Extracted-pole Filters

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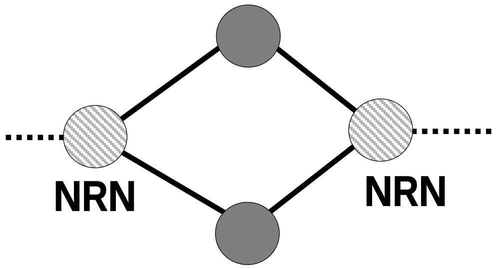
A new element in the low-pass prototype

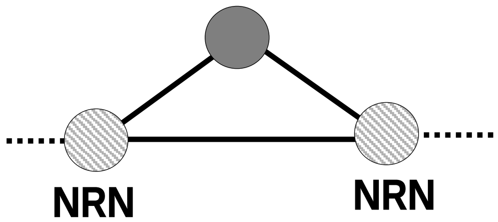
- Transmission zeros in cross-coupled filters are obtained by introducing in the filter topology multiple paths between input and output
- Amari proposed in 2004 to use in such paths, other than resonating nodes, also nodes not featuring resonating properties (**not resonating node, NRN**)
- From a circuit point of view a NRN is constituted by a frequency-invariant reactance (susceptance)
- NRNs allows to increase the number of zeros for a given number of resonators. Also, more compact structures may be obtained
- The synthesis of filters with NRN can be performed in the low pass domain. De-normalization does not modify the value of NRNs (with respect unit loads)

Examples



Doublet

The doublet extracts 1 transmission zero (not possible with 2 resonating nodes)



Singlet

The singlet is implemented with only 1 resonating node (1 zero is extracted)

Synthesis of low-pass prototype with NRNs

- NRN are frequency-invariant reactances, so the synthesis of networks including NRN is in general possible as far as positive functions are concerned
- When NRNs are embedded into a low-pass prototype the coupling matrix as previously defined is no more meaningful
- As a consequence, the technique based on coupling matrix reconfiguration cannot be employed
- The low-pass equivalent circuit is generally synthesized with 'ad-hoc' techniques or is obtained by means of network transformation (tee-to-pi, star-to-triangle, etc)
- The most important use of NRNs is however found in the 'extracted poles synthesis', which allows the synthesis of the network with a specified topology

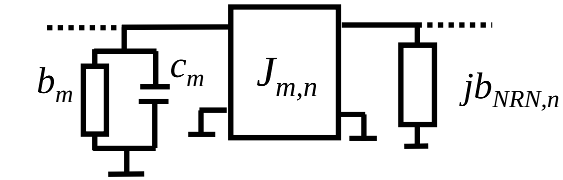
Generalized coupling coefficients

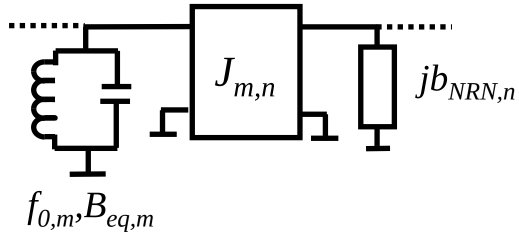
- Once a low-pass prototype including NRNs has been synthesized, there are in general several degrees of freedom which allow to assign a priori some elements of the de-normalized circuit, obtaining all the others through suitable reference parameters which remain unchanged
- Such parameters generalize the concept of coupling coefficient previously introduced for coupled resonators filters and can be also exploited for the practical dimensioning of the physical structures implementing the filter with NRNs (although not as immediately as the classical coupling coefficients)

Expressions for generalized couplings coefficients

Low-Pass

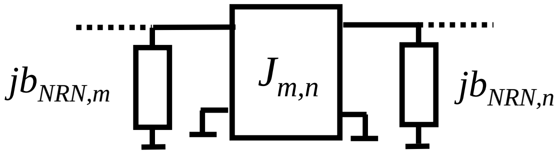
Band-Pass

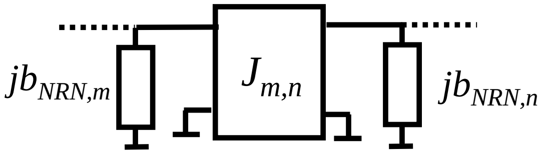




$$B_{eq,m} = \left(\frac{f_0}{B}\right) \cdot c_m$$

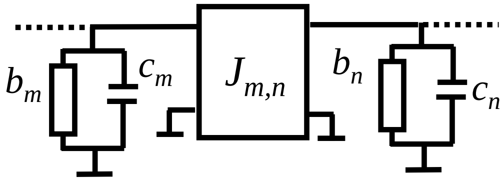
$$B_{eq,n} = \left|b_{NRN,n}\right|$$

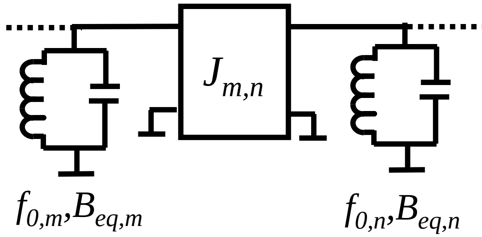




$$B_{eq,m} = \left|b_{NRN,m}\right|$$

$$B_{eq,n} = \left|b_{NRN,n}\right|$$





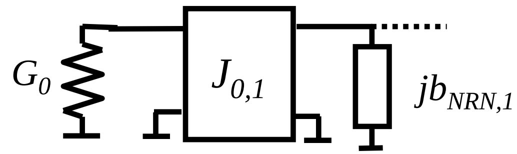
$$B_{eq,m} = \left(\frac{f_0}{B}\right) \cdot c_m$$

$$B_{eq,n} = \left(\frac{f_0}{B}\right) \cdot c_n$$

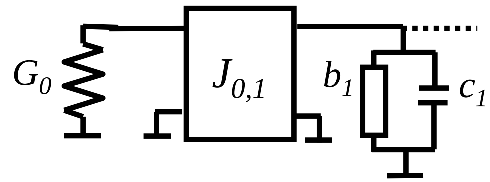
$$k_{m,n} = \frac{J_{m,n}}{\sqrt{B_{eq,m} \cdot B_{eq,n}}}$$

The signs of NRNs must also be conserved

Generalized external Q



$$B_{eq} = \left| b_{NRN,1} \right|$$



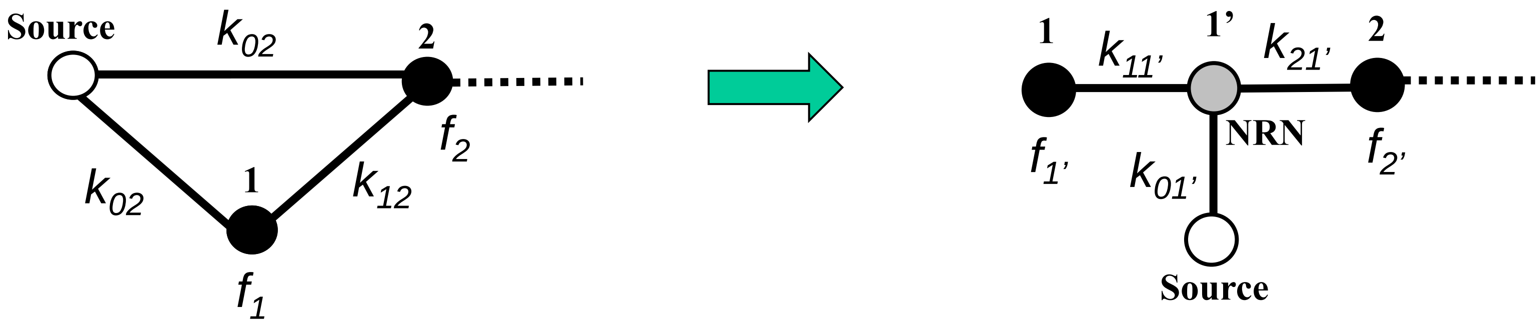
$$B_{eq} = \left(\frac{f_0}{B} \right) \cdot c_1$$

$$Q_{E,(0,1)} = \frac{B_{eq}}{J^2_{01}} G_0$$

Note: Sometime, the inverse of external Q is used:

$k_{0,i}=1/Q_{E(0,i)}$

Example of synthesis based on network transformation



$$k_{01'} = \frac{\sqrt{|k_{02} \cdot k_{01}|}}{k_{12}} \qquad k_{11'} = \sqrt{\sqrt{\left|\frac{k_{01}}{k_{02}}\right|}\left(\frac{k_{12}^2 + |k_{02} \cdot k_{01}|}{|k_{12}|}\right)} \qquad k_{21'} = \sqrt{\sqrt{\left|\frac{k_{02}}{k_{01}}\right|}\left(\frac{k_{12}^2 + |k_{02} \cdot k_{01}|}{|k_{12}|}\right)}$$

$$a_1 = -\left(\frac{f_1}{f_0} - \frac{f_0}{f_1}\right) - sign\left(\frac{k_{01}}{k_{02}}\right) k_{12}\sqrt{\frac{k_{01}}{k_{02}}} \qquad a_2 = -\left(\frac{f_2}{f_0} - \frac{f_0}{f_2}\right) - sign\left(\frac{k_{02}}{k_{01}}\right) k_{12}\sqrt{\frac{k_{02}}{k_{01}}}$$

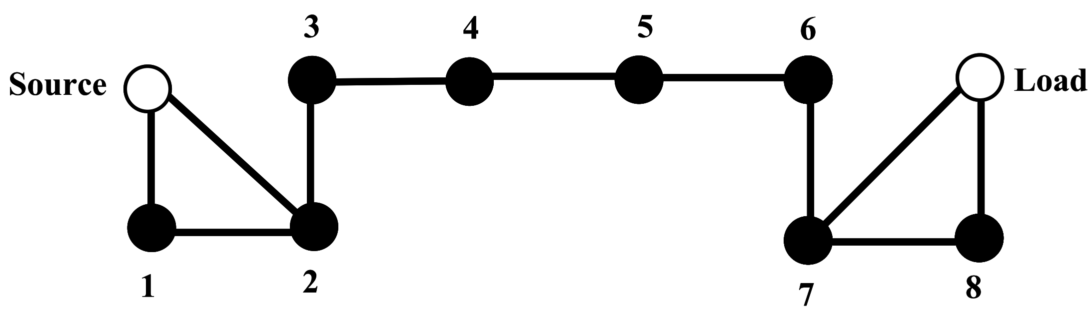
$$f_{1'} = f_0\left(\frac{\sqrt{a_1^2 + 4} - a_1}{2}\right) \qquad f_{2'} = f_0\left(\frac{\sqrt{a_2^2 + 4} - a_2}{2}\right)$$

Synthesis of a filter with 'appended' resonators

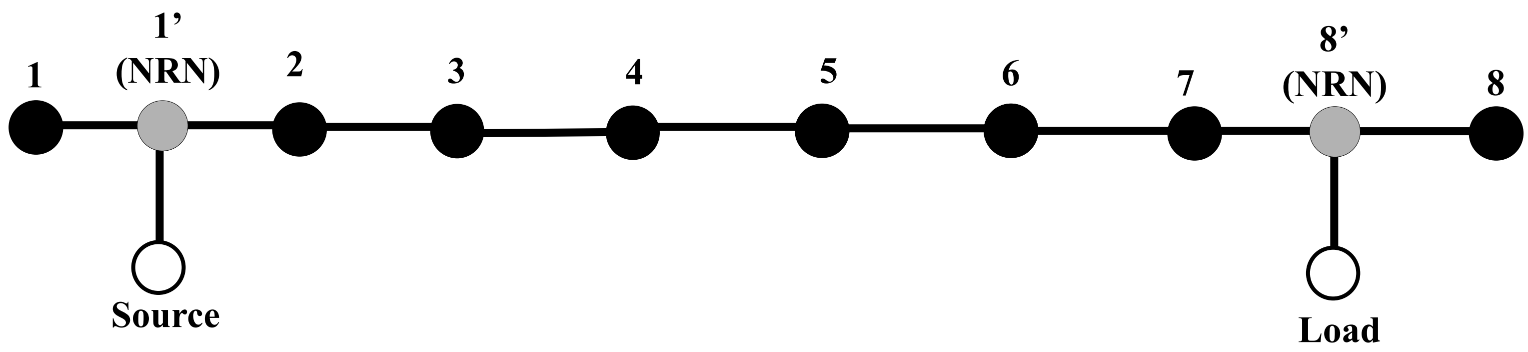
- In this example a technique is shown for realizing an in-line filters without cross couplings which presents 2 transmission zeros
- Such zeros are obtained by introducing 2 NRNs with the transformation shown in the previous slide
- Filter specifications:

  n=8, f_0=1880.2 MHz, B=70 MHz, RL=23 dB,
  Zeri=[1830, 1928.5] MHz

- Cascaded-blocks synthesis (2 triplets starting from Source and Load):



Transformed topology



Generalized Coefficients

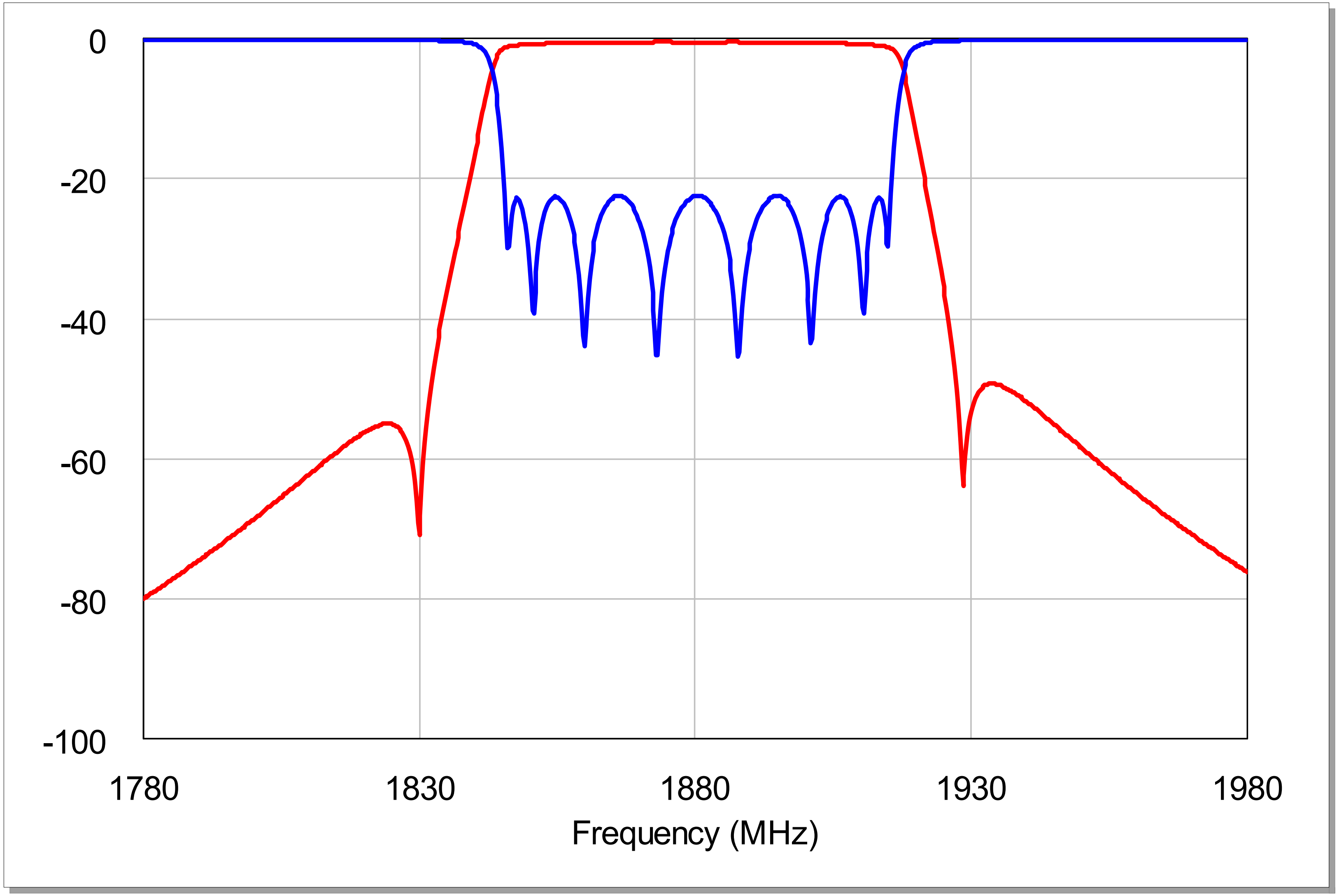
$k_{i,i+1}$: [0.217, 0.18, 0.024, 0.021,0.02, 0.021,0.024,0.2055, 0.211]

$Q_{ext,S}$=0.78, $Q_{ext,L}$=0.63

$f_{ris,i}$: [1830, 1881.96, 1882.59, 1880.63, 1879.56, 1877.51, 1877.68, 1928.5]

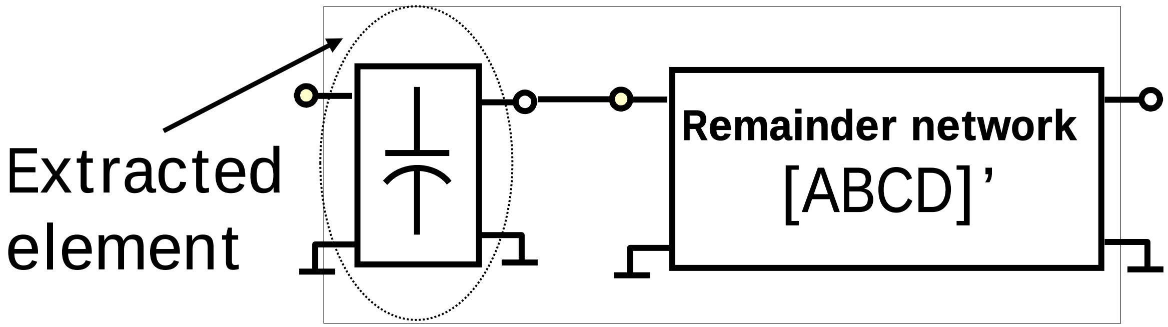
Sign of NRNs: 1’ positive, 8’ negative

Computed Response (Q_0=3000)



Synthesis of the lowpass prototype with ‘extracted poles’

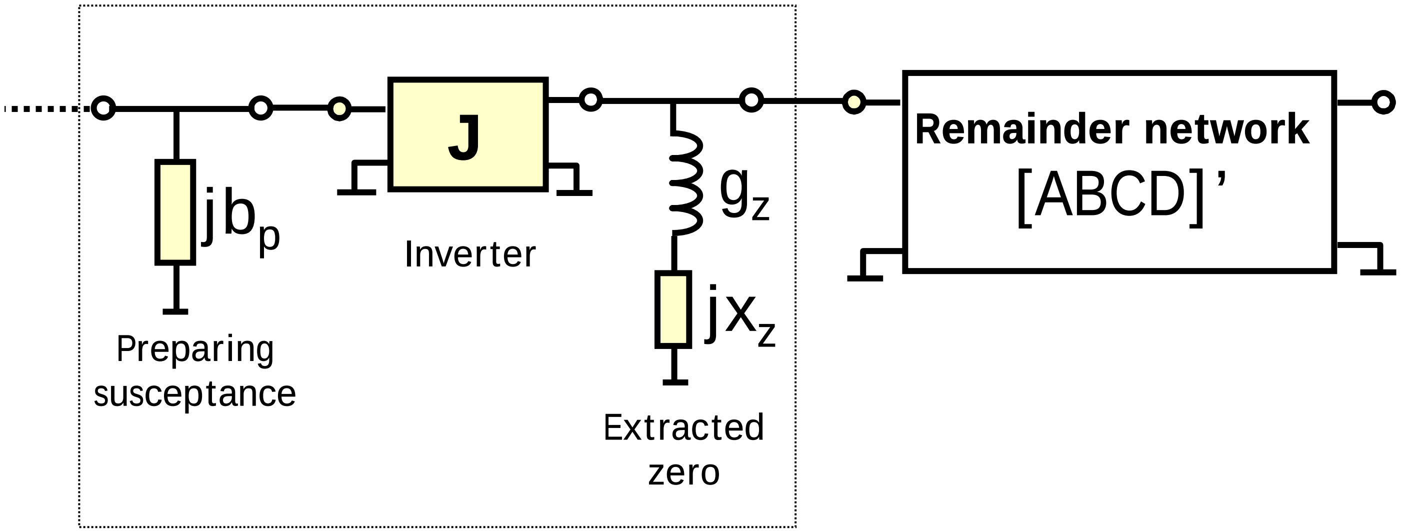
- The synthesis is based on subsequent extractions of the circuit elements from the [ABCD] matrix evaluated by the characteristic polynomials as previously seen:



Extraction of transmission zeros (‘extracted pole’) must be however suitably ‘prepared’ by extracting before a frequency invariant component

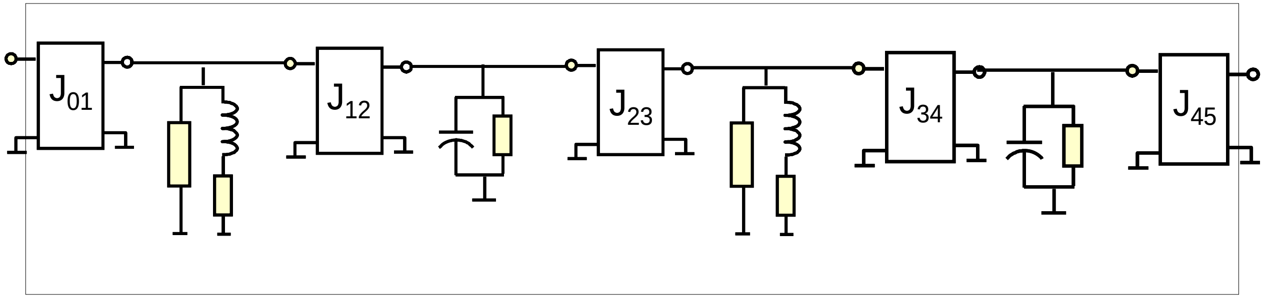
Extraction of a transmission zero

- With reference to the shunt representation of the prototype, the zero is produced by a shunt connected inductance in series with a frequency invariant reactance. The extraction is prepared by extracting before a frequency invariant susceptance:

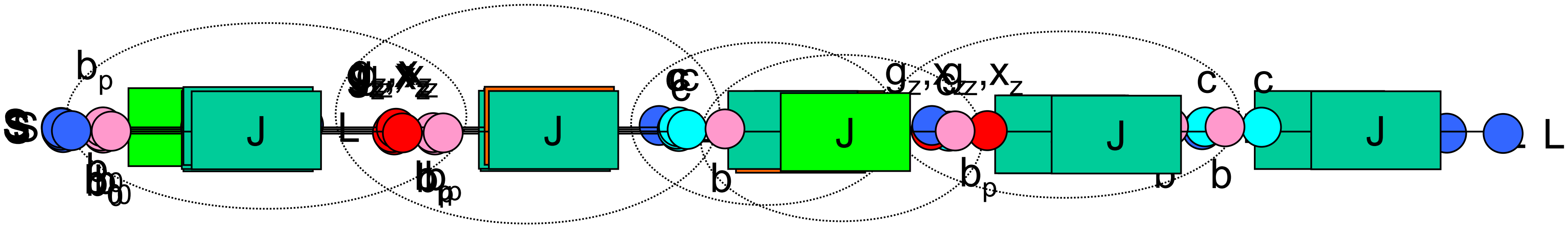


Extracted Block

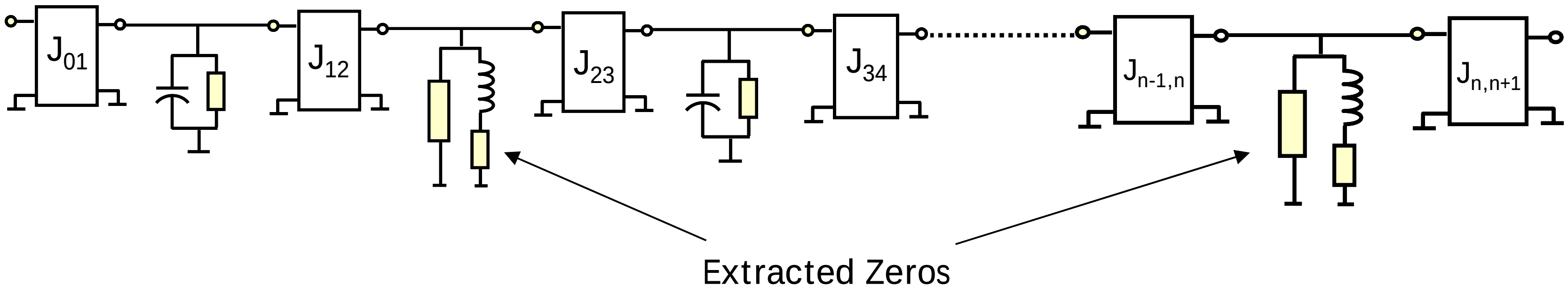
Example of extraction



C Matrix



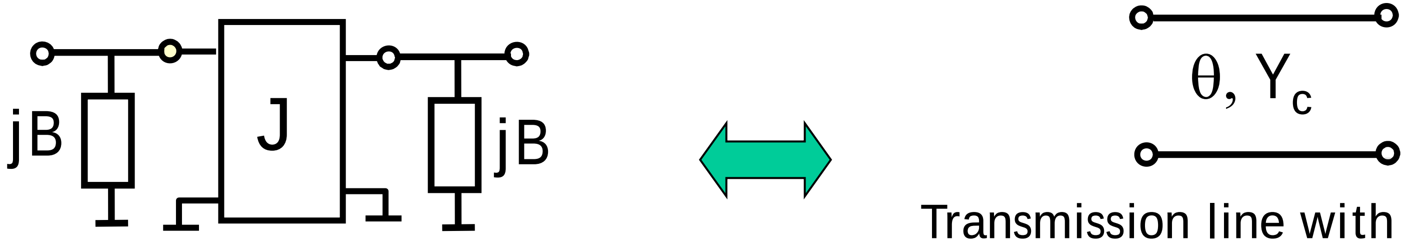
General topology of the lowpass prototype



- The topology is essentially in-line
- Up to *n* imaginary transmission zeros can be extracted
- The position of the extracted zeros is arbitrary. However, if a zero is extracted in first (last) position, a frequency invariant element appears in parallel to source (load)
- The synthesis procedure can be easily programmed (starting from the characteristic polynomials)

Alternative topology

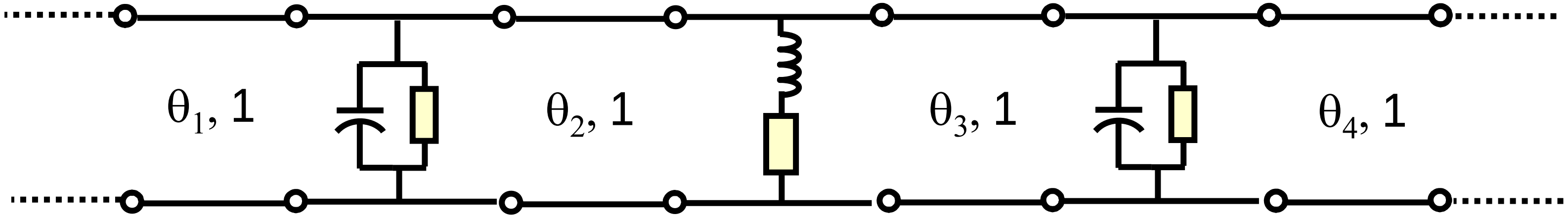
- Consider the following equivalence:



Transmission line with frequency-invariant electrical length (θ)

$$\frac{B}{J} = -\cos(\theta), \qquad Y_C = J \cdot \sin(\theta)$$

- Substituting in the previous scheme (imposing Y_c=1):



NOTE: This topology can be also directly synthesized using the transmission line sections θ_i for preparing the zeros extraction

What is a transmission line with frequency-invariant electrical length in the lowpass domain?

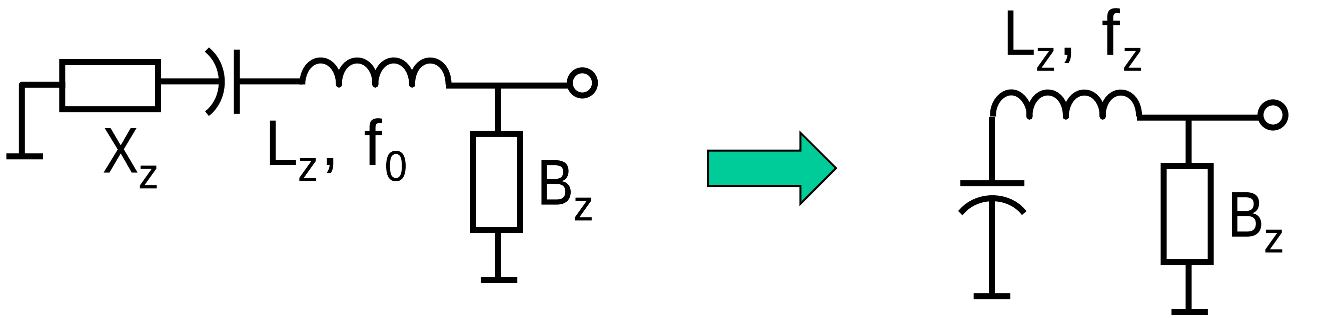
- Scattering matrix (assuming $Y_0=Y_c=1$):

$$S = \begin{vmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{vmatrix}$$

From a circuit point of view, the transmission line with frequency-invariant electrical length is a **phase shifter with constant shift angle.** When the lowpass prototype is de-normalized, the phase shifter **remains unchanged**

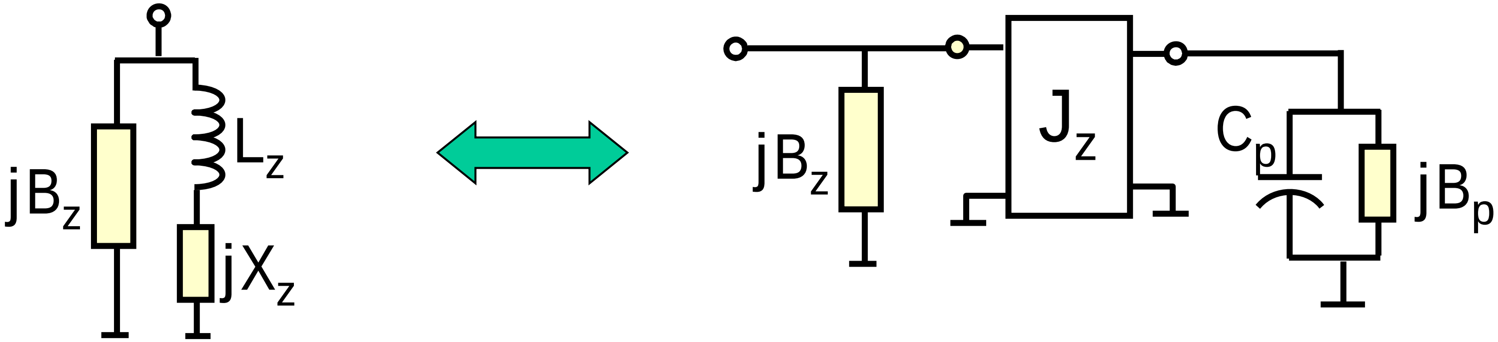
De-normalization of the lowpass prototype

- The coupling elements (inverters, phase shifters) remain unchanged
- The capacitances in parallel with frequency-invariant susceptance become resonator slightly detuned with respect f_0
- The extracted zero blocks becomes (approximately) series resonators (tuned at the transmission zeros frequencies) in parallel with a frequency invariant susceptance (unchanged):



De-normalized extracted-zero block

Transformation of the extracted-pole block



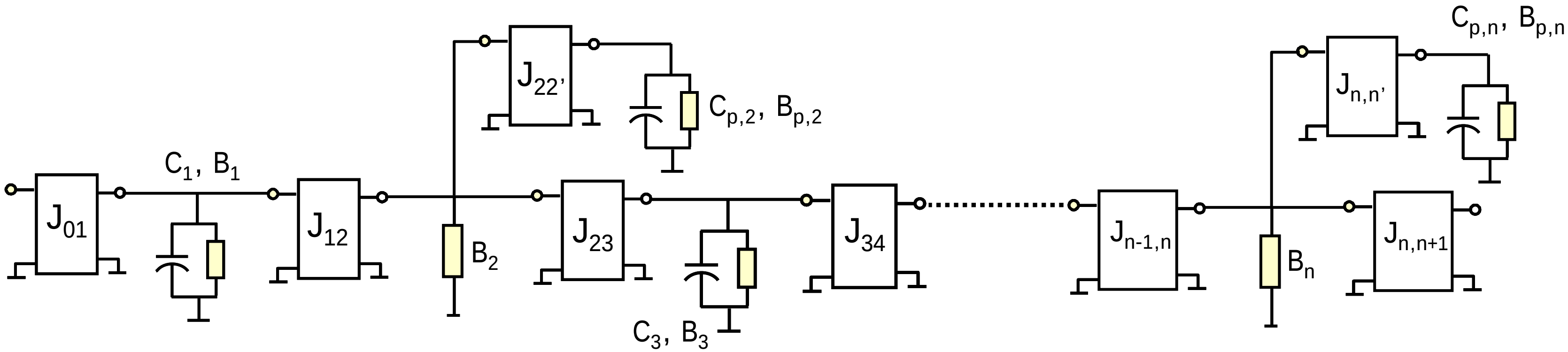
$$B_{in} = B_z - \frac{1}{X_z + \Omega \cdot L_z}$$

$$B_{in} = B_z - \frac{J_z^2}{B_p + \Omega \cdot C_p}$$

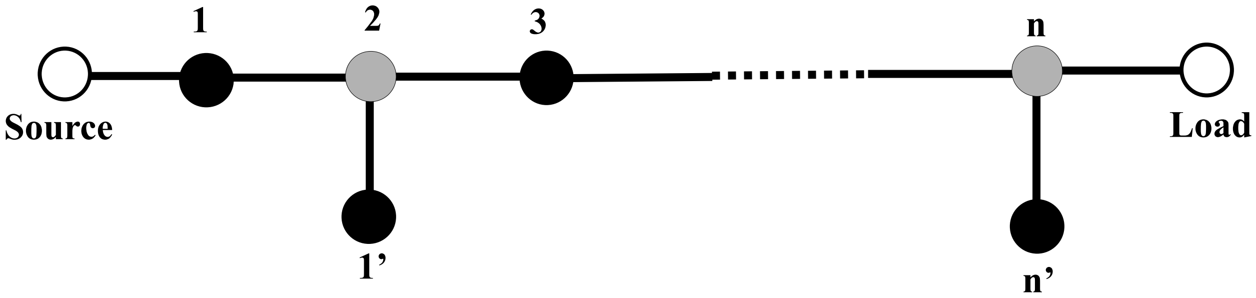
$$C_p = J_z^2 L_z, \qquad B_p = J_z^2 X_z$$

Note: J_z is arbitrary (it can be selected in order to obtain C_p=1)

Extracted-pole filters with coupled-resonators topology



If all the extracted capacitances ($C_{p,k}$, C_k) are imposed unitary, the above structure can be described in the same way of the multiple-coupled resonators prototype:



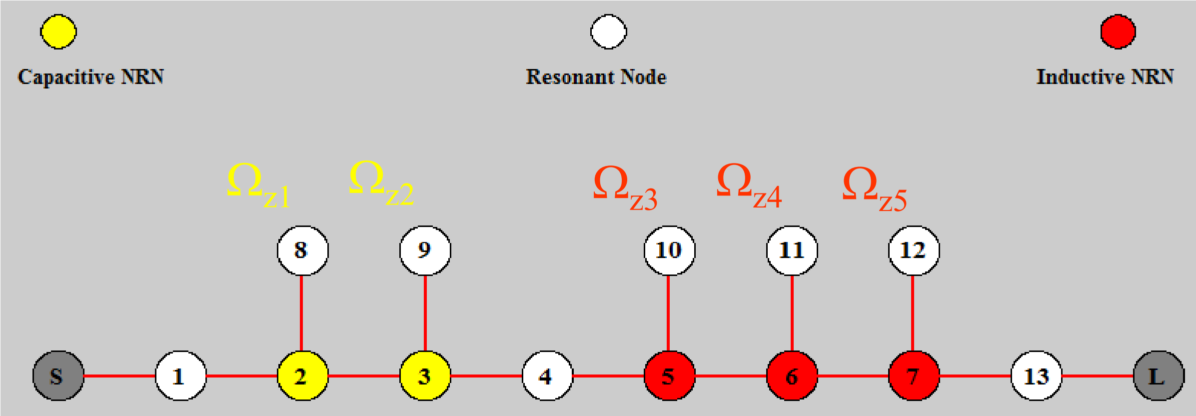
The prototype can be still characterized through a <u>normalized coupling matrix</u> of order ($n+n_z$). It is however necessary to specify the capacitances connected to each node (0 for NRN)

Example: extracted-pole syntesis

Specifications:

n=8, RL=26, Ω_z=[-1.33i, -1.63i, 1.075i, 1.138i, 1.38i]

Extracted topology:



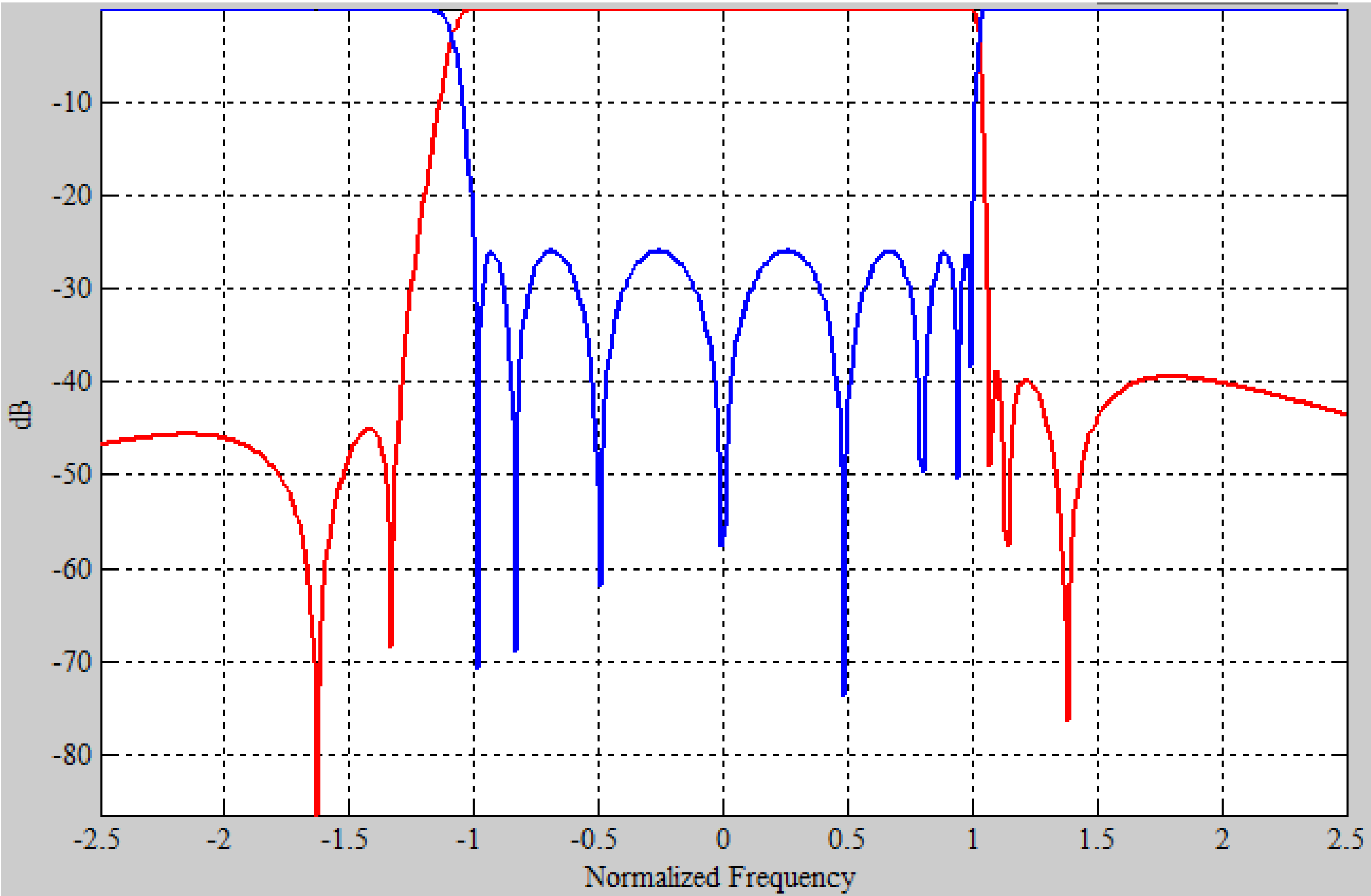
Coupling Matrix (S=L=1):

$M_{i,i+1}$=[1.205,1.205,1,.444, 0.4067, 1, 1,1.205,1.205], $M_{2,8}$=1.46, $M_{3,9}$=1, $M_{5,10}$=1.06, $M_{6,11}$=0.769, $M_{7,12}$=0.31

$M_{i,i}$=[0.5234, 2.533, 1.713, 0.1176, -1.782, -2.639, -1.183, 1.63, 1.33, -1.38, -1.138, -1.075, -1.415]

Capacitances: [1, 0, 0, 1, 0, 0, 0, 1,1,1,1,1, 1]

Frequency Response



De-normalization

- The band-pass de-normalized filter can be characterized through the generalized coupling coefficients previously introduced
- These parameters are evaluated from the normalized coupling matrix **M** and the node capacitances C_k as follows (B_n is the normalized filter bandwidth):

Nodes i,j type	Coupling coefficient
Resonant-Resonant	$k_{i,j} = B_n \frac{M_{i,j}}{\sqrt{C_i \cdot C_j}}$
Resonant-Not Resonant	$k_{i,j} = \sqrt{B_n} \frac{M_{i,j}}{\sqrt{C_i \cdot \lvert M_{j,j} \rvert}}$
Not Resonant-Not Resonant	$k_{i,j} = \frac{M_{i,j}}{\sqrt{\lvert M_{i,i} \cdot M_{j,j} \rvert}}$

De-normalization (cont.)

External Q (load G_0):

$$Q_{EXT,i} = \begin{cases} \dfrac{C_i / B_n}{M_{0,i}^2} G_0 & \text{Node } i \text{ resonant} \\[2ex] \dfrac{|M_{i,i}|}{M_{0,i}^2} G_0 & \text{Node } i \text{ non-resonant} \end{cases}$$

NOTE

Once the generalized parameters are known, a de-normalized network can be set up by assigning arbitrarily some circuit elements and obtain the others from the coupling parameters definition

Example: Assign resonators and NRN parameters

Let assume in the previous example B=70 MHz, f_0=1800 MHz

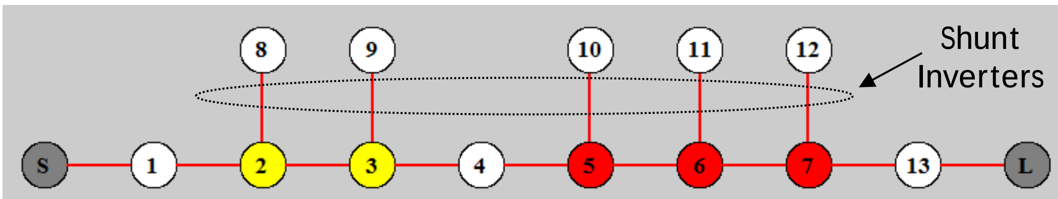
Synthesis Results

Main Coupling Coefficients: 0.1632 0.4678 0.0869 0.1013 0.6203 0.4558 0.1420
Zeros Coupling Coefficients: 0.0000 0.1374 0.1935 0.0000 0.0742 0.0887 0.1401 0.0000
Sign of NRN: 0 1 1 0 -1 -1 -1 0
Resonant Frequencies (MHz): 1768.796 1753.712 1743.514 1808.290 1837.678 1839.930 1848.608 1826.818
External Qs: 21.436302 21.436302

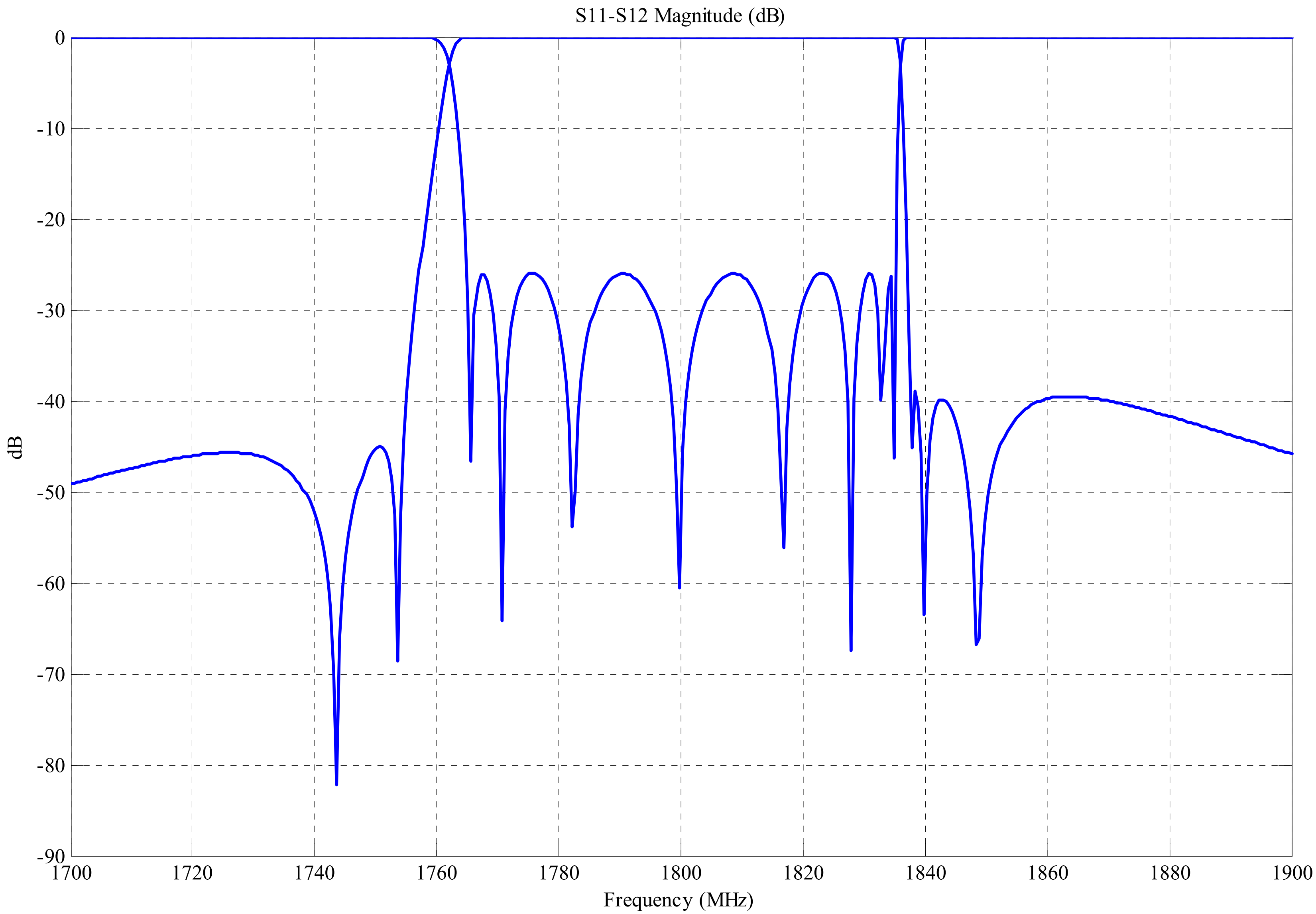
Assign now: B_{eq}=0.02 (all resonators), abs(NRN)=0.01. The inverters are then computed from the definition $J_{i,j} = k_{i,j} \cdot \sqrt{B_{eq,i} \cdot B_{eq,j}}$, with $B_{eq,i}$ equal to 0.02 or 0.01 depending on the node type. The computed parameters are the following:

Main Inverters parameter: 4.3197e-003 2.3086e-003 4.6777e-003 1.2285e-003 1.4328e-003 6.2027e-003 4.5581e-003 2.0082e-003 4.3197e-003

Shunt Inverters parameter: 0 1.94e-003 2.74e-003 0 1.05e-003 1.25e-003 1.98e-003 0



De-normalized response



Design of a single-sided fully canonical filter

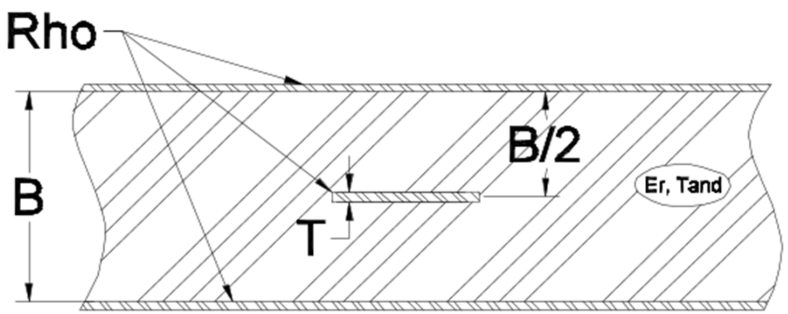
- Single-sided filters are defined by a single passband and a single stopband
- Using all the possible TZs, a very high attenuation can be realized even with few resonators (fully canonical)
- Extracted-pole configuration is particularly convenient, especially with a planar technology implementation
- The design of a stripline single-sided filter of order 3 will be illustrated in the following

Specifications

- Passband: 950-1000 MHz, Return loss: 15 dB
- Stopband: 1020-1100, Amin: 50 dB

These requirements can be satisfied with a fully canonical response of order 3.

- Technology: Stripline with air as dielectric

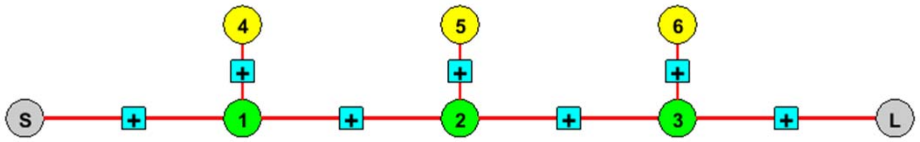


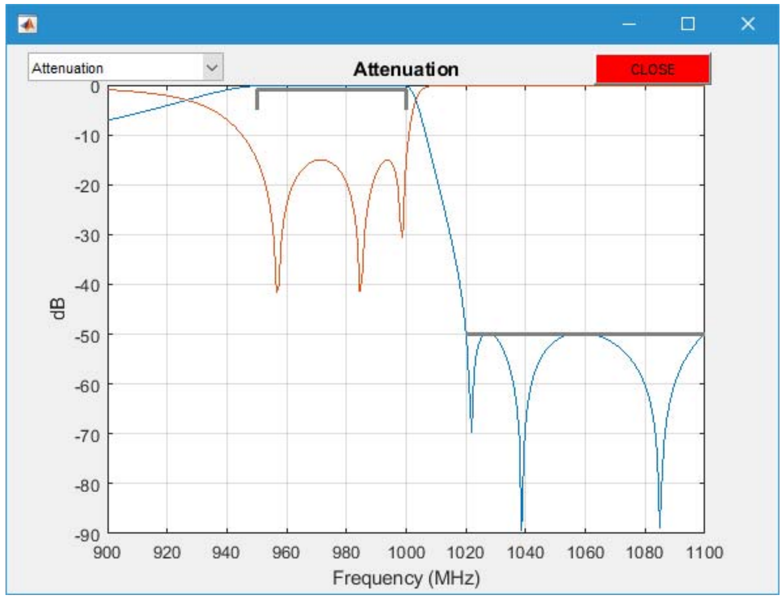
Er=1
B=5mm
T=1mm

Evaluation of the Universal Parameters (SynFil)

Node	Resonance Frequency (MHz)
1	-NRN
2	-NRN
3	-NRN
4	1085.0416
5	1021.6851
6	1038.7455

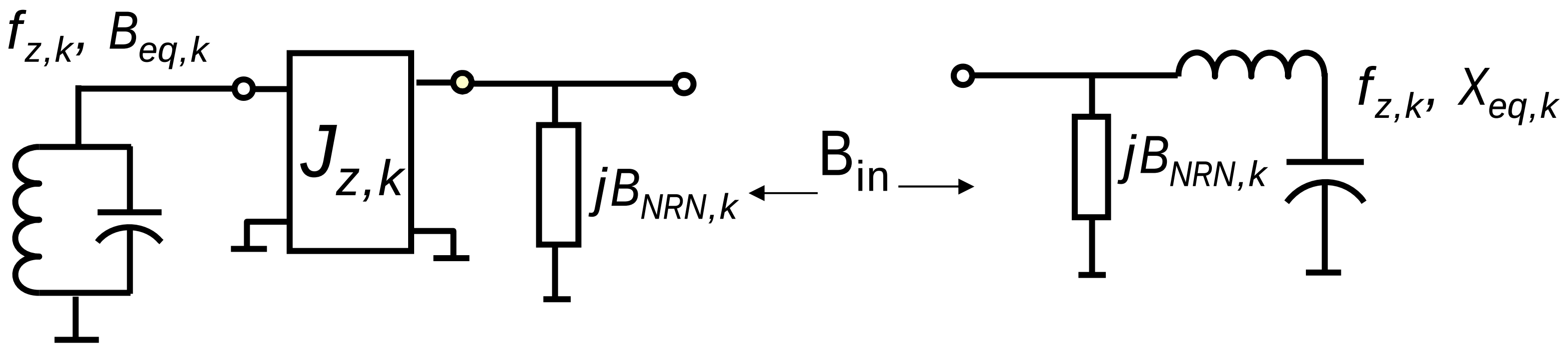
Coupling Coefficient	Nodes
0.20649	S - 1
0.31486	1 - 2
0.40819	2 - 3
0.44043	1 - 4
0.26027	2 - 5
0.31789	3 - 6
0.33865	3 - L





Assigning parameters

- The blocks constituted by a NRN coupled to a resonator can be transformed as:



$$k_{z,k}^2 = \frac{J_{z,k}^2}{B_{eq,k}\left|B_{NRN,k}\right|} = \frac{1}{X_{eq,k}\left|B_{NRN,k}\right|} \quad \Rightarrow \quad \left|B_{NRN,k}\right| = \frac{1}{k_{z,k}^2 X_{eq,k}}$$

- We assume to realize the series resonators with $\lambda_{z,k}/4$ open stubs presenting $X_{eq,k}=Z_{c,k}(\pi/4)$. $Z_{c,k}$ is assigned a priori so that the stubs can be physically realized (<100 Ω)

Main inverters

- Main inverters are supposed to be realized by $\lambda_0/4$ transmission line sections whose characteristic impedance is related to NRNs by the following relations:

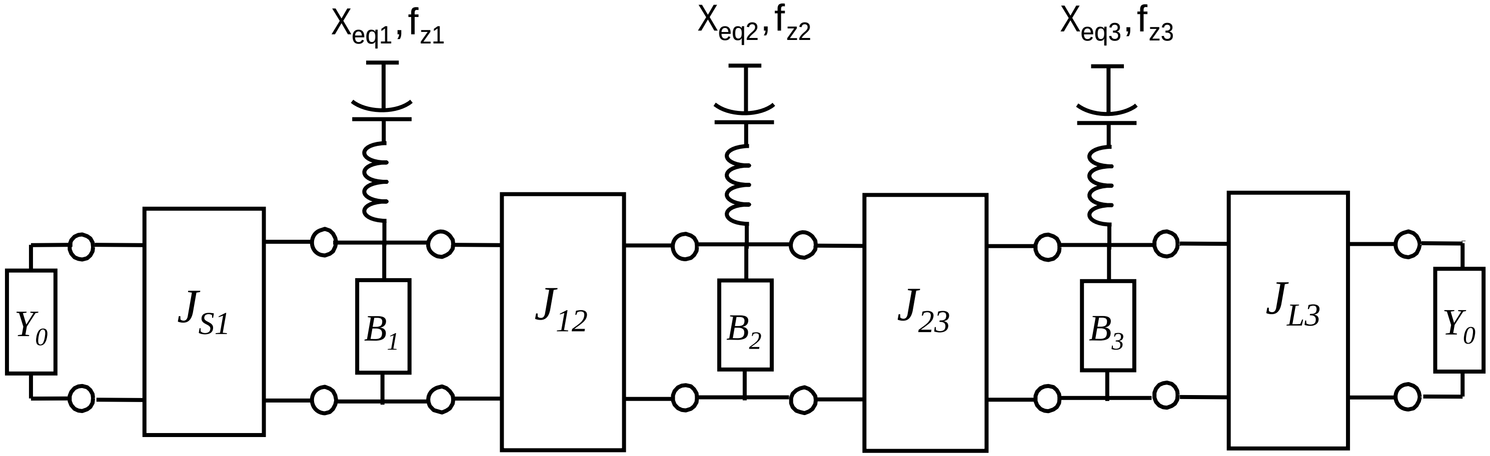
$$k_{i,i+1} = \frac{Y_{c,i,i+1}}{\sqrt{\left|B_{NRN,i} \cdot B_{NRN,i}\right|}}, \quad k_{S1} = \frac{Z_0 Y_{c,S1}^2}{\left|B_{NRN,1}\right|}, \quad k_{L3} = \frac{Z_0 Y_{c,L3}^2}{\left|B_{NRN,3}\right|}$$

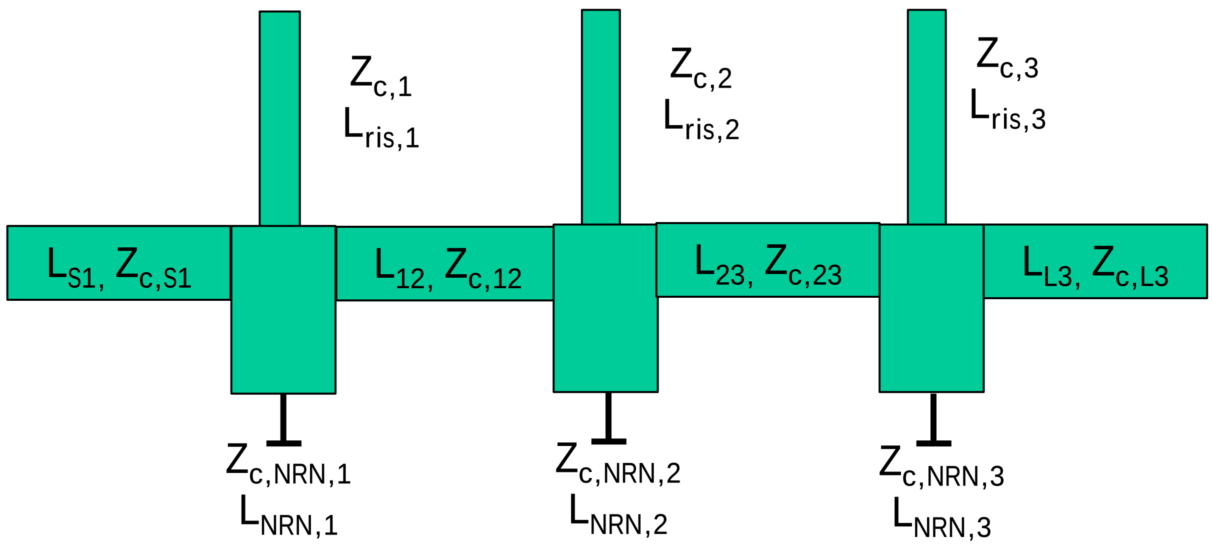
$$\vdots$$

$$Z_{c,i,i+1} = \frac{1}{k_{i,i+1}\sqrt{\left|B_{NRN,i} \cdot B_{NRN,i}\right|}}, \quad Z_{c,S1} = \sqrt{\frac{Z_0}{\left|B_{NRN,1}\right| k_{S1}}}, \quad Z_{c,L3} = \sqrt{\frac{Z_0}{\left|B_{NRN,3}\right| k_{L3}}}$$

- The above impedances must also be realizable in the considered technology ($15\ \Omega < Zc < 150\ \Omega$)
- The NRNs ($B_{NRN,i}$) are implemented as short circuited stubs with assigned Zc and length determining the required susceptance value.

Filter overall scheme





Parameters evaluation

- We assign $Z_{c,1}$ so that $Z_{c,S1}$ is equal to 50 (this allow to remove the first inverter). We get:

$$\left|B_{NRN,1}\right| = \frac{Z_0}{Z_{c,S1}^2 k_{S1}} = \frac{1}{Z_0 k_{S1}} = 0.0969, \quad X_{eq,1} = \frac{1}{k_{z,1}^2 \left|B_{NRN,1}\right|} = 53.2249, \;\; Z_{c,1} = \frac{4}{\pi} X_{eq,1} = 67.7681$$

- Applying the same for $Z_{c,L3}$ we discover $Z_{c,3}$=213.3425, outside the realizable range of Zc. The last inverter cannot then be removed. Assigning $Z_{c,3}$=100 we get:

$$X_{eq,3} = \frac{\pi}{4} Z_{c,3} = 78.54, \quad \left|B_{NRN,3}\right| = \frac{1}{X_{eq,3} k_{z,3}^2} = 0.126, \quad Z_{c,L3} = \sqrt{\frac{Z_0}{\left|B_{NRN,3}\right| k_{L3}}} = 34.232$$

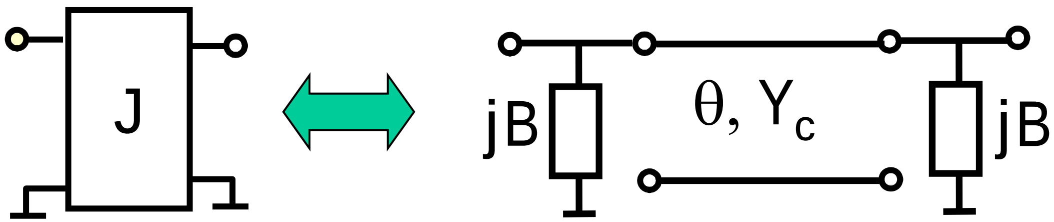
- Assigning $Z_{c,2}$=100, we get B_{nrn3}, $Z_{c,12}$ and $Z_{c,23}$:

$$X_{eq,2} = \frac{\pi}{4} Z_{c,2} = 78.54, \quad \left|B_{NRN,2}\right| = \frac{1}{X_{eq,2} k_{z,2}^2} = 0.188, \quad Z_{c,12} = \frac{1}{k_{12}\sqrt{\left|B_{NRN,1} \cdot B_{NRN,2}\right|}} = 23.539$$

$$Z_{c,23} = \frac{1}{k_{23}\sqrt{\left|B_{NRN,2} \cdot B_{NRN,3}\right|}} = 15.9195$$

Transformation of $\lambda_0/4$ inverters

- It can be observed that $Z_{c,12}$ and $Z_{c,23}$ are difficult to implement. We than apply the following transformation:



$$B = J\cos(\theta)$$

$$Y_C = J \cdot \sin(\theta)$$

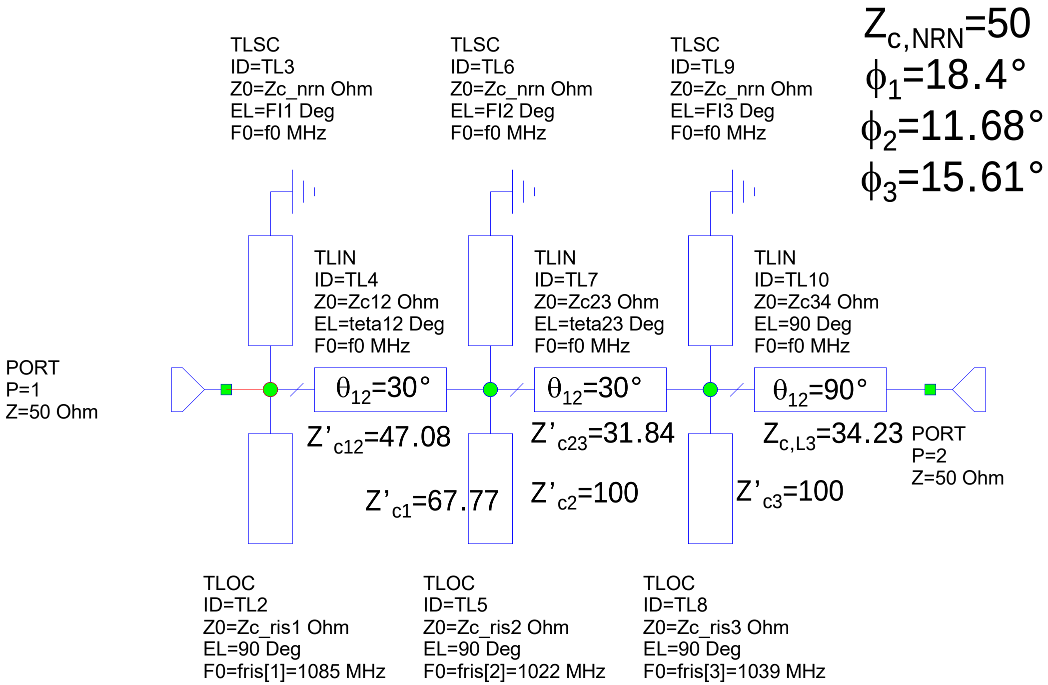
- Imposing θ=30° we get:
  B_{12}=0.0368, Z'_{c12}=47.0778, B_{23}=0.0544, Z'_{c23}=31.839
- The new susceptances add to B_{nrn} giving:

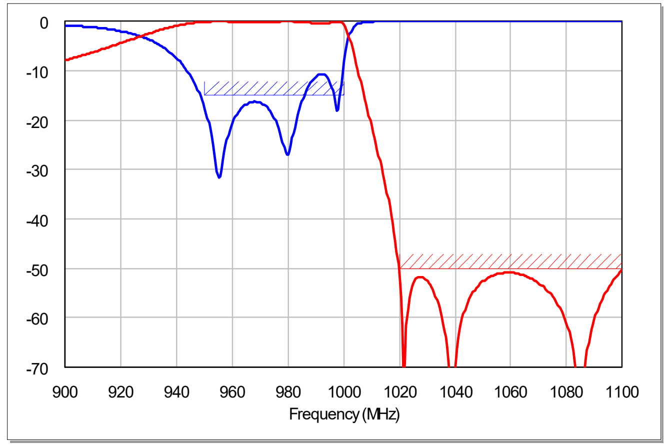
$$B_1 = B_{NRN,1} + B_{12} = \text{-}0.0601, B_2 = B_{NRN,2} + B_{12} + B_{23} = \text{-}0.0968,$$

$$B_3 = B_{NRN,3} + B_{23} = \text{-}0.0716$$

- We implement the B_i with short circuited stub with Zc=50 Ω and electrical length: ϕ_1=18.4°, ϕ_2=11.68°, ϕ_3=15.61°

Resulting ideal scheme

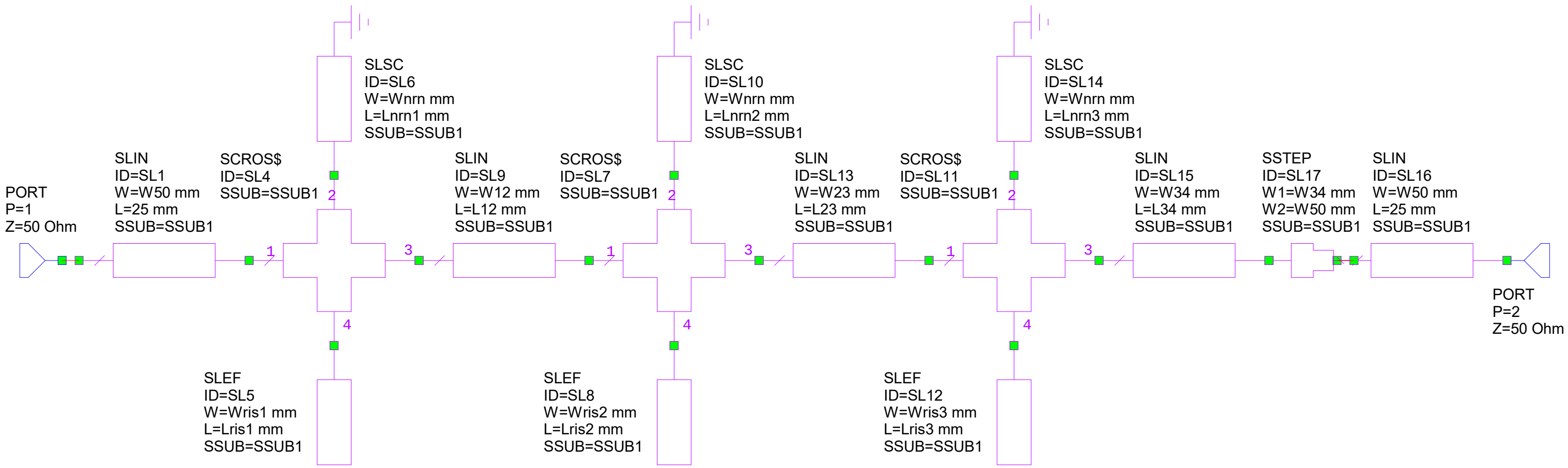


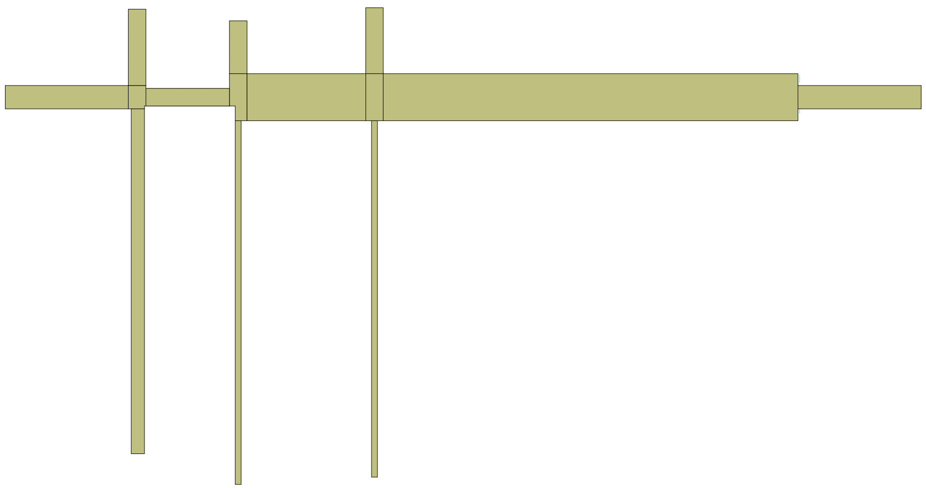


The not perfect response in the passband is due to the lines frequency dispersion

- The length of the stubs (NRNs) and main lines is referred to f_0 (974.6794 MHz). That of open circuited stub is 90° at the resonance frequencies (TZs)

Stripline Implementation





Final response (tuned)