
Design of various types of all-pole microwave filters

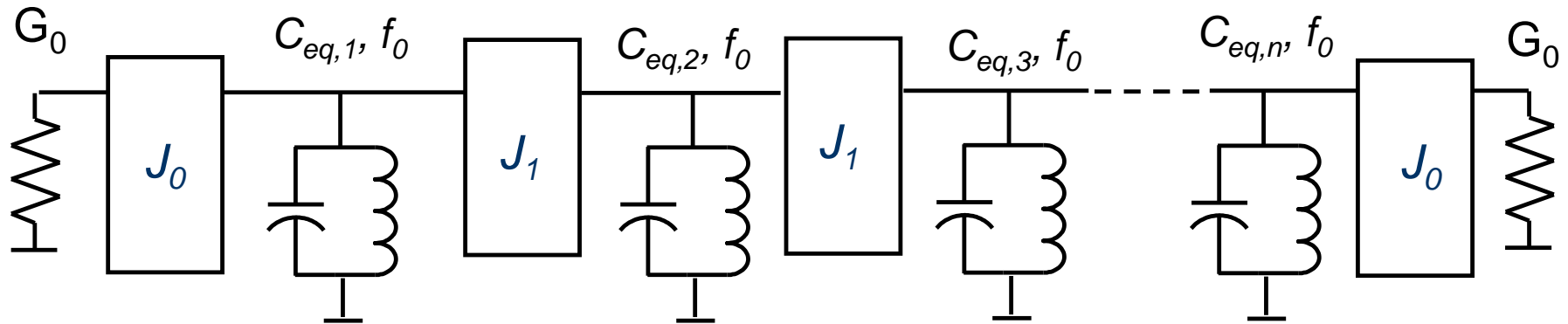
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In-line filters with assigned inverters



$$J_0 = \sqrt{G_0 \cdot J_1} \quad k_{q,q+1} = B_n \sqrt{\frac{1}{g_q g_{q+1}}}, \quad Q_E = \frac{g_1}{B_n}$$

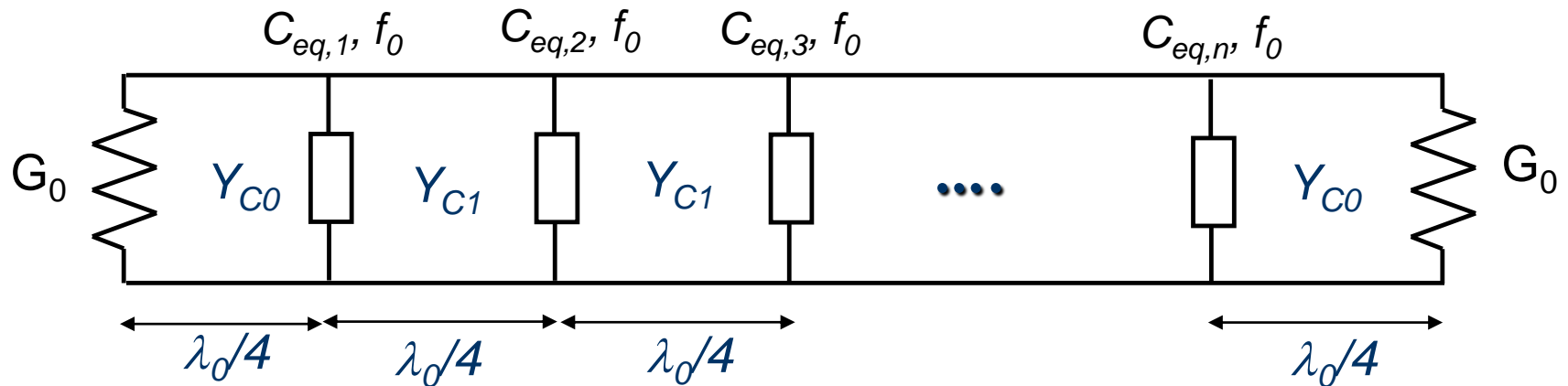
From the definition of the universal parameters:

$$Q_E = \frac{\omega_0 C_{eq,1}}{J_0^2 / G_0} = \frac{g_1}{B_n} \Rightarrow \omega_0 C_{eq,1} = J_1 \frac{g_1}{B_n}, \quad k_{12} = \frac{J_1}{\sqrt{\omega_0 C_{eq1} \cdot \omega_0 C_{eq2}}} = B_n \sqrt{\frac{1}{g_1 g_2}} \Rightarrow$$

$$\omega_0 C_{eq2} = J_1 \frac{g_2}{B_n}, \quad \dots \dots \dots, \quad \omega_0 C_{eq,q} = J_1 \frac{g_q}{B_n}$$

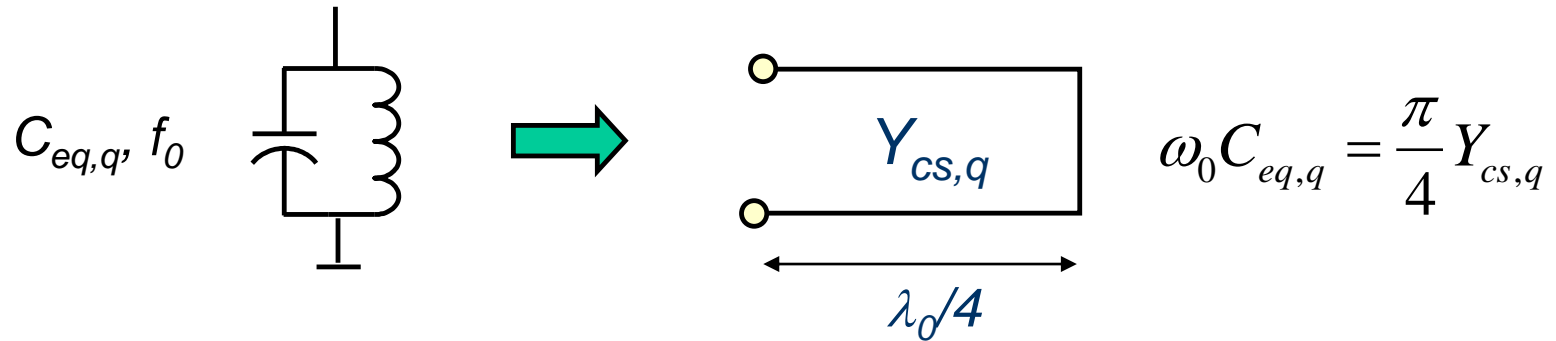
$$\omega_0 C_{eq,N-q+1} = \omega_0 C_{eq,N-q}$$

Inverters with $\lambda_0/4$ line sections



- 1) Assign $Y_{c1} = J_1$
- 2) Compute $Y_{c0} = \sqrt{G_0 \cdot Y_{c1}}$
- 3) Compute $\omega_0 C_{eq,q} = Y_{c1} \frac{g_q}{(B/f_0)}$

Resonators with shorted transmission lines



$$Y_{cs,q} = Y_{c1} \frac{4g_q}{\pi(B/f_0)}$$

NOTE: The choice of Y_{c1} affects the computed values of $Y_{cs,q}$. These latter must be physically realizable with the fabrication technology of the filter

Example: filter in microstrip technology

Let assume the following specifications:

$$f_0=1 \text{ GHz}, B/f_0=0.1, RL=15 \text{ dB}, n=5$$

Assuming the Chebycheff characteristic:

$$g=\{1.2327 \quad 1.3592 \quad 2.0599 \quad 1.3592 \quad 1.2327\}$$

Now suppose that the range of realizable characteristic impedance is 20-150 Ohm.

Then we assume $Y_{c1}=1/150=0.0067$. Using the previous formula:

$$Y_{cs,q}=\{0.1046 \quad 0.1154 \quad 0.1748 \quad 0.1154 \quad 0.1046\} \Rightarrow$$

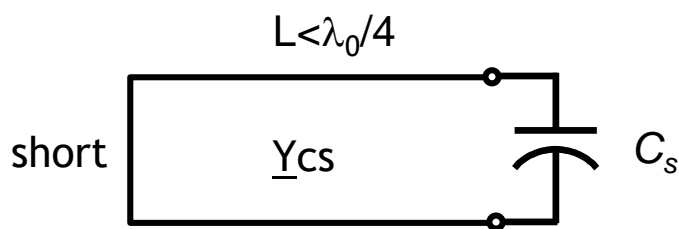
$$\Rightarrow 1/Y_{cs,q}=\{9.5570 \quad 8.6676 \quad 5.7192 \quad 8.6676 \quad 9.5570\}$$

The computed impedances are not realizable with microstrip technology!

Possible solution to allow the feasibility

- Increase the imposed return loss (but this reduces stopband attenuation)
- Increase the bandwidth (increase the filter order for the same stopband attenuation)
- Use two stubs in parallel with half $Y_{cs,q}$
- Choice a different implementation of resonators (for obtaining smaller $Y_{cs,q}$ for the same equivalent capacitance). For instance, using open-circuited, $\lambda_0/2$ stubs the characteristic impedances are doubled.

Another choice is the use of capacity-loaded resonators:



$$\omega_0 C_{eq} = \frac{Y_{cs}}{4} [2 + \pi] \Rightarrow$$

$$Y_{cs,q} = Y_{c1} \frac{4g_q}{\pi(B/f_0)} \left(\frac{\pi}{2 + \pi} \right)$$

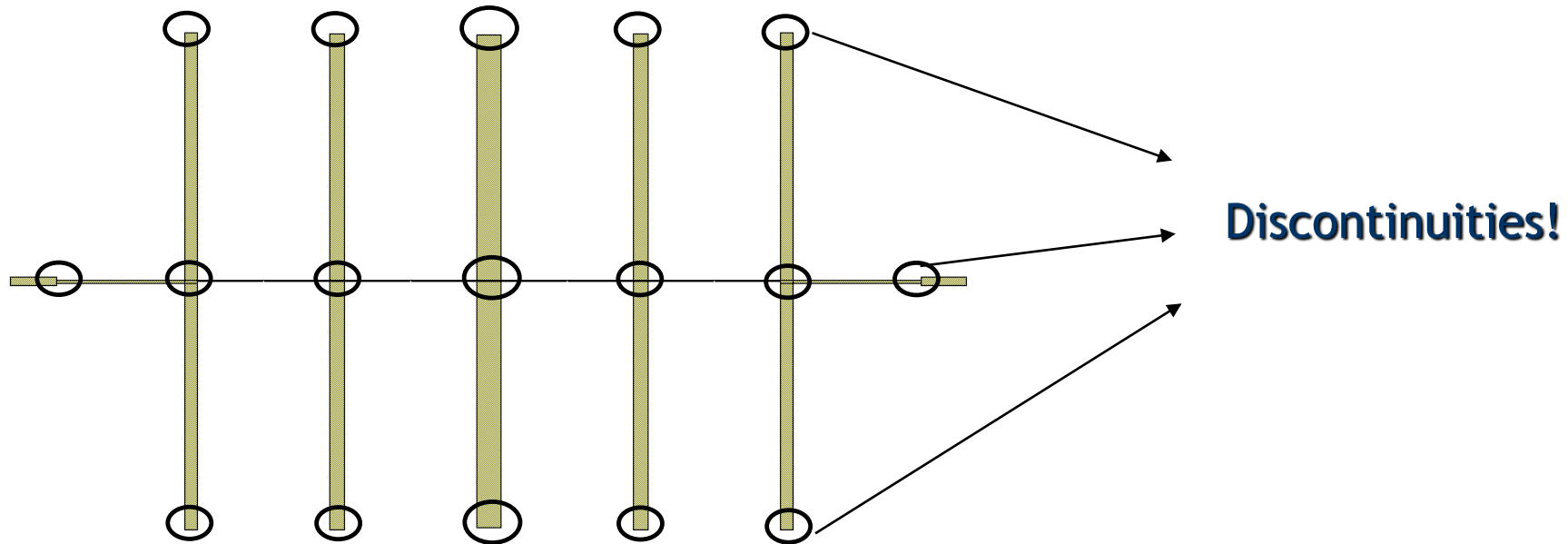
Implementation with double $\lambda_0/2$ resonators

Substrate: Duroid ($\epsilon_r=2.16$, $H=1.2$ mm, $t=50\mu$)

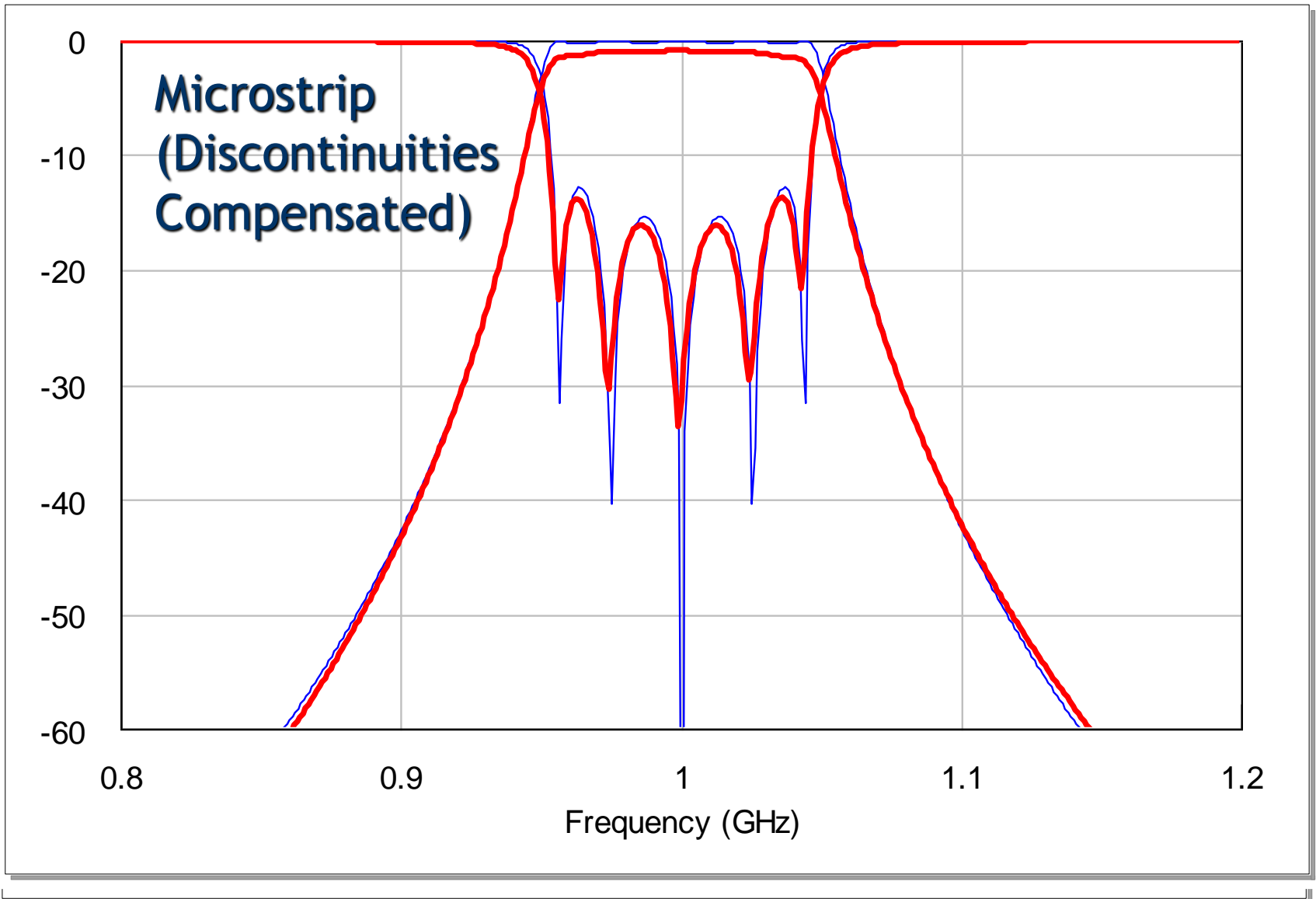
$1/Y_{cs,q}=\{38.23, 34.67, 22.88, 34.67, 38.23\}$

$Y_{c0} = \sqrt{G_0 \cdot Y_{c1}} = 0.0115$ ($1/Y_{c0} = 86.6$) $Y_{c1} = 1/150 = 0.0067$

Layout:



Computed response



Attenuation produced by losses

An estimate of the overall attenuation at center frequency f_0 can be obtained with the following formula:

$$A_{dB} = \sum_{q=1,n} 20 \log \left(1 + \frac{Q_q}{Q_0} \right)$$

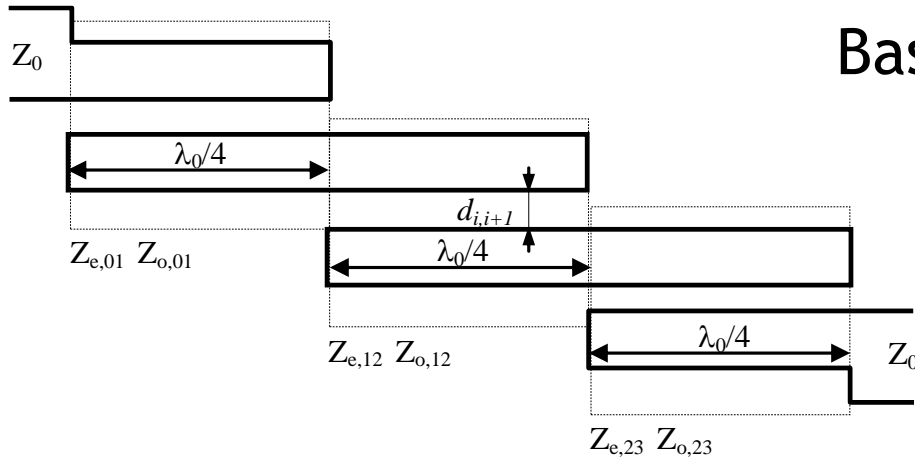
Where Q_q is expressed as function of the prototype parameters g_q :

$$Q_q = \frac{1}{2} \frac{g_q}{B/f_0}$$

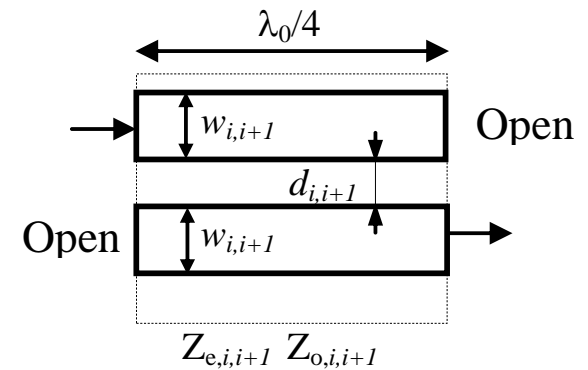
Note: Narrow band filters require high Q_0 resonators to reduce passband attenuation

Coupled-line filters

Physical structure:



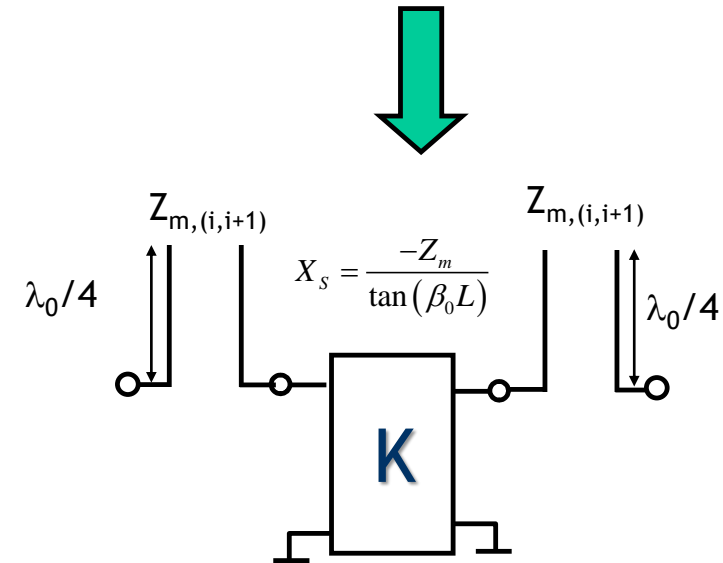
Basic Block:



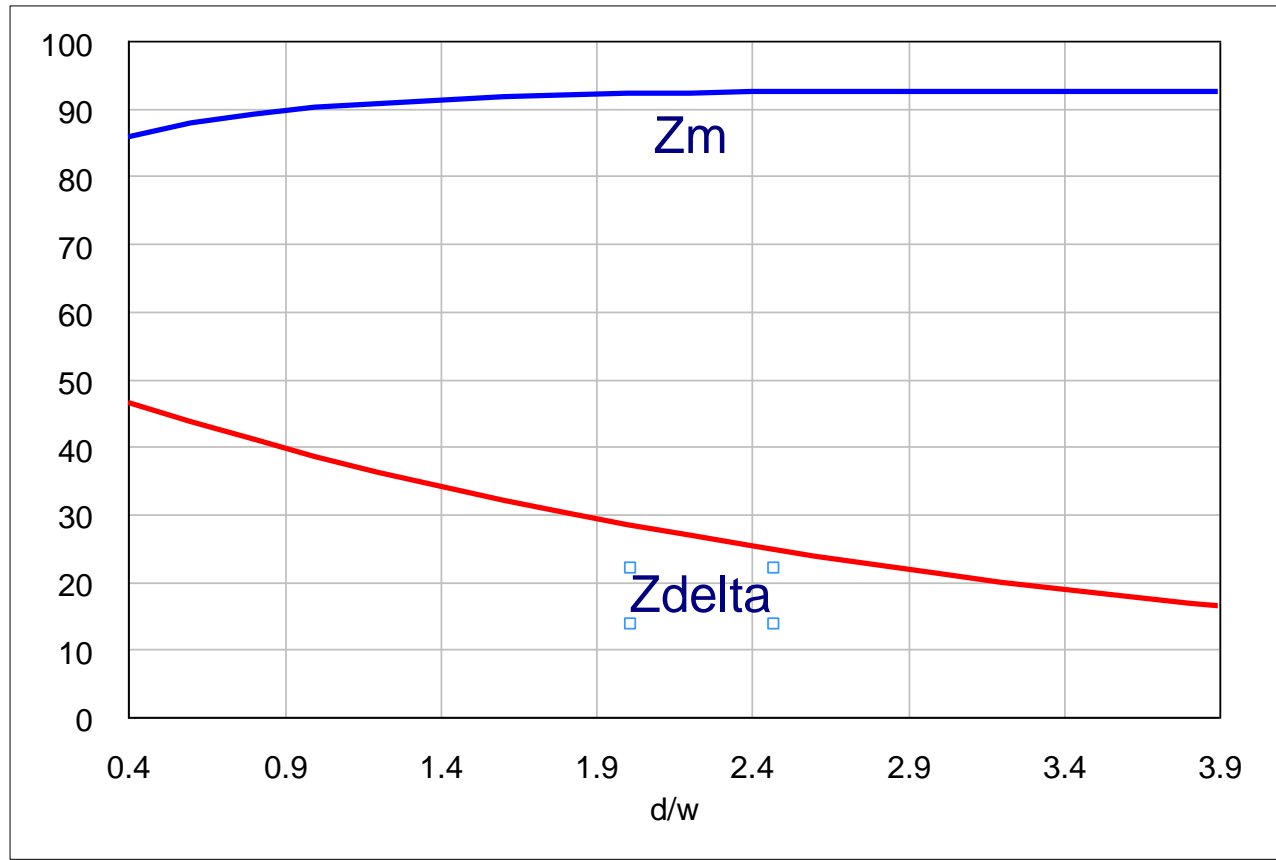
$$Z_{m,(i,i+1)} = \frac{1}{2} (Z_{e,(i,i+1)} + Z_{o,(i,i+1)}), \quad m_{i,i+1} = \frac{Z_{e,(i,i+1)} - Z_{o,(i,i+1)}}{Z_{e,(i,i+1)} + Z_{o,(i,i+1)}}$$

$$K_{(i,i+1)} = \frac{Z_{\Delta,(i,i+1)}}{\sin(\beta_0 L)} = \frac{1}{\sin(\beta_0 L)} \left(\frac{Z_{e,(i,i+1)} - Z_{o,(i,i+1)}}{2} \right) =$$

$$K_{i,i+1} \Big|_{\beta_0 L = \frac{\pi}{2}} = m_{i,i+1} Z_{m,(i,i+1)}$$

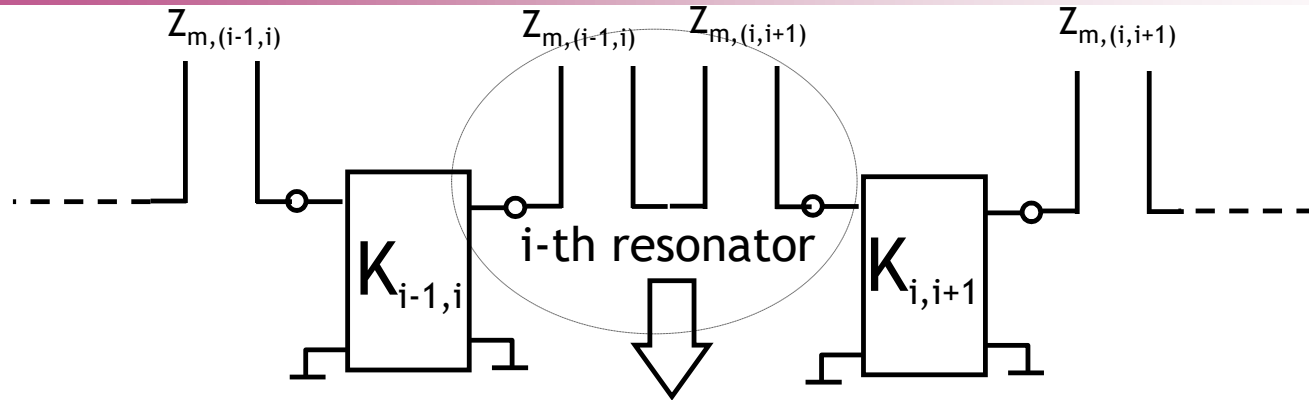


Typical dependance of Z_m and Z_Δ on d/w

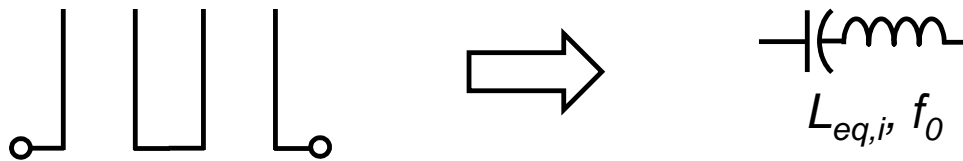


For $d/w > 1$ Z_m is practically independent on d for assigned w and practically coincides with the characteristic impedance of the isolated line with the same w .

Equivalent circuit (inner blocks)



$$X_{S,i} = -\frac{Z_{m,(i-1,i)} + Z_{m,(i,i+1)}}{\tan(\beta_0 L)} \Rightarrow \omega_0 L_{eq,i} = \frac{\pi}{4} (Z_{m,(i-1,i)} + Z_{m,(i,i+1)}) \approx \frac{\pi}{2} Z_c$$

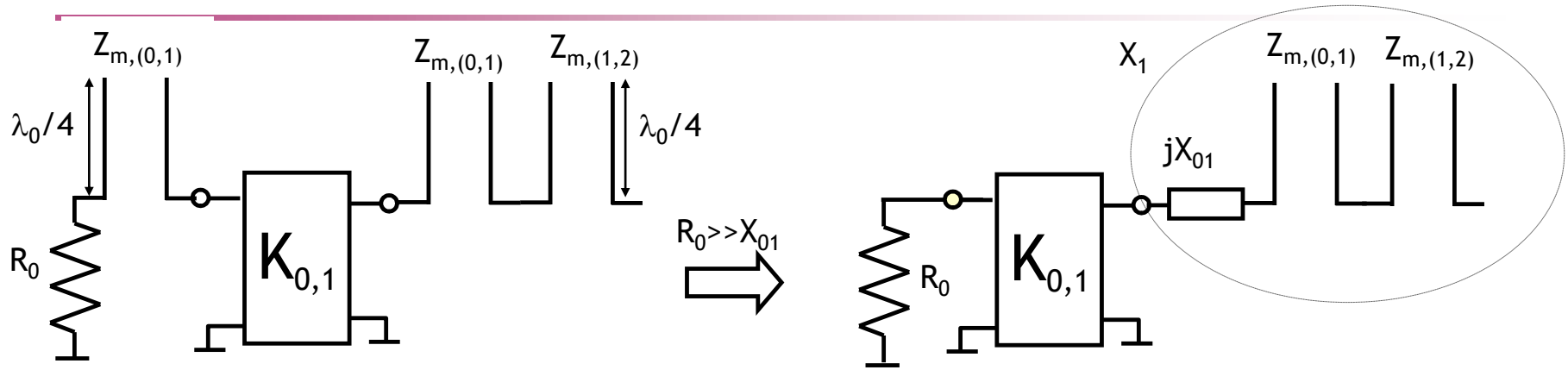


Design Equations:

$$k_{q,q+1} = \frac{K_{q,q+1}}{\sqrt{(\omega_0 L_{eq,q}) \cdot (\omega_0 L_{eq,q+1})}} = \frac{2 Z_{\Delta,(i,i+1)}}{\pi Z_{m,(i,i+1)}} = \frac{2}{\pi} m_{i,i+1} \Rightarrow$$

$$m_{i,i+1} = \frac{\pi}{2} k_{i,i+1}$$

First (last) block



$$Q_E = \frac{\omega_0 L_{eq,1}}{K_{0,1}^2 / R_0} = \frac{\omega_0 L_{eq,1}}{m_{0,1}^2 \cdot Z_{m,(0,1)}^2} R_0$$

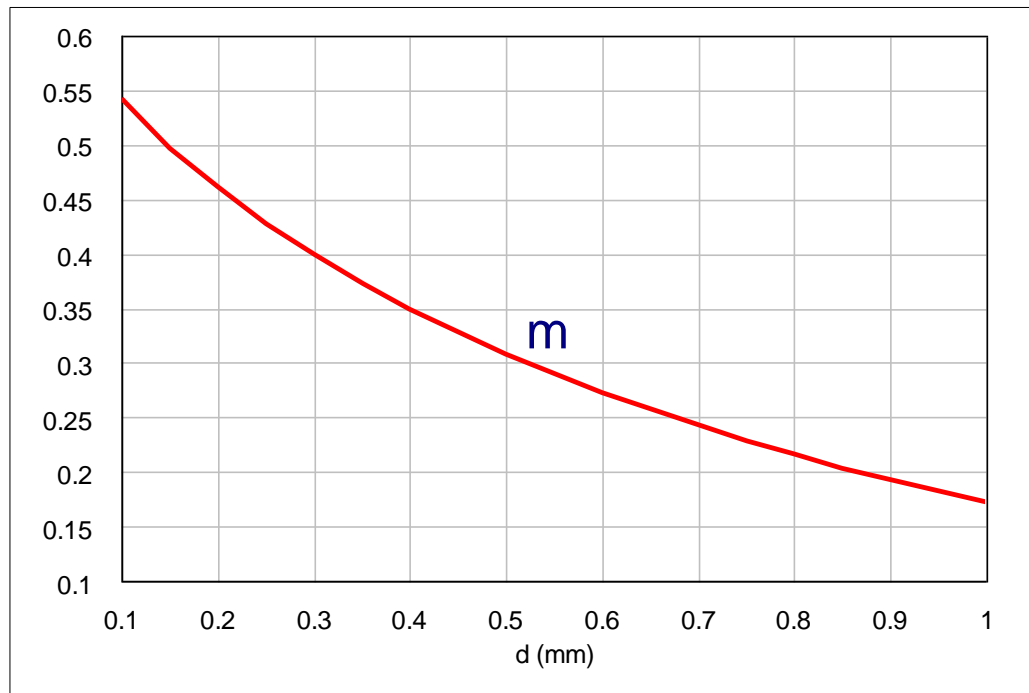
$$X_{01} = \left(\frac{K_{01}}{R_0} \right)^2 \left(Z_{m,(0,1)} \cot \left(\frac{\pi f}{2 f_0} \right) \right)$$

$$X_1 = X_{01} - \cot \left(\frac{\pi f}{2 f_0} \right) (Z_{m,(0,1)} + Z_{m,(1,2)}) \approx - \left[2 - \left(\frac{K_{01}}{R_0} \right)^2 \right] Z_c \cot \left(\frac{\pi f}{2 f_0} \right)$$

$$\omega_0 L_{eq,1} \approx \frac{(\pi/2) Z_c}{1 + \frac{\pi}{4} \frac{Z_c}{R_0 \cdot Q_E}}, \quad m_{0,1} \approx \frac{\sqrt{(\omega_0 L_{eq,1} \cdot R_0) / Q_E}}{Z_c}, \quad m_{1,2} \approx \frac{(\pi/2) k_{1,2}}{\sqrt{1 + \frac{\pi}{4} \frac{Z_c}{R_0 \cdot Q_E}}}$$

Dimensioning of the filter

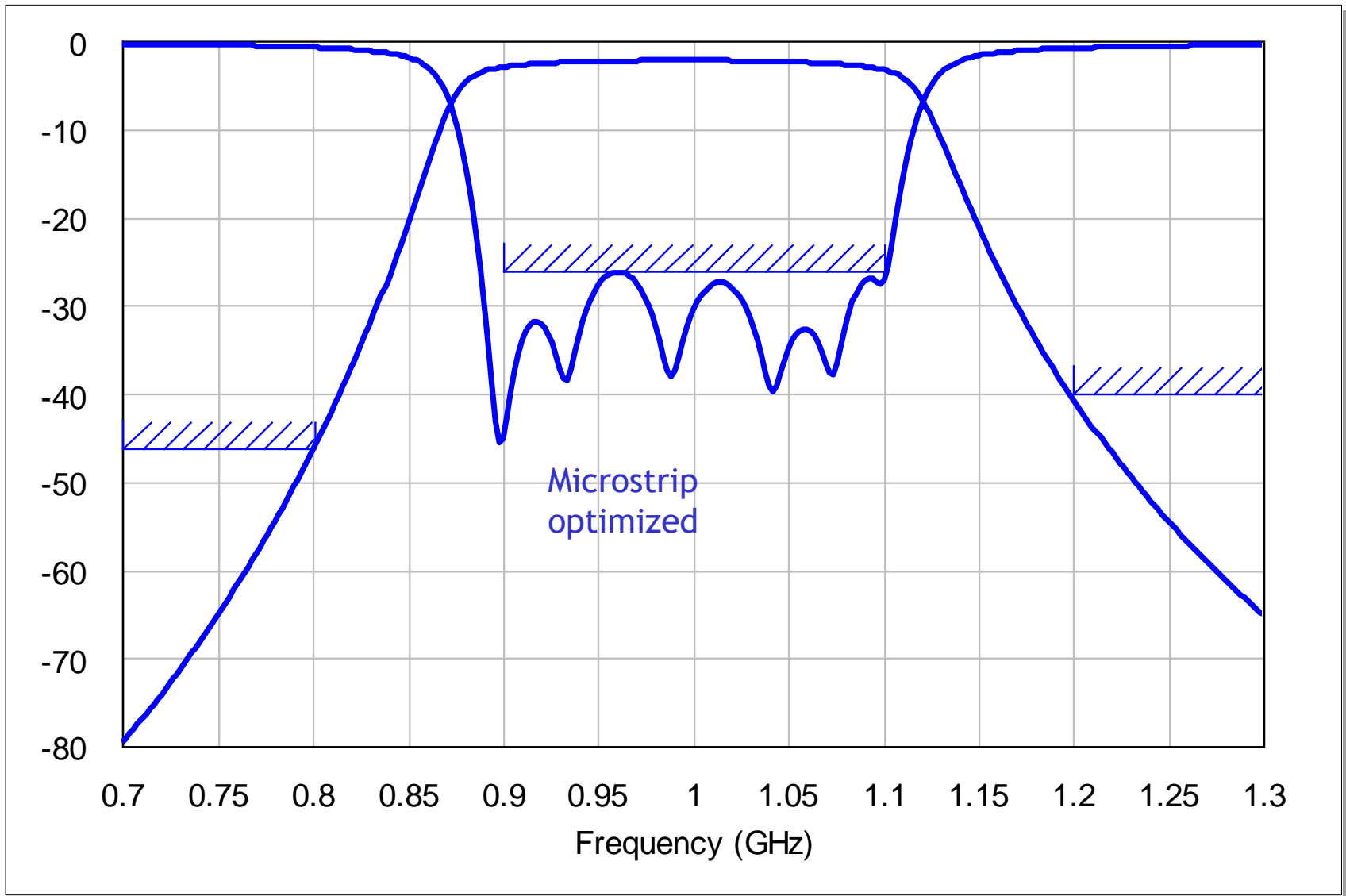
1. The dimension w of the lines is first assigned
2. The values of $m_{i,i+1}$ are computed with the previous formulas
3. Using a graph of m vs. d (like the one shown below) the distances $d_{i,i+1}$ are evaluated



Example of microstrip filter design

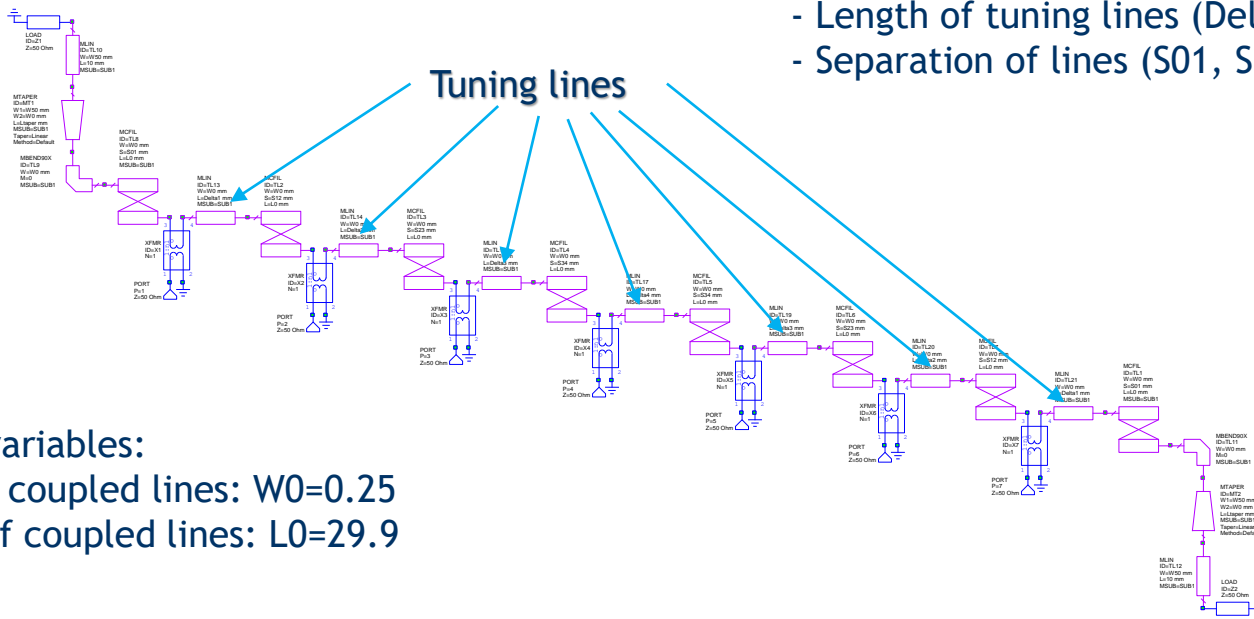
- Specifications:
 $f_0=1$ GHz, $B=200$ MHz, $RL=26$ dB, $n=7$
 $g=\{0.807, 1.397, 1.758, 1.634, 1.758, 1.397, 0.807\}$
- Assigned substrate parameters:
 $\epsilon_r=10$, $H=1.2$ mm, $t=10\mu$
- Assigned width of lines (for $Z_c\approx 85$ Ohm): $w=0.25$ mm
- Computed $m_{i,j+1}$:
 $m=\{0.415, 0.258, 0.201, 0.186, 0.186, 0.201, 0.258, 0.415\}$
- Computed distances $d_{i,j+1}$ (with the previous graph):
 $d=\{0.272, 0.649, 0.861, 0.93, 0.93, 0.861, 0.649, 0.272\}$
- Evaluation of length of blocks (about $\lambda_0/4$ for not coupled lines): $L=30.44$ mm

Computed filter response (circuit model)



Design of the filter with MWOOffice

N-port model:



Note: each resonator is a half-wavelength open-circuited resonator. At center a series resonance is observed, then the required port must be connected in series (through e 1:1 transformer)

Output Equations for the dimensioning

Z11 = Filtro Coupled Lines N porte:Z(1,1)
 Z22 = Filtro Coupled Lines N porte:Z(2,2)
 Z33 = Filtro Coupled Lines N porte:Z(3,3)
 Z44 = Filtro Coupled Lines N porte:Z(4,4)

Z12 = Filtro Coupled Lines N porte:Z(1,2)
 Z23 = Filtro Coupled Lines N porte:Z(2,3)
 Z34 = Filtro Coupled Lines N porte:Z(3,4)
 Z45 = Filtro Coupled Lines N porte:Z(4,5)

X11=imag(Z11) K12=imag(Z12)
 X22=imag(Z22) K23=imag(Z23)
 X33=imag(Z33) K34=imag(Z34)
 X44=imag(Z44) K45=imag(Z45)

F= FREQ*1e-6
 I0=find_index(F,f0)

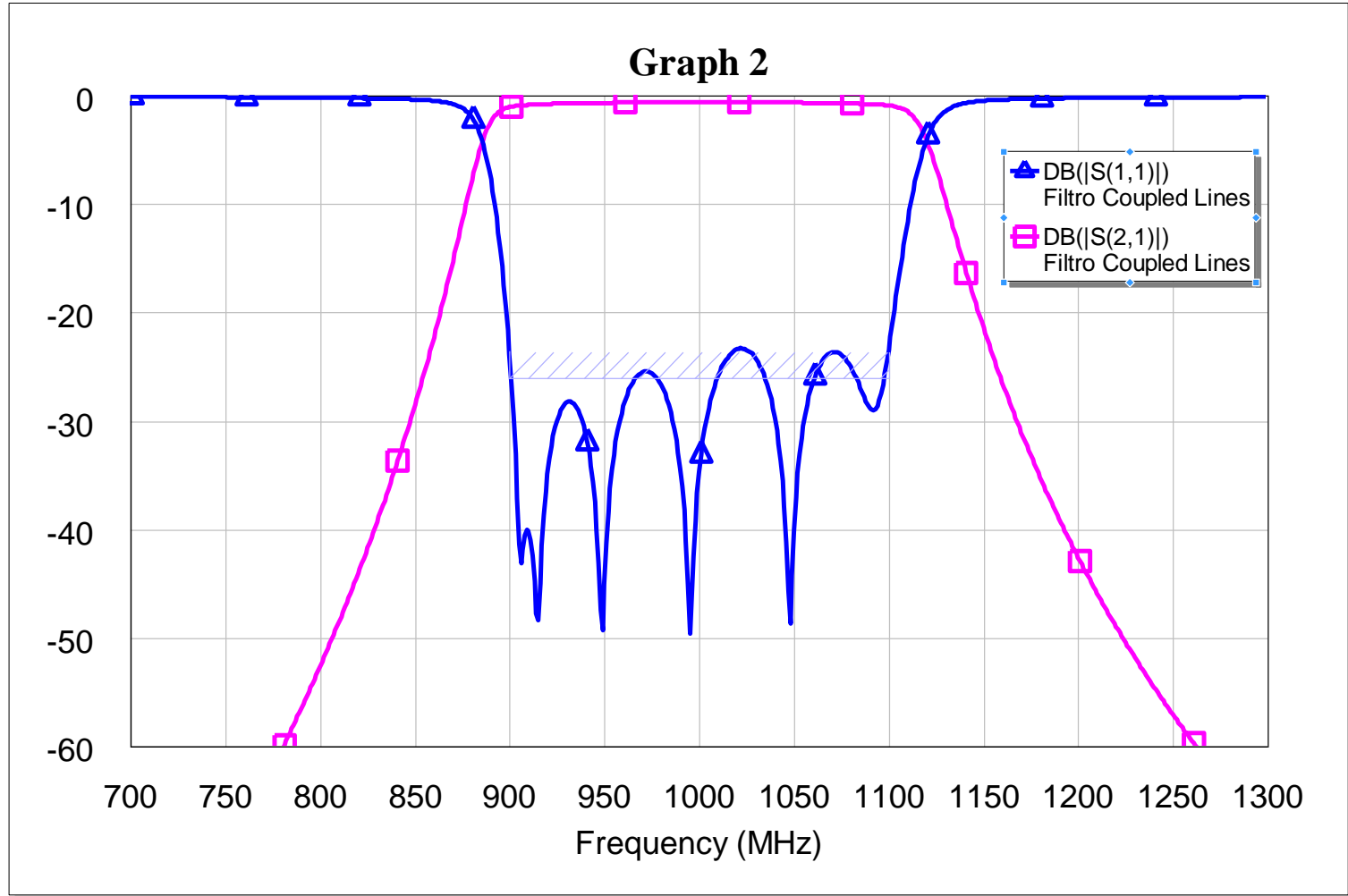
I1=find_index(X11,0) F[I1]: 995 Xeq1=0.5*der(X11,F) Xeq3=0.5*der(X33,F)
 I2=find_index(X22,0) F[I2]: 995 Xeq1=Xeq1[I1]*F[I1] Xeq3=Xeq3[I3]*F[I3]
 I3=find_index(X33,0) F[I3]: 995 Xeq2=0.5*der(X22,F) Xeq4=0.5*der(X44,F)
 I4=find_index(X44,0) F[I4]: 995 Xeq2=Xeq2[I2]*F[I2] Xeq4=Xeq4[I4]*F[I4]

Qext=Xeq1/real(Z11[I1]) 1/Qext: 0.2489 **(0.2491)**
 k12=K12[I0]/sqrt(Xeq1*Xeq2) k12: 0.1896 **(0.1893)**
 k23=K23[I0]/sqrt(Xeq3*Xeq2) k23: 0.1282 **(0.1283)**
 k34=K34[I0]/sqrt(Xeq3*Xeq4) k34: 0.1189 **(0.1186)**

Result of design

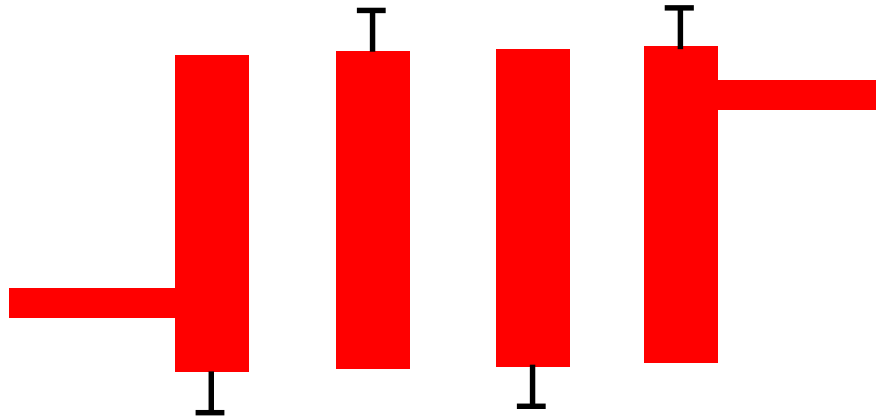
S01=0.3593 Delta1=0.523
 S12=0.752 Delta2=0.1
 S23=0.9882 Delta3=0.1621
 S34=1.068 Delta4=0.1765

Response of the dimensioned filter

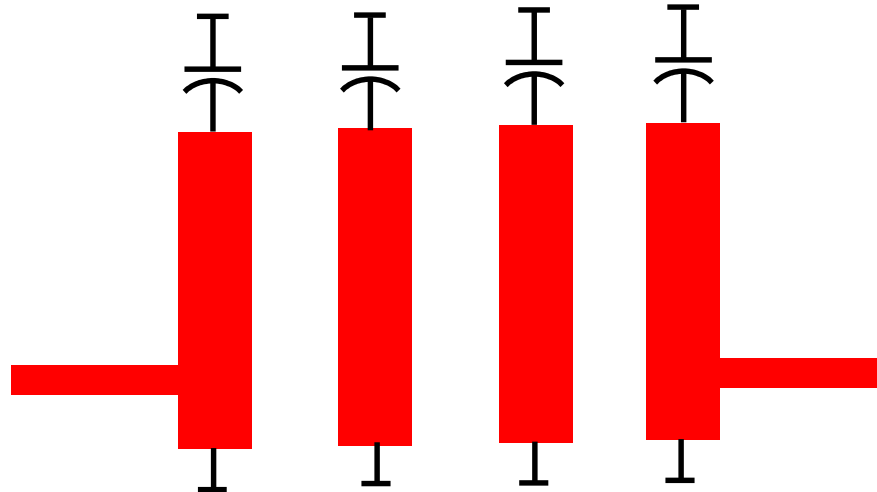


Filters with an array of coupled-lines

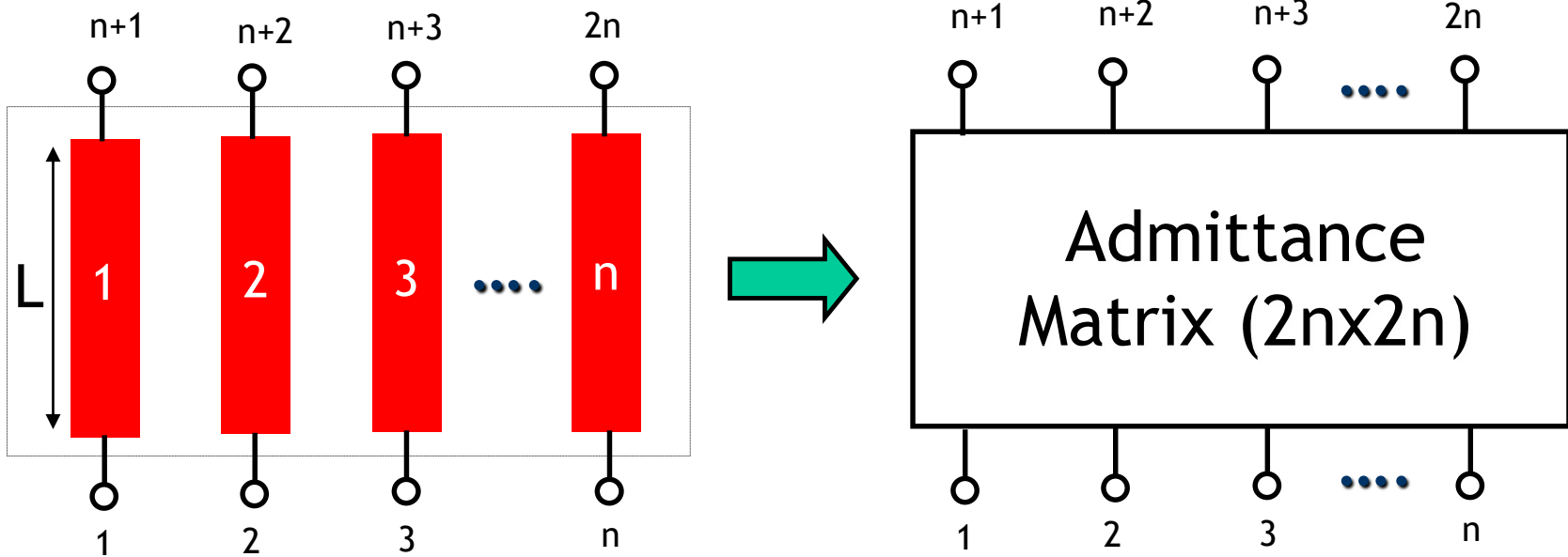
□ Interdigital:



□ Comb:



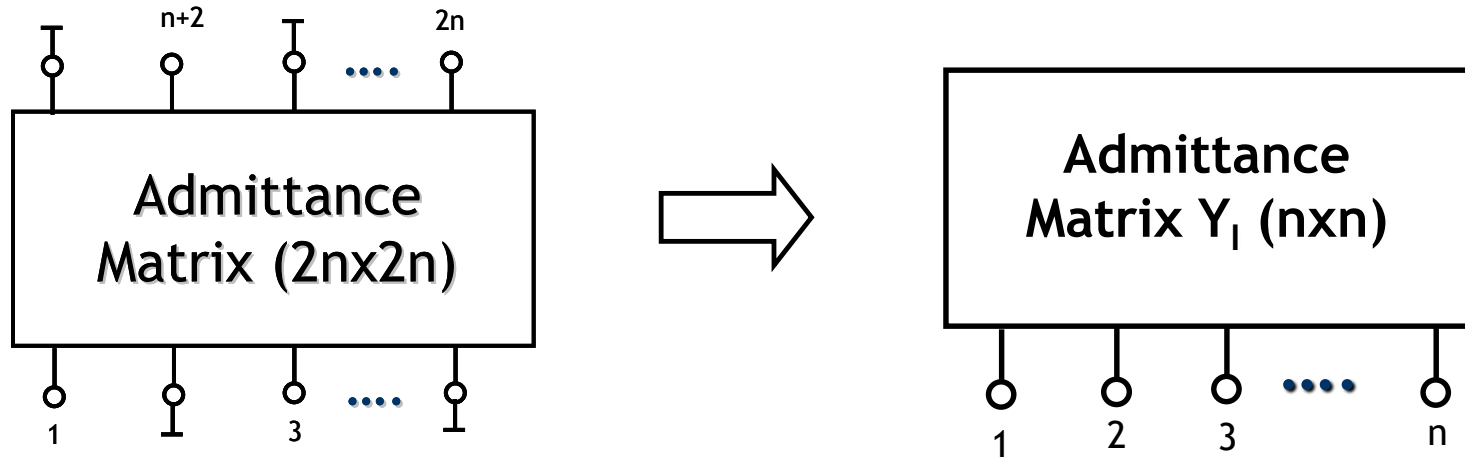
Basic block: array of coupled lines



The admittance matrix \mathbf{Y} ($2n \times 2n$) can be evaluated from the static capacitance matrix p.u.l \mathbf{C} (dimension $n \times n$):

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}' & \mathbf{Y}'' \\ \mathbf{Y}'' & \mathbf{Y}' \end{bmatrix}, \quad \mathbf{Y}' = \frac{v \cdot \mathbf{C}}{j \tan(\beta L)}, \quad \mathbf{Y}'' = \frac{v \cdot \mathbf{C}}{j \sin(\beta L)}$$

Matrix Y of Interdigital Array

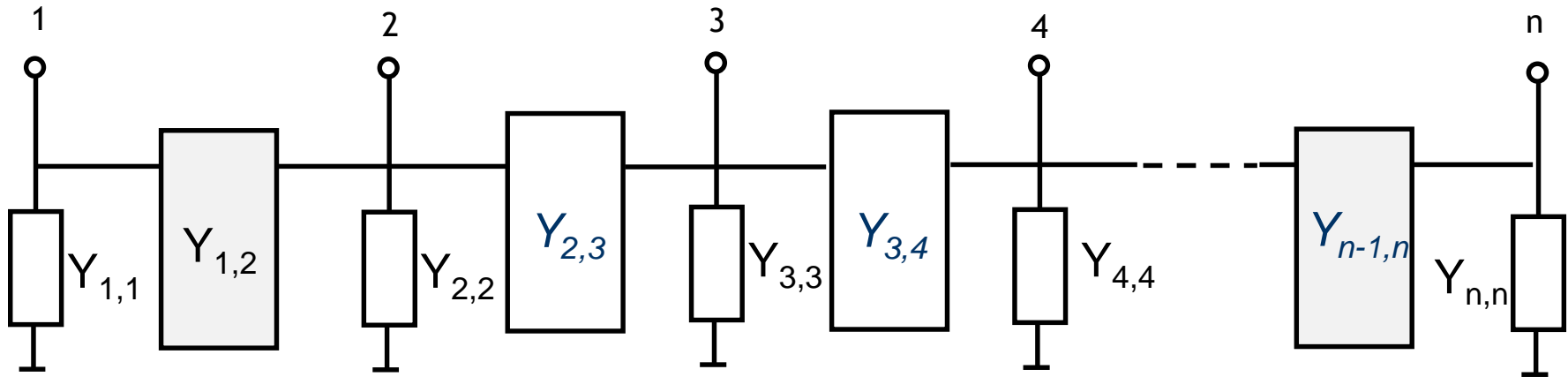


Let assume that $C_{i,j}=0$ for $j>i+1$ (no coupling between non-adjacent lines). Elements of matrix Y_I are then given by (v is the propagation velocity):

$$y_I(i, j) = \begin{cases} \frac{v \cdot C_{i,i}}{j \tan(\beta L)} & i = j \\ \frac{v \cdot C_{i,i+1}}{j \sin(\beta L)} & j = i + 1 \\ 0 & j > i + 1 \end{cases}$$

Matrix Y_I is **tri-diagonal**

Condition on the Y matrix to represent a filter



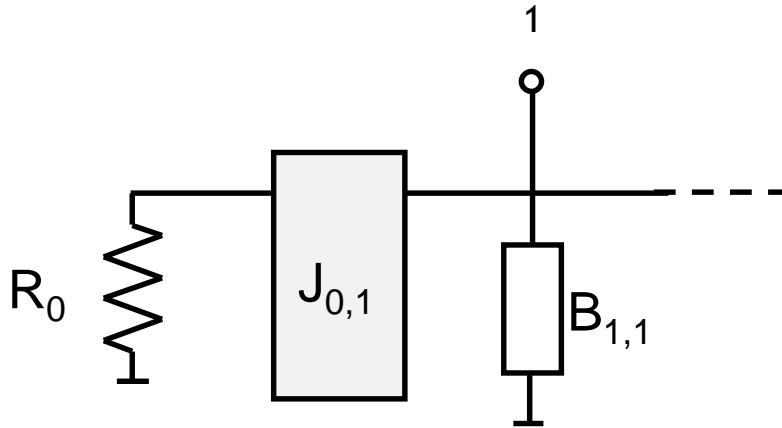
Equivalent parameters ($Y_{i,j} = jB_{i,j}$):

$$B_{i,i}(\omega_0) = -\frac{v \cdot C_{i,i}}{\tan(\beta_0 L)} = 0 \Rightarrow \beta_0 L = \frac{\pi}{2} \Rightarrow L = \frac{\lambda_0}{4}$$

$$J_{i,i+1} = B_{i,i+1}(\omega_0) = -\frac{v \cdot C_{i,i+1}}{\sin(\beta_0 L)} = -v C_{i,i+1}$$

$$\omega_0 C_{eq,i} = \omega_0 \left. \frac{1}{2} \frac{\partial B_{i,i}(\omega)}{\partial \omega} \right|_{\omega=\omega_0} = \frac{\pi}{4} v C_{i,i}$$

Coupling with external loads



$$Y_{1,1} = \frac{J_{0,1}^2}{G_0} + jB_{1,1}$$

$$Q_E = \frac{\omega_0 C_{eq,1}}{J_{0,1}^2 / G_0} = \frac{\omega_0 \frac{1}{2} \frac{\partial B_{1,1}(\omega)}{\partial \omega} \Big|_{\omega=\omega_0}}{\text{Re}\{Y_{1,1}\}_{\omega=\omega_0}} = \frac{\frac{\pi}{4} \nu C_{i,i}}{\text{Re}\{Y_{1,1}\}_{\omega=\omega_0}}$$

Equations for interdigital filters design

$$k_{i,i+1} = k_{i,i+1} = \frac{J_{i,i+1}}{\omega_0 \sqrt{C_{eq,i} \cdot C_{eq,j}}} = \frac{4}{\pi} \frac{C_{i,i+1}}{\sqrt{C_{i,i} C_{i+1,i+1}}}, \quad Q_E = \frac{\frac{\pi}{4} \nu C_{1,1}}{\operatorname{Re}\{Y_{1,1}\}_{\omega=\omega_0}}$$

Typically, the coupled lines are assumed of equal width (assigned a priori). The unknowns are then represented by the distances $d_{i,i+1}$ between the lines and by the dimensional parameter (d_e) of the external coupling structure.

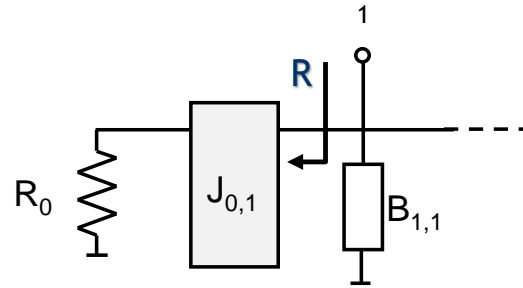
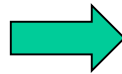
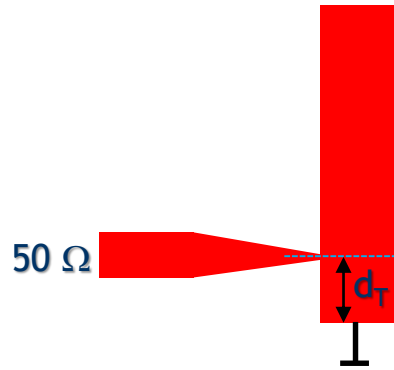
The solution is obtained by solving the system of non-linear equations in the variables $d_{i,i+1}$ and d_e :

$$k_{i,i+1} - \frac{4}{\pi} F(d_{i,i+1}) = 0, \quad Q_E - \frac{\pi}{4} \nu \cdot U(d_e, d_{1,2}) = 0$$

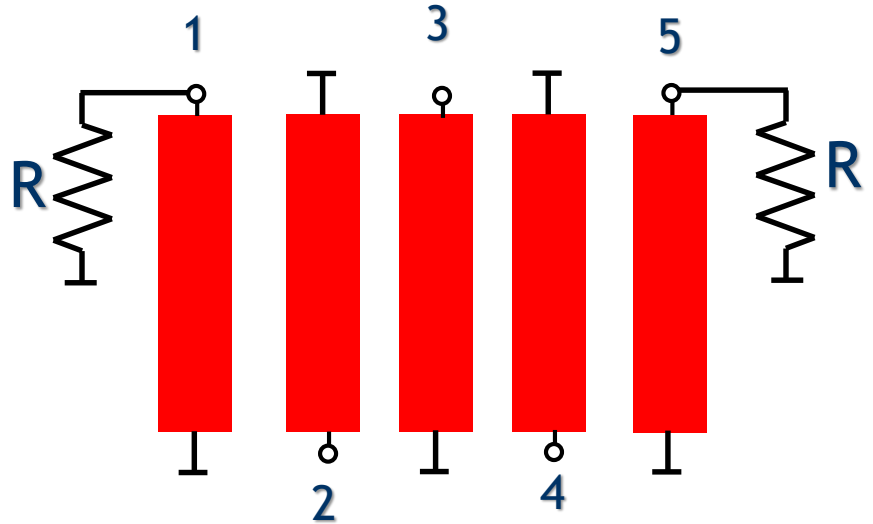
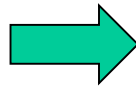
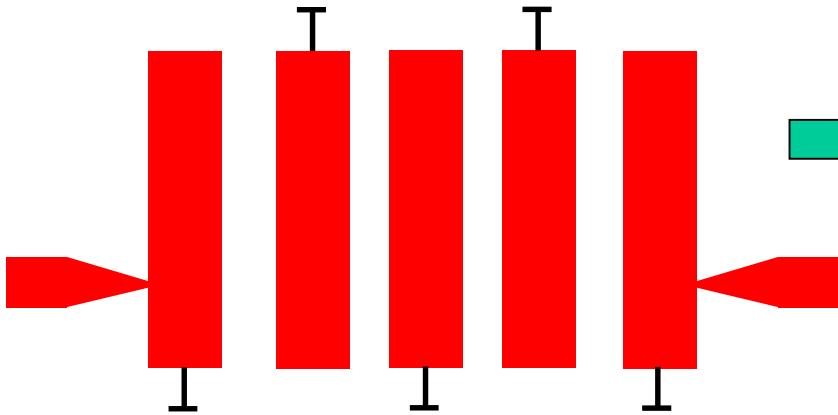
Example of interdigital filter in microstrip

- Passband: 1800-2000 MHz, Return Loss: 18 dB
- Filter order: 5
- Result of the synthesis:
 $f_{\text{ris}}=1900$ MHz,
 $k_{12}=k_{45}=0.087$, $k_{23}=k_{34}=0.0653$, symmetric
 $Q_{\text{ext}}=10.1$
- Substrate parameters:
 $\epsilon_r=2.2$, $H=0.508$ mm, $t=35\mu$
- Assigned dimensions:
Width of coupled strips: 1mm
Ideal length of coupled strips: 29.25mm ($\lambda_0/4$)
Width of 50 Ohm lines: 1.52mm

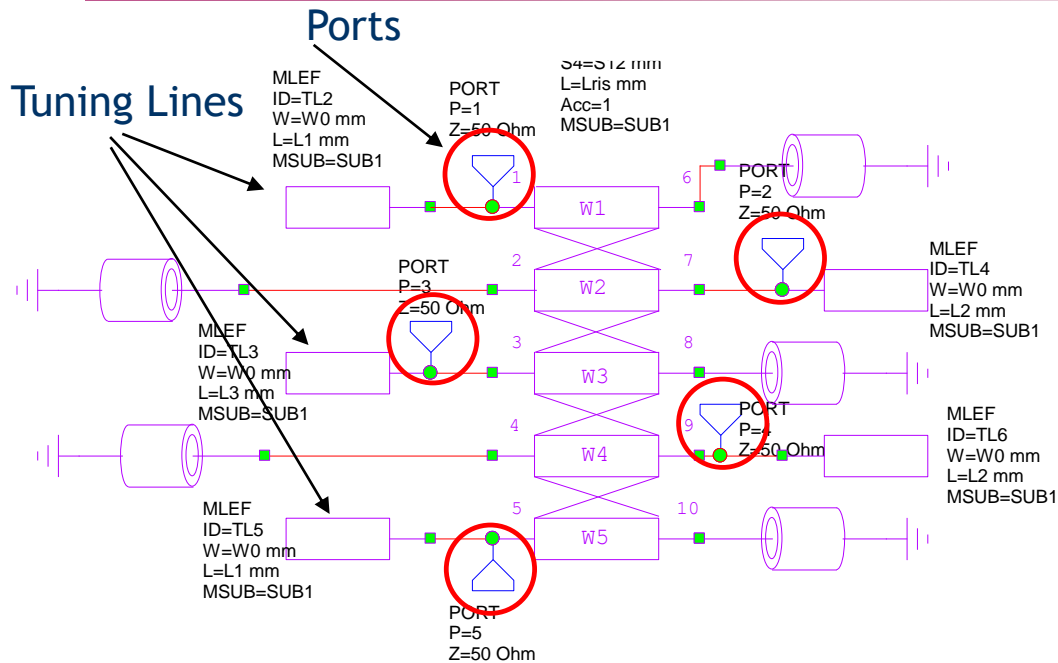
In-out coupling modelling



$$Q_E = \frac{B_{eq,1}}{J_{0,1}^2 R_0} \Rightarrow R = \frac{1}{J_{0,1}^2 R_0} = \frac{Q_E}{B_{eq,1}}$$



Extraction of universal parameters



$$B_{eq,i} = \omega_0 C_{eq,i} = \omega_0 \left. \frac{1}{2} \frac{\partial \text{Im}[Y_{i,i}(\omega)]}{\partial \omega} \right|_{\omega=\omega_0},$$

$$J_{i,i+1} = \text{Im}[Y_{i,i+1}(\omega_0)]$$

Geometrical unknown parameters:

- separation of lines (s_{12}, s_{23})
- Length of tuning lines (L_1, L_2, L_3)

Assigned dimensions:

- width of coupled lines ($w_0=1\text{mm}$)
- Length of coupled lines ($L_0=28\text{mm}$)

The design is carried out by means of the tuning facility of MWOoffice assigning initially $L_i=0$ and $s_{12}=s_{23}=1\text{mm}$.

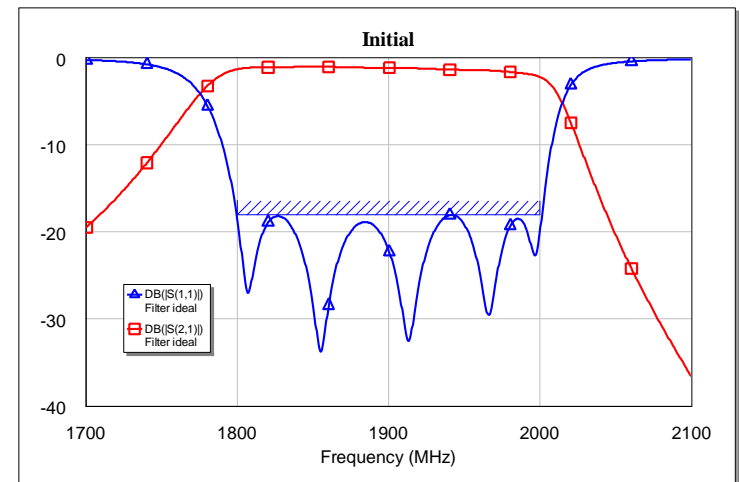
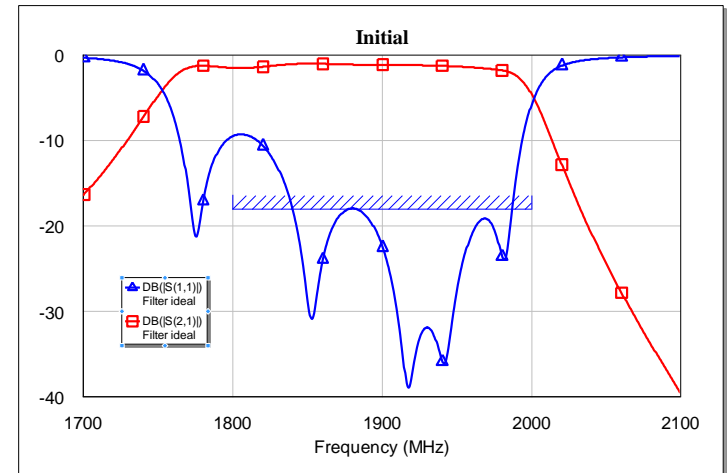
Result of design (circuit model)

Once the tuning of the multi-port circuit has been performed, the following values have been computed:

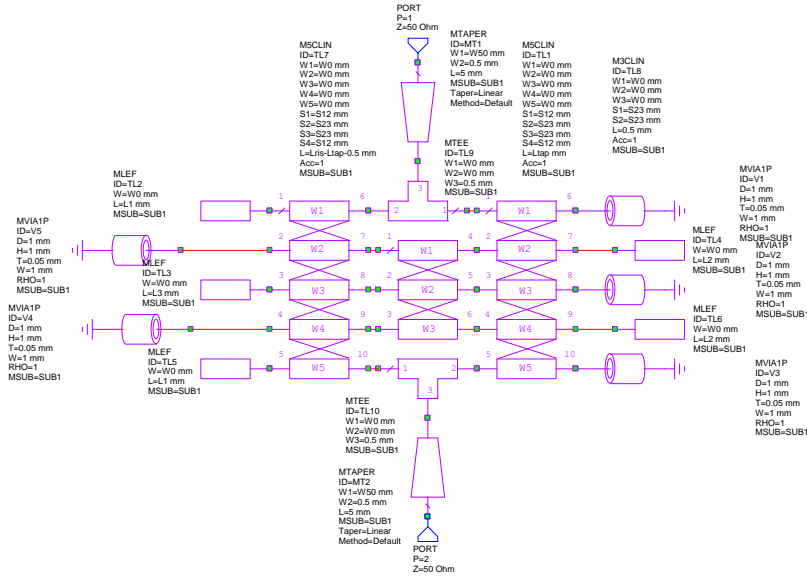
$$\begin{aligned} L1 &= 0.238 & S12 &= 1.092 \\ L2 &= 0.33 & S23 &= 1.342 \\ L3 &= 0.31 & & \end{aligned}$$

The filter response is however not satisfactory. This is due to the coupling between not adjacent lines not included in the synthesis model. The response of the filter must be improved by manual tuning.

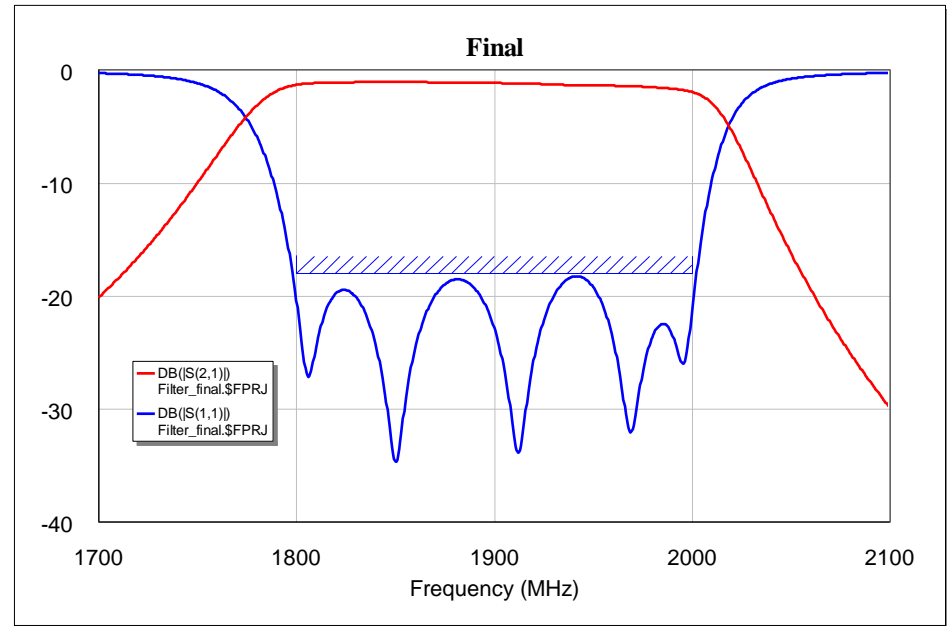
$$\begin{aligned} L_{ris} &= 27.74 & L1 &= 0.4155 \\ & & L2 &= 0.285 \\ & & L3 &= 0.25 \\ & & S12 &= 1.1 \\ & & S23 &= 1.431 \end{aligned}$$



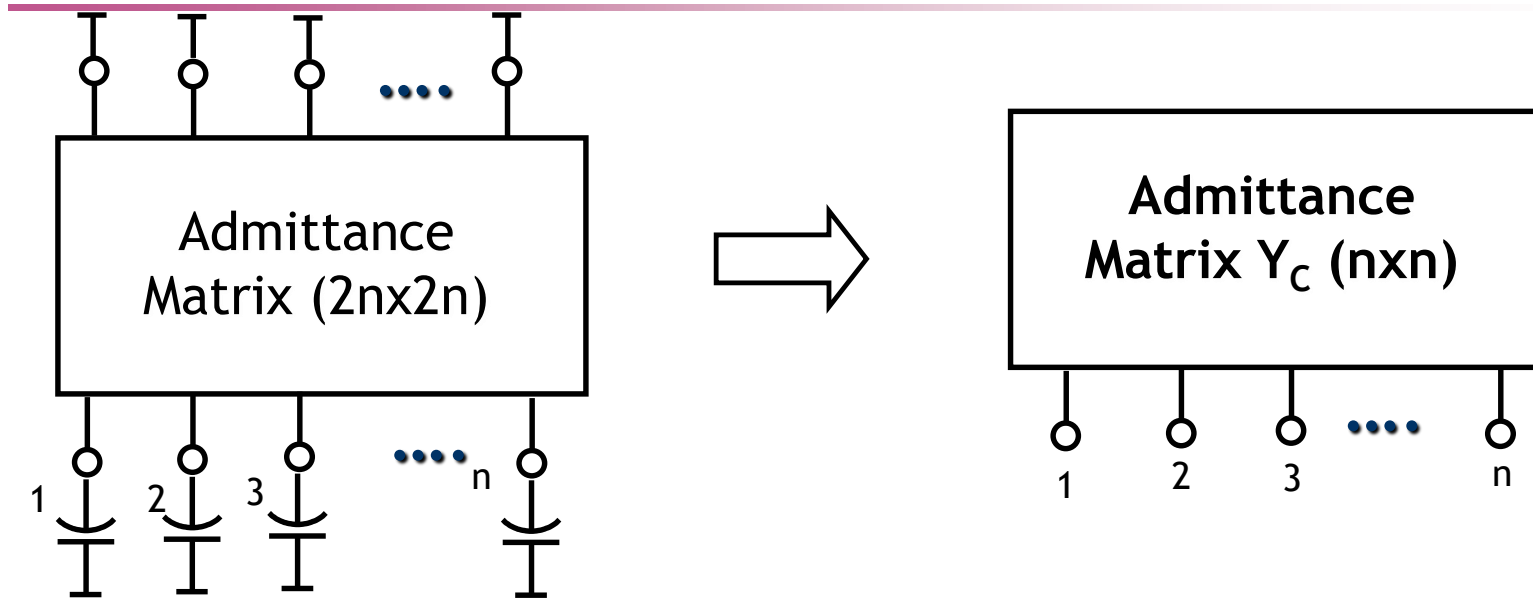
Dimensioning of in-out couplings (tap)



$L_{ris}=27.94$
 $L1=1.08$ $S12=1.086$
 $L2=0.016$ $S23=1.413$
 $L3=0$
 $L_{tap}=4.16$



Matrix Y of Comb Array



With no coupling between non-adjacent lines:

$$y_{C,i,j}(\omega) = \begin{cases} j \left(\omega C_{s,i} - \frac{v \cdot C_{i,i}}{\tan\left(\frac{\omega L}{v}\right)} \right) & i = j \\ \frac{v \cdot C_{i,i+1}}{j \tan\left(\frac{\omega L}{v}\right)} & j = i+1 \\ 0 & j > i+1 \end{cases}$$

To get $y_{C,i,j}(\omega_0) \neq 0$:

$$L < \frac{\lambda_0}{4}$$

Equations for comb filters design ($\beta l = \pi/4$)

$$J_{i,i+1} = B_{i,i+1} \Big|_{\omega=\omega_0} = -v C_{i,i+1}, \quad \omega_0 C_{eq,i} = \left(\frac{2+\pi}{4} \right) v C_{i,i}$$

$$k_{i,i+1} = \frac{4}{2+\pi} \frac{C_{i,i+1}}{\sqrt{C_{i,i} C_{i+1,i+1}}}, \quad Q_E = \frac{\left(\frac{2+\pi}{4} \right) v C_{1,1}}{\text{Re}\{Y_{1,1}\}_{\omega=\omega_0}}$$

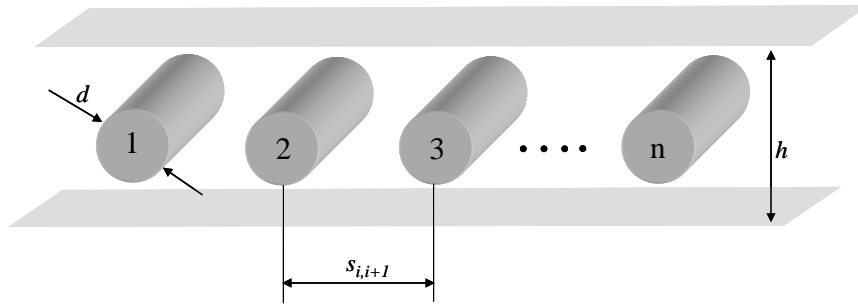
As in the case of interdigital filters, assigning all the lines with the same width, the solution is obtained by solving the system of non-linear equations in the variables $d_{i,j+1}$ and d_e :

$$k_{i,i+1} - \frac{4}{2+\pi} F(d_{i,i+1}) = 0, \quad Q_E - \frac{2+\pi}{4} v \cdot U(d_e, d_{1,2}) = 0$$

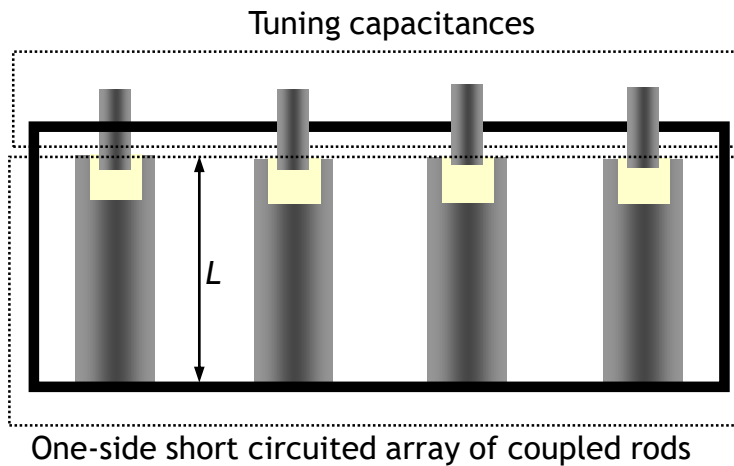
The tuning capacitances are obtained by the resonance condition:

$$\omega_0 C_{s,i} = \frac{v \cdot C_{i,i}}{\tan(\pi/4)} = v \cdot C_{i,i}$$

Example: Comb Filters in slab-line



Array of coupled rods
(coupled slab-lines)



Inline comb filter configuration

