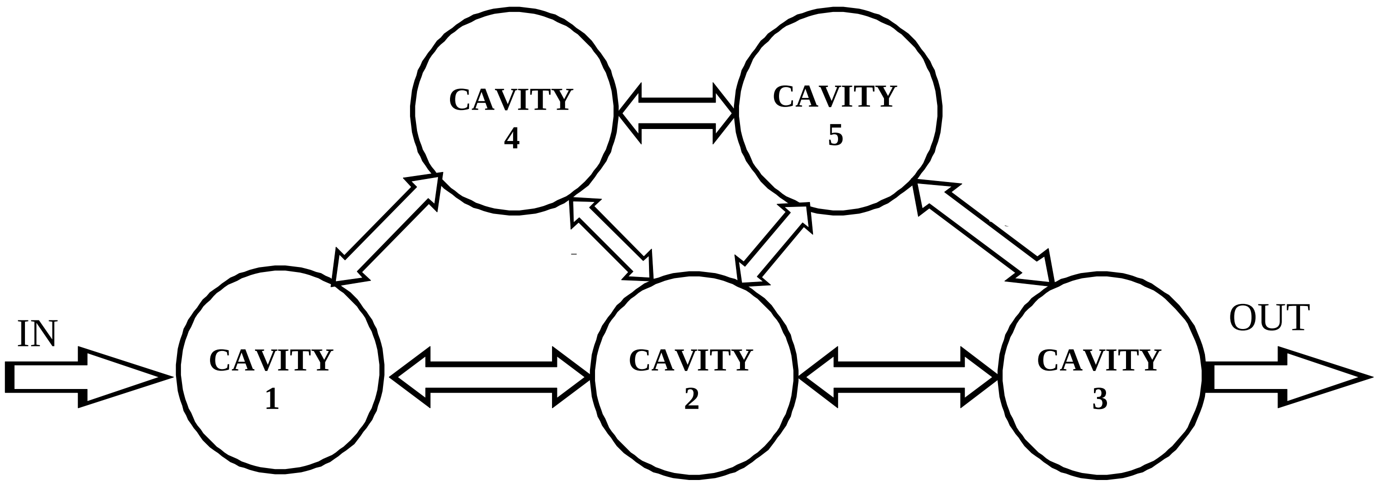
- Image parameters methods (old technique based on attenuation produced by basic blocks, which are suitably interconnected).
- Synthesis of networks composed by commensurate transmission line sections and stubs (suited for broadband filters; poses strong constraints on the filter configuration)
- **Synthesis method based on equivalent circuits,** exploits the equivalence between the real structure (distributed components) and a lumped-element filter network (suitable for narrow/moderate bandwidth). It is the most used technique today

Advantages of the Synthesis Method

- Accurate control of the curve representing the filter response (at least in the passband)
- The synthesis is developed in the lumped-element world, where well established, analytical and numerical procedures are available
- The results of the synthesis can be expressed by means of universal parameters which maintain the same meaning both for lumped-element circuit and distributed microwave networks
- A first-order dimensioning of the physical structure can be easily performed using these universal parameters

Recalls on Microwave Circuits for Filter Design

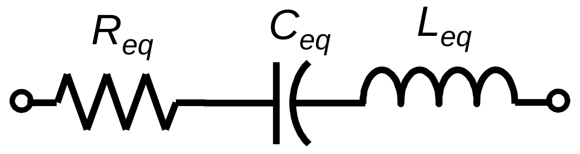
General physical structure of a microwave filter

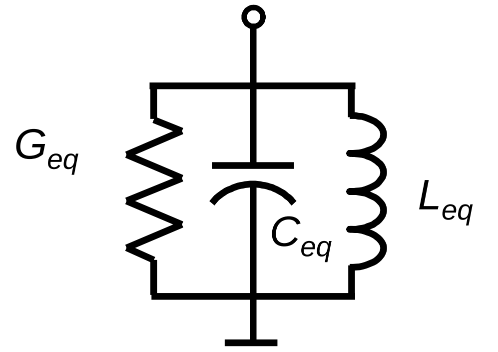


Basic components:
- Cavities (resonating on a specific mode)
- Coupling structures (represented as ⟺)

Not always cavities and couplings can be identified separately

Basic equivalent circuit of the cavity



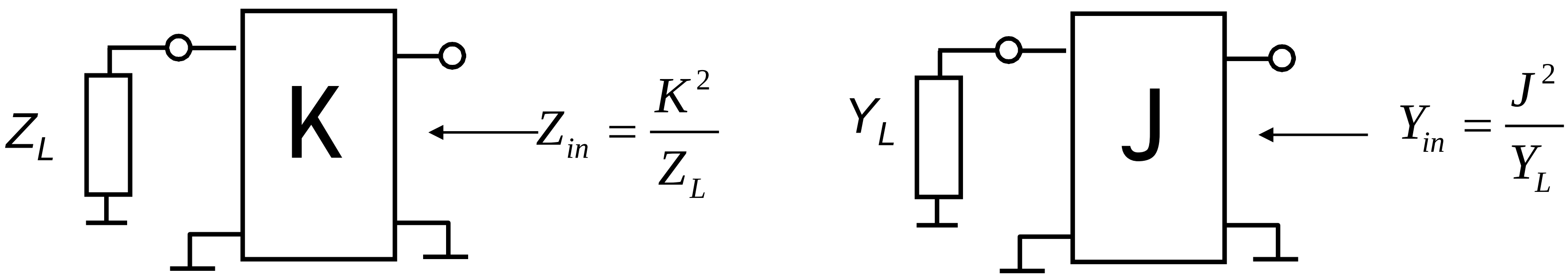


Series or parallel resonator

<u>Characteristic parameters</u>:
- Mode Resonant frequency (f_0)
- Equivalent slope parameter (L_{eq}, C_{eq})
- Loss parameter (R_{eq}, G_{eq})

Basic equivalent circuit of the coupling

Couplings are generally modeled with impedance or admittance inverters



The inverter has two basic functions:
- Operate as impedance transformer
- Change the nature of the load

The inverter is an ideal component!
It can only be approximated by real elements
(in a limited frequency range)

S parameters of the Impedance Inverter

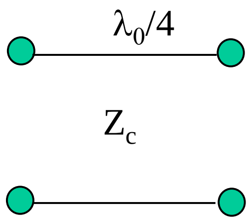
$$S_{11} = S_{22} = \frac{\frac{K^2}{Z_0} - Z_0}{\frac{K^2}{Z_0} + Z_0} = \frac{K^2 - Z_0^2}{K^2 + Z_0^2}$$

Real (positive or negative)

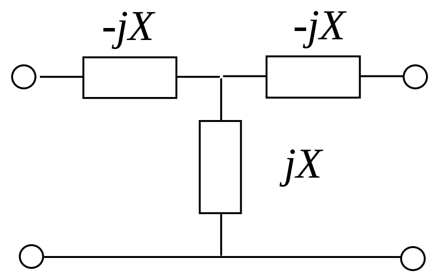
$$\phi_{11} - \phi_{12} = \pm\frac{\pi}{2} \quad \Rightarrow \quad \phi_{12} = \pm\frac{\pi}{2}$$

The impedance inverter is a symmetrical and reciprocal 2-port network. It behaves as an ideal $\pi/2$ phase shifter.
Imposing the lossless condition and the K value it is not sufficient to identify univocally the network (the ± sign of ϕ_{12} remains undetermined)

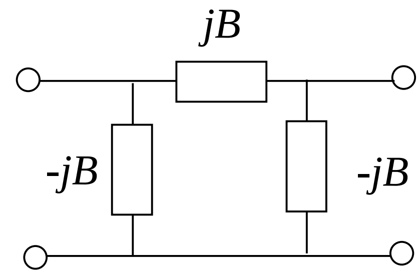
Equivalent circuit for the impedance inverter



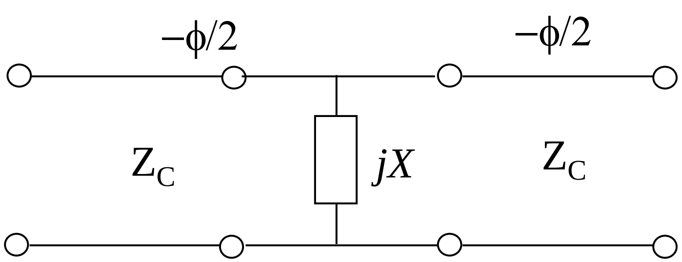
K=Zc



K=X



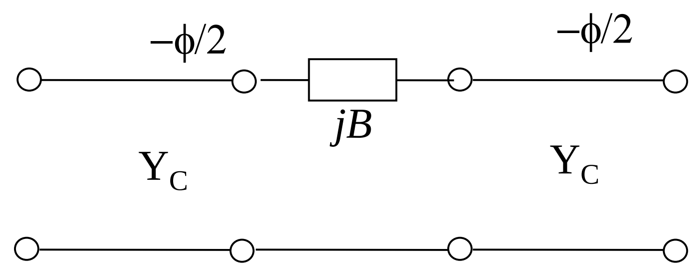
J=1/K=B



$$K = \left| Z_C \tan(\phi/2) \right|$$

$$\frac{\phi}{2} = \frac{1}{2}\tan^{-1}\left(\frac{2X}{Z_C}\right)$$

$$\frac{X}{Z_C} = \frac{K/Z_C}{1-(K/Z_C)^2}$$

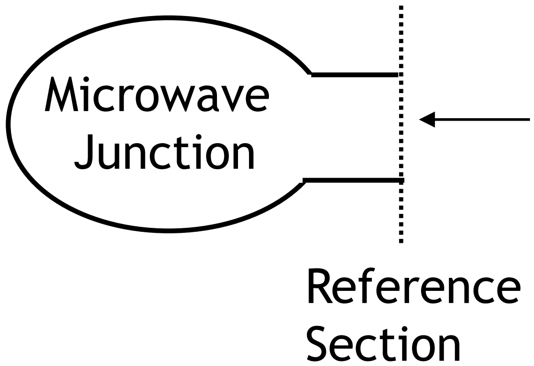


$$J = \left| Y_C \tan(\phi/2) \right|$$

$$\frac{\phi}{2} = \frac{1}{2}\tan^{-1}\left(\frac{2B}{Y_C}\right)$$

$$\frac{B}{Y_C} = \frac{J/Y_C}{1-(J/Y_C)^2}$$

Modeling of a microwave junction with a lumped equivalent circuit (1-port)



$Z_J = R_J(\omega) + jX_J(\omega) \approx Z_J = R_J(\omega_0) + jX_J(\omega)$

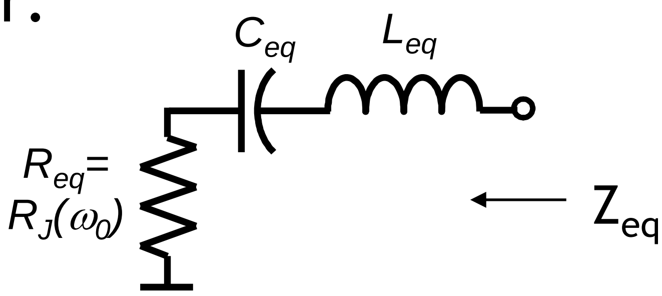
Goal: model of the junction in a defined frequency range B around f_0 ($B \ll f_0$)

The model is represented by an equivalent lumped-element impedance $Z_{eq} = R_{eq} + X_{eq}$ obtained by imposing:

- Constant real part: $R_{eq} = R_J(\omega_0)$
- Same value at ω_0 of the imaginary part: $X_{eq}(\omega_0) = X_J(\omega_0)$
- Same value of derivative at ω_0: $\partial X_{eq}(\omega_0)/\partial\omega = \partial X_J(\omega_0)/\partial\omega$

Equivalent impedance Z_{eq}

The most elementary network exhibiting Z_{eq} is a series resonator:



$$Z_{eq} = R_{eq} + j\left(\omega L_{eq} - \frac{1}{\omega C_{eq}}\right)$$

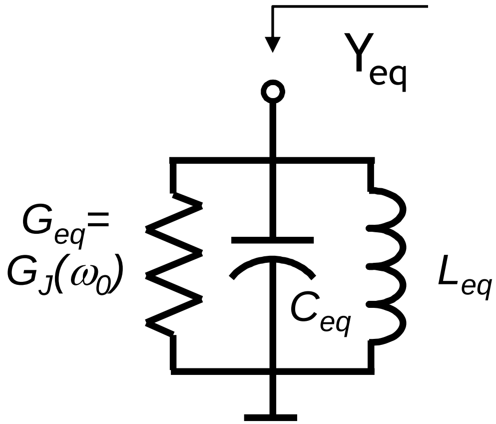
Imposing the previous conditions:

$$X_{eq}(\omega_0) = \omega_0 L_{eq} - \frac{1}{\omega_0 C_{eq}} = X_J(\omega_0)$$

$$\left.\frac{\partial X_{eq}(\omega)}{\partial \omega}\right|_{\omega=\omega_0} = L_{eq} + \frac{1}{\omega_0^2 C_{eq}} = \left.\frac{\partial X_J(\omega)}{\partial \omega}\right|_{\omega=\omega_0}$$

$$L_{eq} = \frac{1}{2}\left[\left.\frac{\partial X_J(\omega)}{\partial \omega}\right|_{\omega=\omega_0} + \frac{X_J(\omega_0)}{\omega_0}\right], \qquad \frac{1}{\omega_0^2 C_{eq}} = \frac{1}{2}\left[\left.\frac{\partial X_J(\omega)}{\partial \omega}\right|_{\omega=\omega_0} - \frac{X_J(\omega_0)}{\omega_0}\right]$$

Equations for the Equivalent Admittance



$$Y_{eq} = G_{eq} + j\left(\omega C_{eq} - \frac{1}{\omega L_{eq}}\right)$$

Imposing the previous conditions:

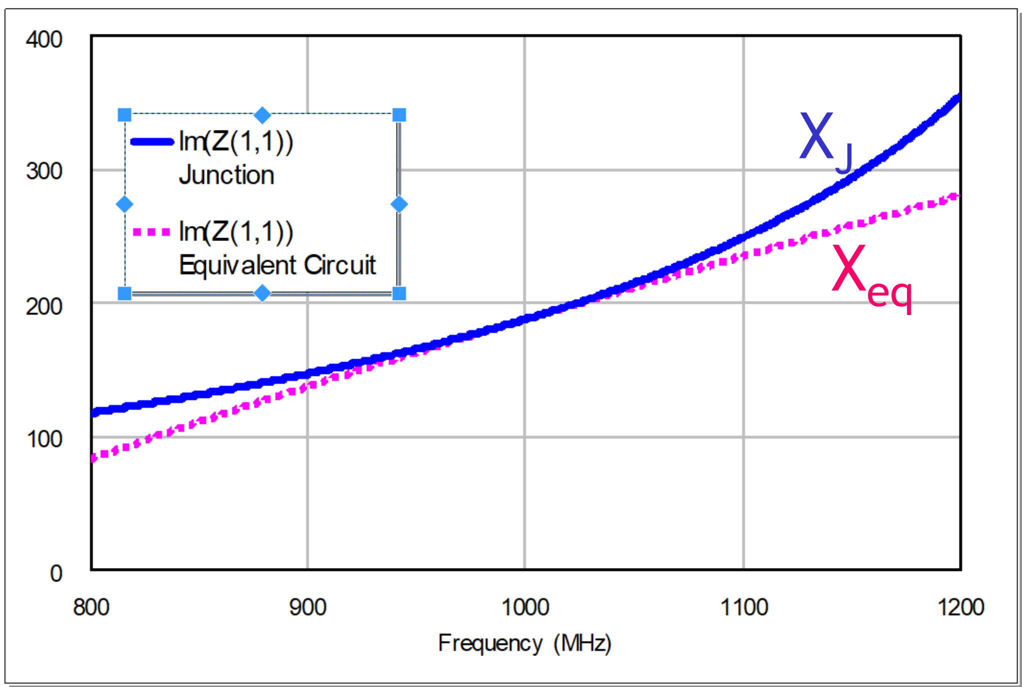
$$B_{eq}(\omega_0) = \omega_0 C_{eq} - \frac{1}{\omega_0 L_{eq}} = B_J(\omega_0)$$

$$\left.\frac{\partial B_{eq}(\omega)}{\partial \omega}\right|_{\omega=\omega_0} = C_{eq} + \frac{1}{\omega_0^2 L_{eq}} = \left.\frac{\partial B_J(\omega)}{\partial \omega}\right|_{\omega=\omega_0}$$

$$C_{eq} = \frac{1}{2}\left[\left.\frac{\partial B_J(\omega)}{\partial \omega}\right|_{\omega=\omega_0} + \frac{B_J(\omega_0)}{\omega_0}\right], \qquad \frac{1}{\omega_0^2 L_{eq}} = \frac{1}{2}\left[\left.\frac{\partial B_J(\omega)}{\partial \omega}\right|_{\omega=\omega_0} - \frac{B_J(\omega_0)}{\omega_0}\right]$$

Remark on the equivalent network

- The equivalence between the microwave junction and the lumped-element equivalent circuit is exact only at f_0
- The deviation between Z_J (Y_J) and Z_{eq} (Y_{eq}) becomes larger and larger with the increase of B



f_0=1000 MHz
$X(f_0)$=188.4 Ω

Special case: Resonant Junction (Cavity)

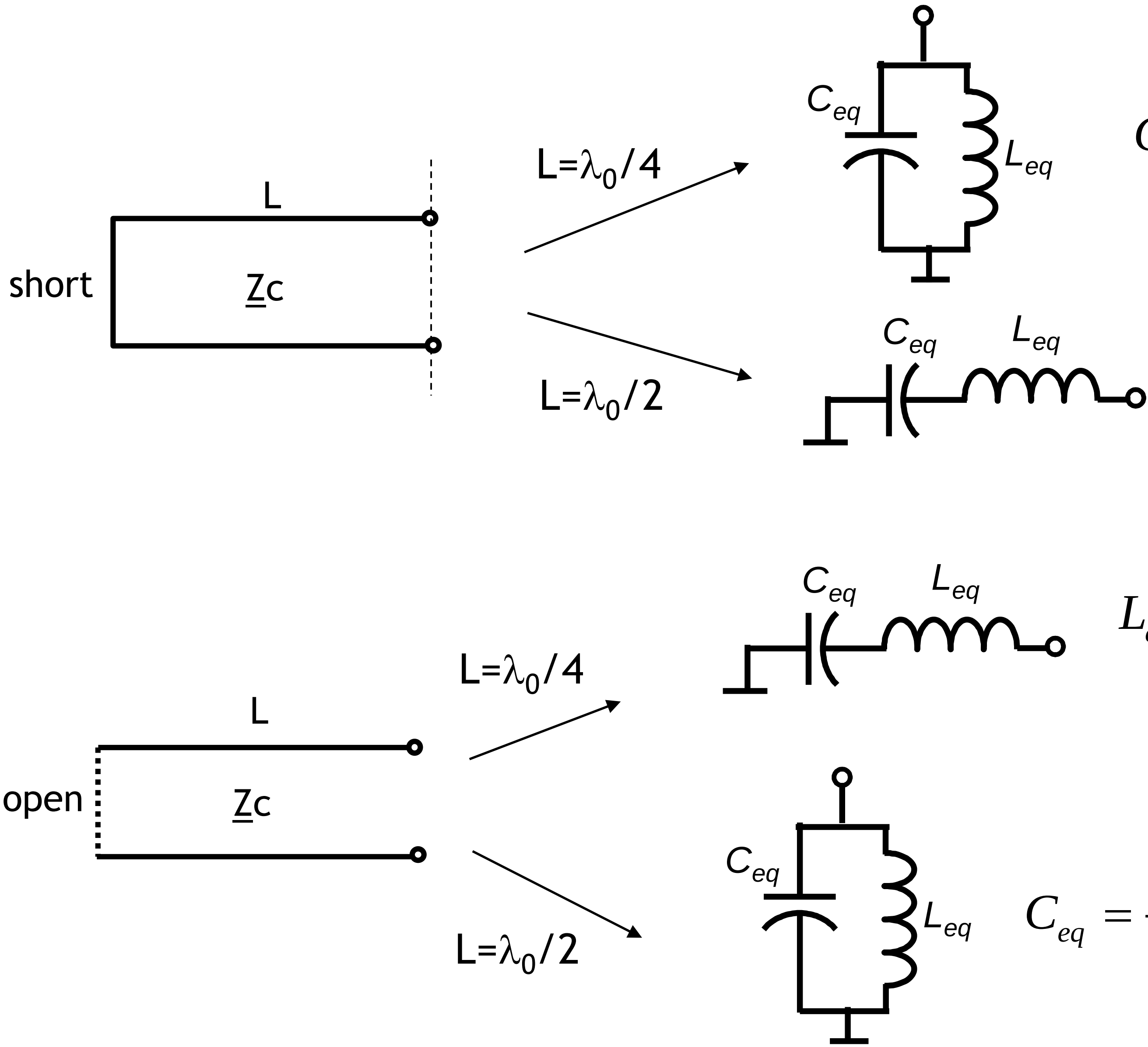
In case of resonant junctions ($X_J(\omega_0)=0$, $B_J(\omega_0)=0$), the previous equations become:

$$L_{eq} = \frac{1}{2}\frac{\partial X_J(\omega)}{\partial \omega}\bigg|_{\omega=\omega_0} \qquad C_{eq} = \frac{1}{\omega_0^2 L_{eq}}$$

$$C_{eq} = \frac{1}{2}\frac{\partial B_J(\omega)}{\partial \omega}\bigg|_{\omega=\omega_0} \qquad L_{eq} = \frac{1}{\omega_0^2 C_{eq}}$$

These equations define an equivalent circuit for a large class of **cavity resonators.**
The type of resonator depends on the **resonant mode** and on the **reference section**

Example: TEM cavity realized with short-circuited/ open-circuited transmission line



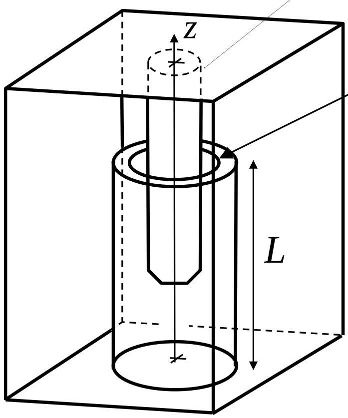
$$C_{eq} = \frac{\pi}{4}\frac{Y_C}{\omega_0}$$

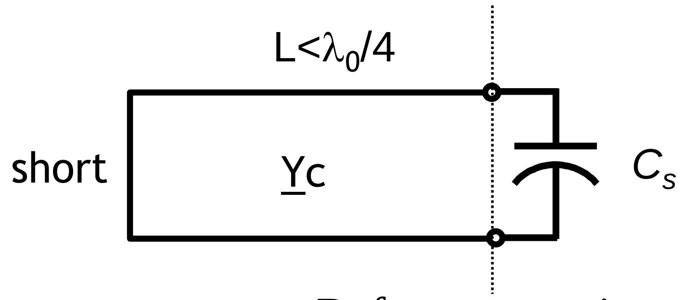
$$L_{eq} = \frac{\pi}{2}\frac{Z_C}{\omega_0}$$

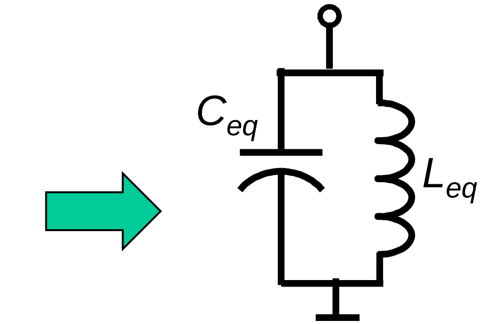
$$L_{eq} = \frac{\pi}{4}\frac{Z_C}{\omega_0}$$

$$C_{eq} = \frac{\pi}{2}\frac{Y_C}{\omega_0}$$

Capacity-loaded coaxial resonator







$$B_{tot} = \omega C_s - Y_C \cot(\beta L)$$

Resonance condition:

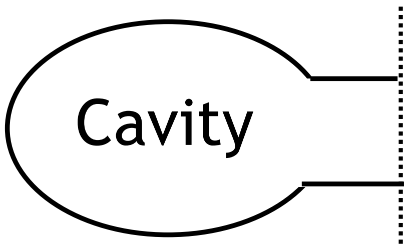
$$B_{tot}(\omega_0) = 0 \quad \Rightarrow \quad \omega_0 C_s = Y_C \cot(\beta_0 L)$$

Equivalent capacitance:

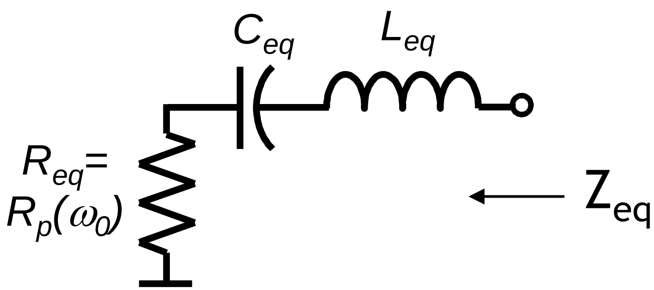
$$\omega_0 C_{eq} = \frac{1}{2} \frac{\partial B_{tot}(\omega)}{\partial \omega}\bigg|_{\omega=\omega_0} = \frac{1}{2} Y_C \left[\cot(\beta_0 L) + \frac{\beta_0 L}{\sin^2(\beta_0 L)} \right] =$$

$$= \frac{1}{2} Y_C \cot(\beta_0 L) \left[1 + \frac{2\beta_0 L}{\sin(2\beta_0 L)} \right] \quad \overset{L=\lambda_0/8}{\Rightarrow} \quad \omega_0 C_{eq} = \frac{Y_C}{2}\left[1 + \frac{\pi}{2}\right]$$

Losses in the cavity: the unloaded Q



$$Q_0 = \omega_0 \frac{\text{Energy stored in the junction}}{\text{Power dissipated in the junction}}$$

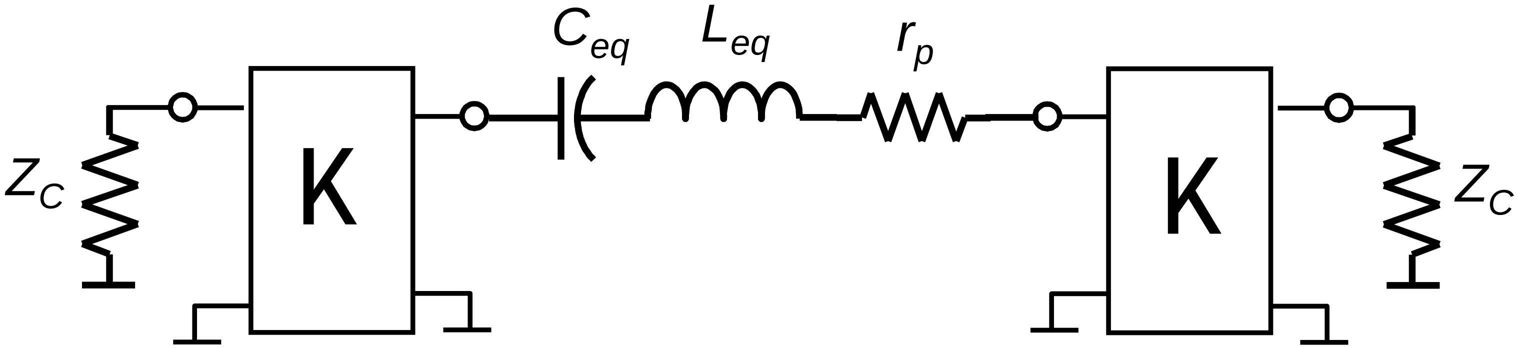


$$Q_0 = \frac{\omega_0 L_{eq}}{R_p} \qquad R_p = R_J + R_\varepsilon$$

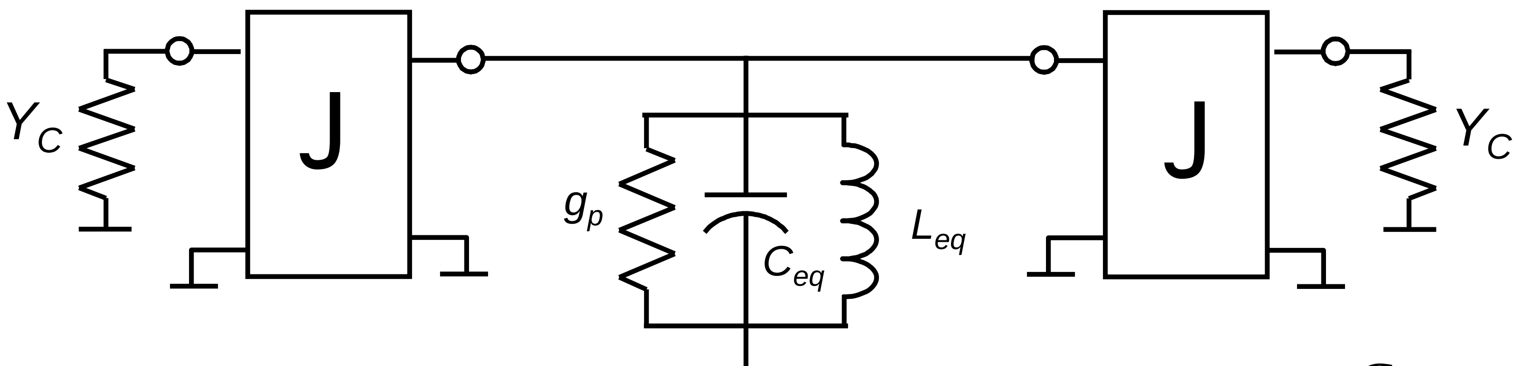
R_J is the sum of two terms: Rp, due to thefinite conductivity of metallic walls and Re to to medium dissipation (dielectric losses). Is has then:

$$\frac{1}{Q_0} = \frac{1}{Q_{0J}} + \frac{1}{Q_{0\varepsilon}}$$

Modeling of a cavity coupled to loads



$$Q_L = \frac{\omega_0 L_{eq}}{2K^2/Z_C} = \frac{f_0}{B_{3dB}} \qquad \left(\text{for } r_p << K^2/Z_C\right) \qquad Q_0 = \frac{\omega_0 L_{eq}}{r_p}$$



$$Q_L = \frac{\omega_0 C_{eq}}{J^2/Y_C} = \frac{f_0}{B_{3dB}} \qquad \left(\text{for } g_p << J^2/Y_C\right) \qquad Q_0 = \frac{\omega_0 C_{eq}}{g_p}$$

Parameters of loaded cavities

The loaded Q (Q_L): determines

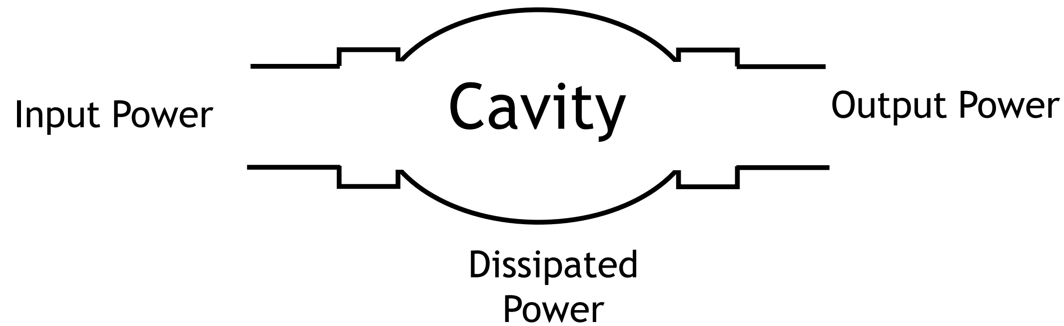
- The 3 dB bandwidth ($B_{3dB}=f_0/Q_L$). For a given cavity (L_{eq} or C_{eq}), a value for K (J) can be evaluated for obtaining the desired B_{3dB}:

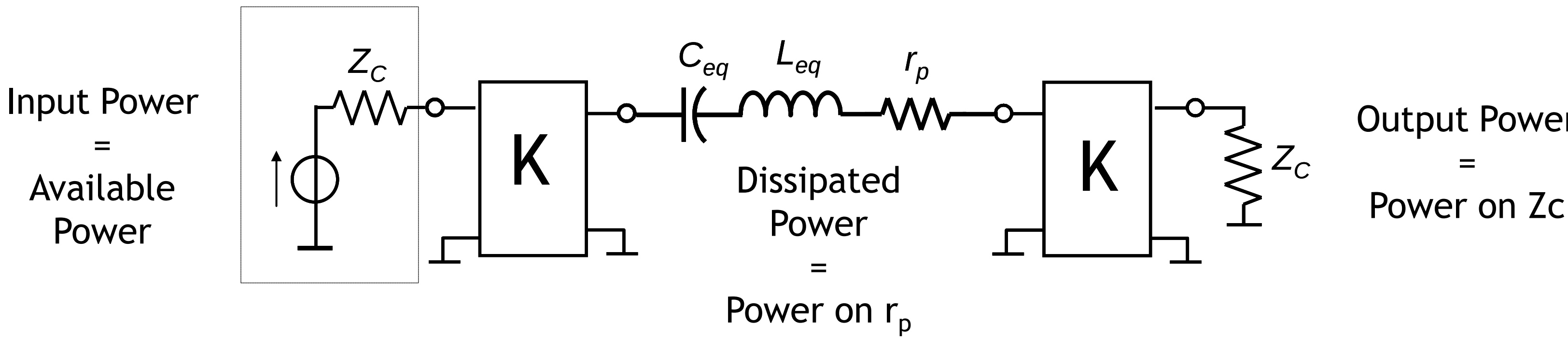
$$K=\sqrt{\frac{Z_C\cdot\omega_0 L_{eq}}{2Q_L}},\qquad J=\sqrt{\frac{Y_C\cdot\omega_0 C_{eq}}{2Q_L}}$$

- The matching band at the input (output) port. For small losses, it can be shown that the bandwidth B_Γ for a given value Γ of the input reflection coefficient is related to B_{3dB} as follows:

$$B_\Gamma \simeq \left|\Gamma\right|\cdot B_{3dB}$$

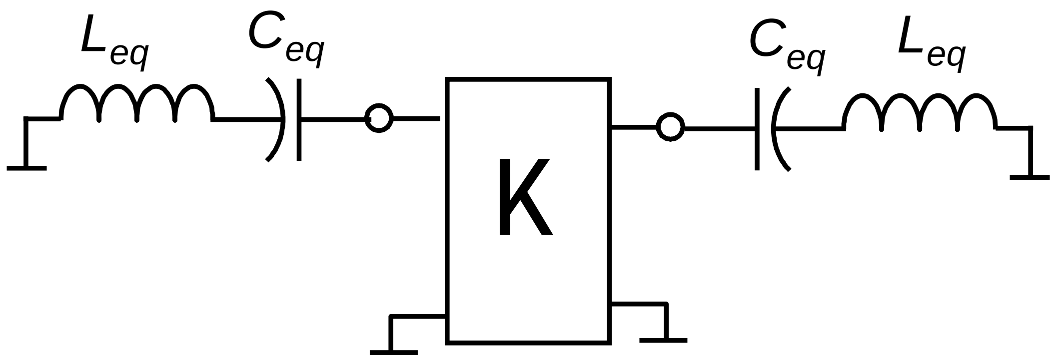
Power transmitted to load (transducer attenuation)





$$A_{dB} = -10\log\left(1-\frac{2Q_L}{Q_0}\right) \cong 20\log\left(1+\frac{Q_L}{Q_0}\right) \qquad \left(Q_0 >> Q_L\right)$$

Coupling of two cavities: equivalent circuit



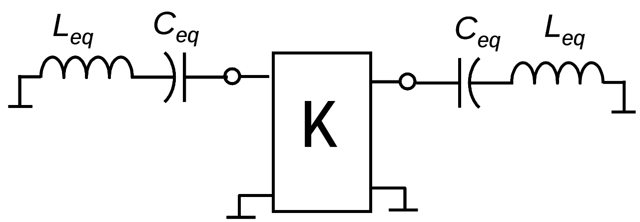
Characteristic parameter: the **coupling coefficient**

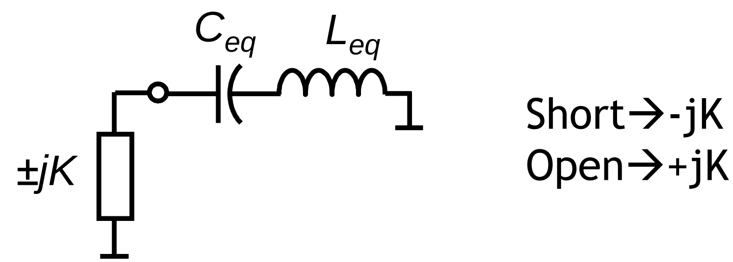
$$k = \frac{K}{\omega_0 L_{eq}} \qquad \left(= \frac{J}{\omega_0 C_{eq}} \text{ for shunt resonators} \right)$$

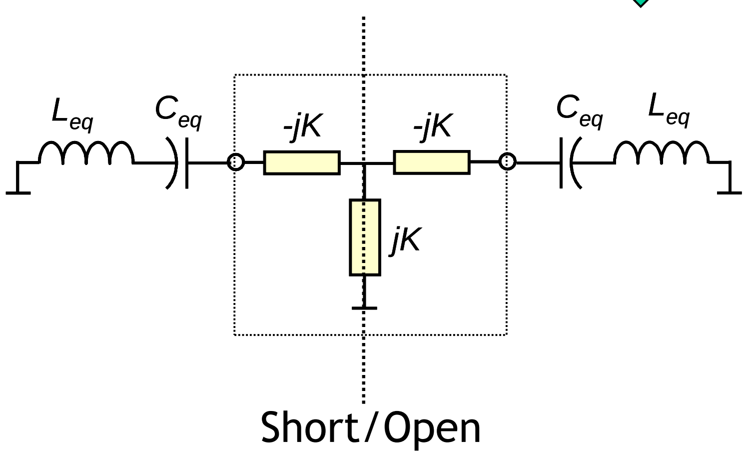
k determines the coupling bandwidth of the two resonators (the larger is *k* the larger is the bandwidth)

Evaluation of k from resonances

- Even and odd resonances of two coupled resonators:







$$f_o = f_0 \left[\sqrt{\left[\left(\frac{K}{2\omega_0 L_{eq}} \right)^2 + 1 \right]} - \frac{K}{2\omega_0 L_{eq}} \right]$$

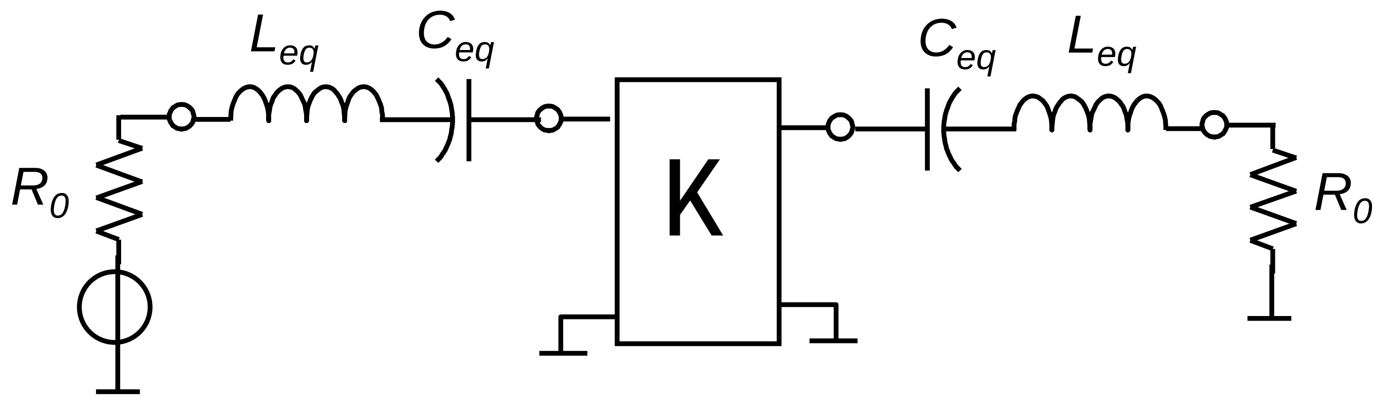
Odd Resonance (-jK)

$$f_e = f_0 \left[\sqrt{\left[\left(\frac{K}{2\omega_0 L_{eq}} \right)^2 + 1 \right]} + \frac{K}{2\omega_0 L_{eq}} \right]$$

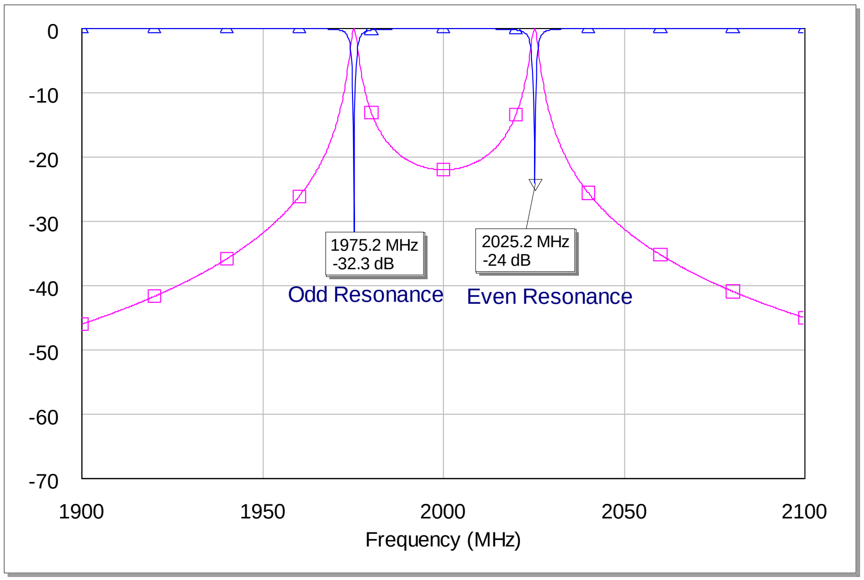
Even Resonance (+jK)

$$f_0 = \sqrt{f_e \cdot f_o}, \qquad k = \frac{K}{\omega_0 L_{eq}} = \frac{f_e - f_o}{f_0}$$

Practical evaluation of *k*



$$\omega_0 L_{eq} >> R_0$$ ➡ Input/output coupling ≈0

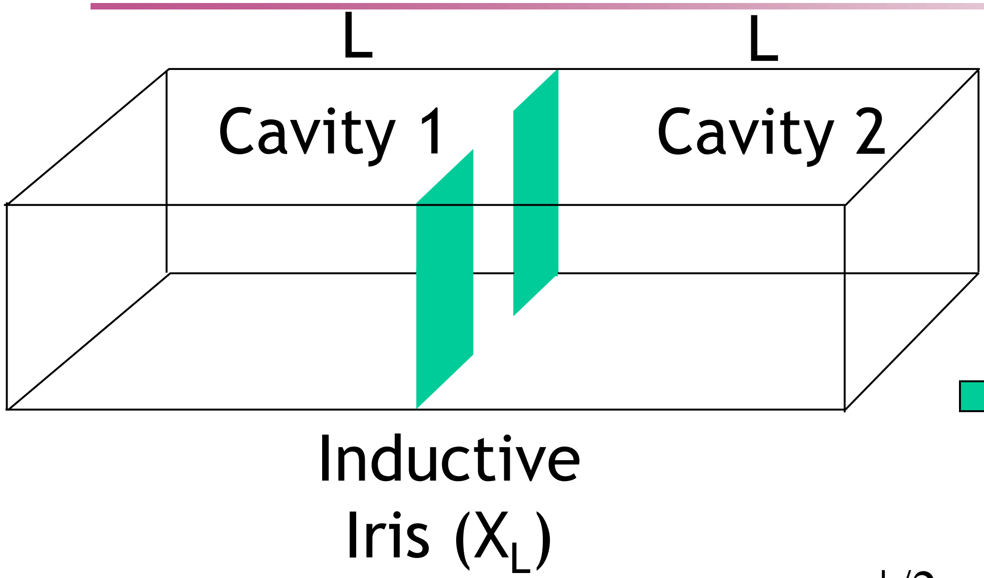


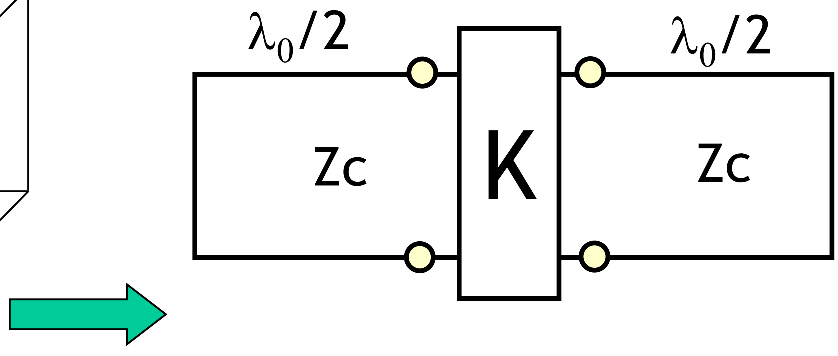
X_{eq}=10, R0=0.01
f_e=2025.2 MHz
f_o=1975.2

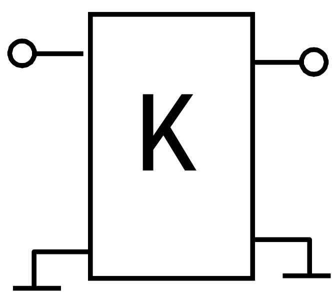
$$f_0 = \sqrt{f_e \cdot f_o} = 2000MHz$$

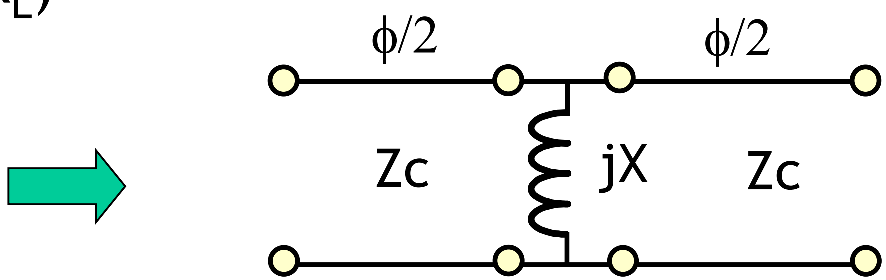
$$k = \frac{f_e - f_o}{f_0} = 0.025$$

Coupled cavities: example 1



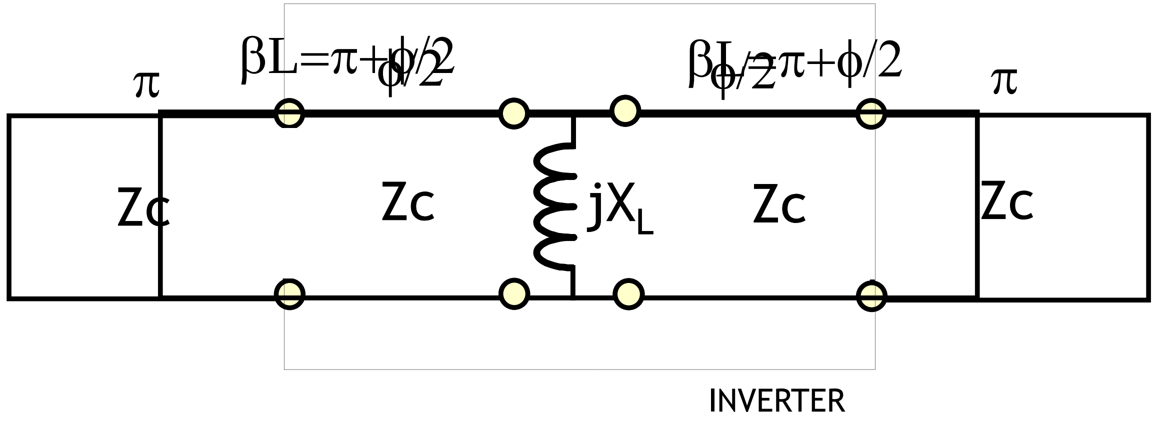






$$X = \frac{K}{1-\left(\frac{K}{Z_C}\right)^2}, \quad \phi = -\tan^{-1}(2X)$$

example 1 (cont.)

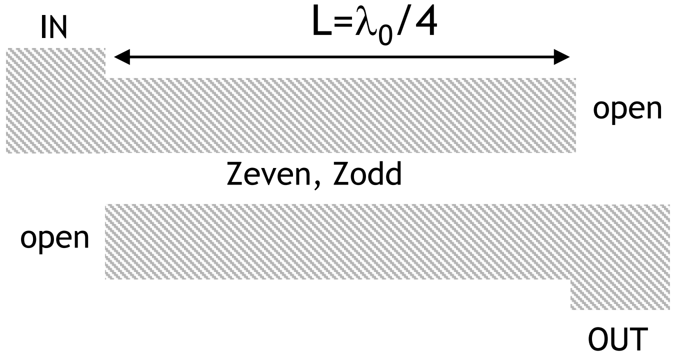


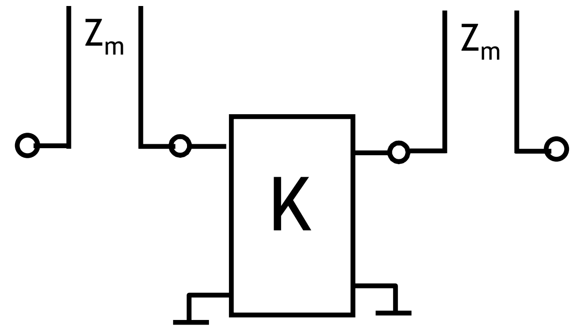
$$\beta L = \pi + \frac{\phi}{2} = \pi - \tan^{-1}\left(2X_L\right) \qquad X_L = \frac{K}{1-\left(\frac{K}{Z_C}\right)^2} \approx K \quad (\text{for } K << Z_C)$$

Equiv. reactance: $\omega_0 L_{eq} = \frac{\pi}{2} Z_C \left(\frac{\lambda_g}{\lambda_0}\right)^2$

Coupling coefficient: $k = \frac{K}{\omega_0 L_{eq}} = \frac{2}{\pi}\frac{X_L}{Z_C}\left(\frac{\lambda_g}{\lambda_0}\right)^2$

Resonators with coupled lines: Interdigital





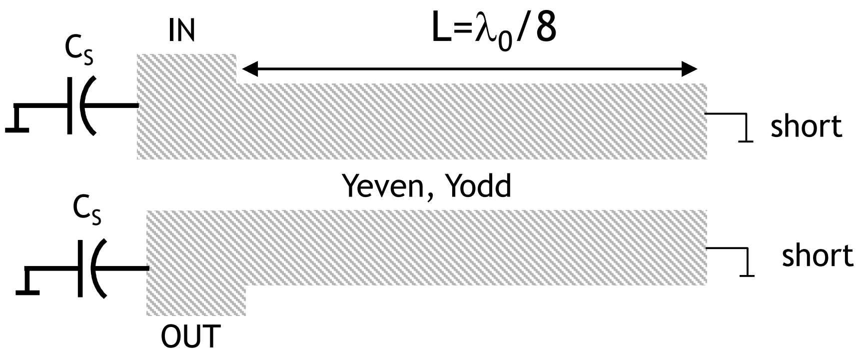
$$X_S = -\frac{1}{2}\left(Z_{even} + Z_{odd}\right)\frac{1}{\tan(\beta L)} = \frac{-Z_m}{\tan(\beta L)}$$

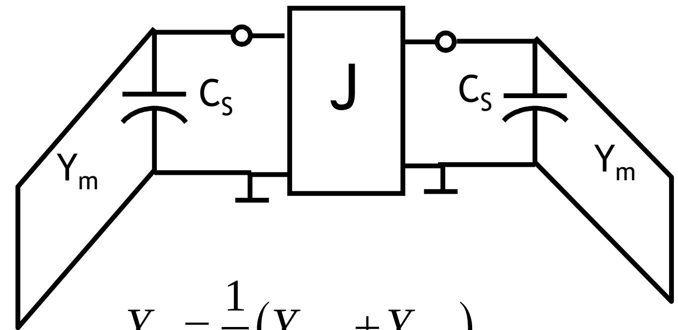
$$K = \frac{1}{2}\left(Z_{even} - Z_{odd}\right)\frac{1}{\sin(\beta L)}$$

$$\omega_0 L_{eq} = \frac{1}{2}\omega_0 \left.\frac{\partial X_S}{\partial \omega}\right|_{\omega=\omega_0} = \frac{\pi}{4} Z_m$$

$$k = \frac{K}{\omega_0 L_{eq}} = \frac{4}{\pi}\left(\frac{Z_{even} - Z_{odd}}{Z_{even} + Z_{odd}}\right) = \frac{4}{\pi} m$$

Resonators with coupled lines: Comb





$$Y_m = \frac{1}{2}\left(Y_{even} + Y_{odd}\right)$$

$$J = \frac{1}{2}\left(Y_{even} - Y_{odd}\right)\frac{1}{\tan\left(\beta L\right)}$$

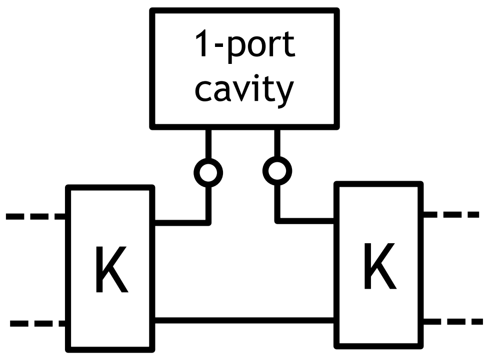
$$\omega_0 C_{eq} = \frac{Y_m}{2}\left[1 + \frac{\pi}{2}\right],$$

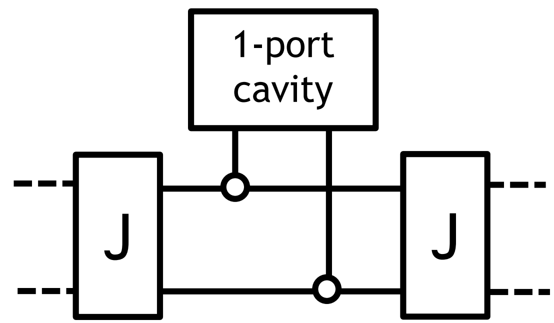
$$k = \frac{J}{\omega_0 C_{eq}} = \frac{2}{\pi + 2}\left(\frac{Y_{odd} - Y_{even}}{Y_{even} + Y_{odd}}\right) = \frac{2m}{\pi + 2}$$

Note: $$\frac{Y_{odd} - Y_{even}}{Y_{even} + Y_{odd}} = \frac{Z_{even} - Z_{odd}}{Z_{even} + Z_{odd}} = m$$

1-port and 2-port Cavities

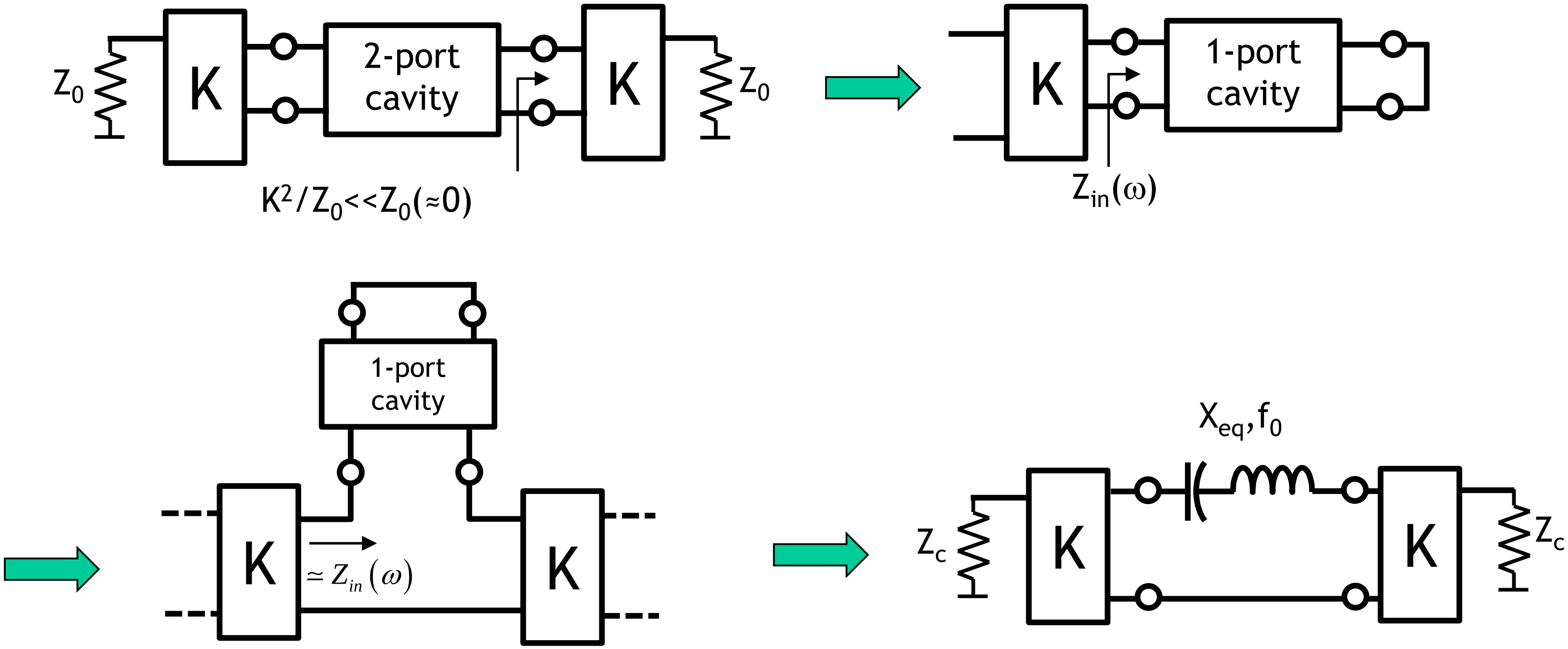
Previous results refer to 1-port cavities that can be represented with a series or shunt resonator:





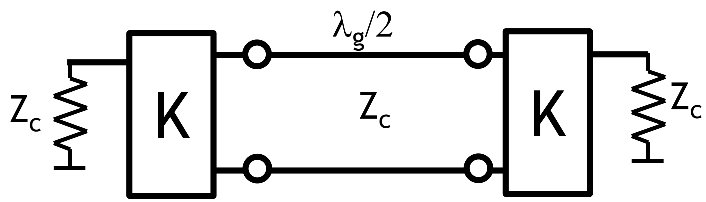
In case of 2-port cavities the equivalence should be imposed on all the elements of S or Z or Y matrix.
It is however possible an approximation allowing to still refer to 1-port resonator

1-port equivalent circuit of 2-port cavities

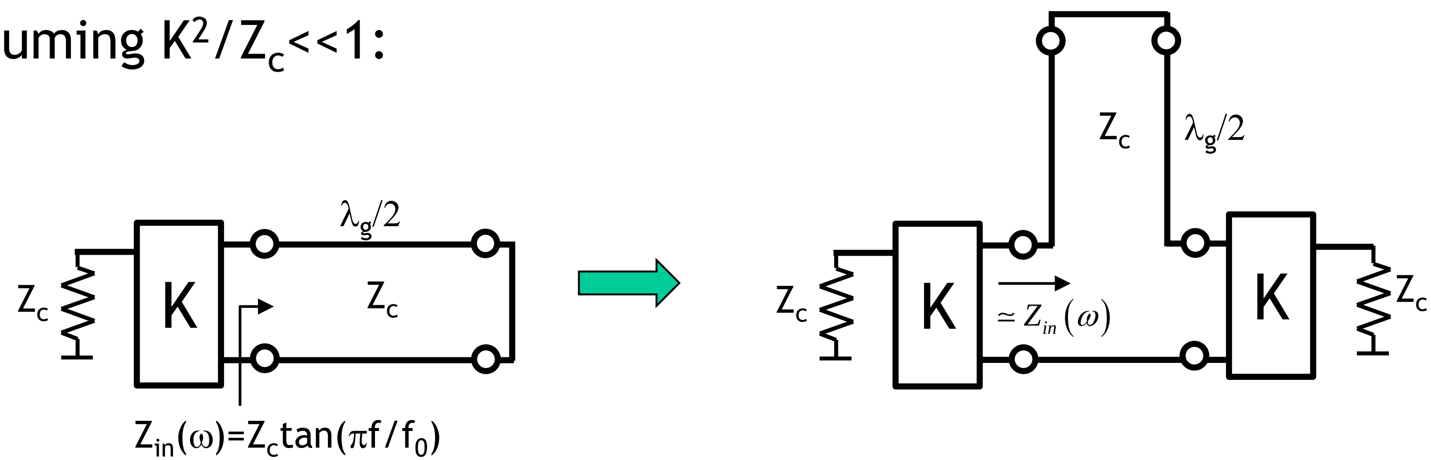


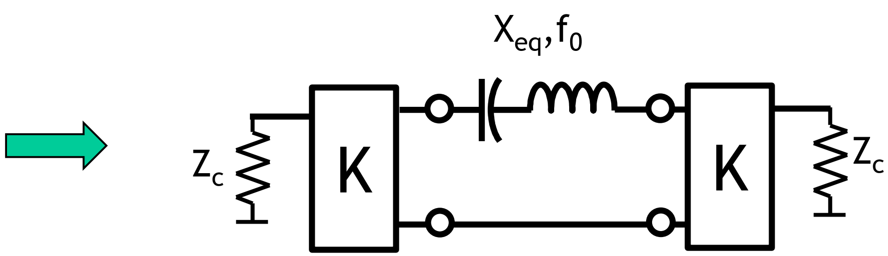
In general, this is acceptable for narrowband filters ($B/f_0 << 1$), but represents a further approximation that is added to the approximation of the cavity with a lumped resonator

Example: Transmission line cavity



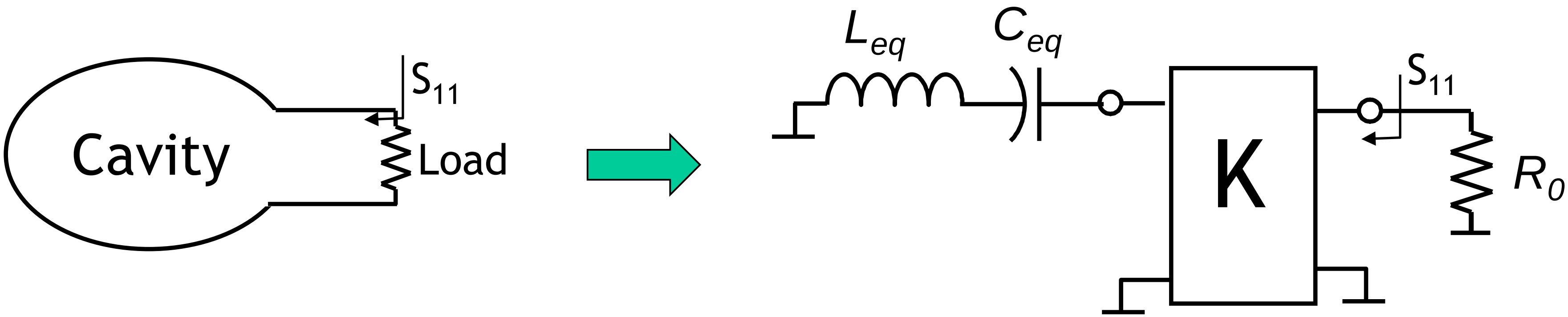
Assuming $K^2/Z_c \ll 1$:





$$X_{eq} = \omega_0 L_{eq} = \frac{\pi}{2} Z_C$$

Coupling a cavity to external load



Universal parameters: External Q (Q_E)

$$Q_E = \frac{\omega_0 L_{eq}}{K^2 / R_0}$$

Q_E can be derived from the phase of S_{11}

Evaluation of Q_E from S_{11}

$$Q_E = \frac{f_0}{f_{+1} - f_{-1}}$$

f_{+1} and f_{-1} represent the frequencies around f_0 where the phase difference of S_{11} is equal to π.