Design of various types of all-pole microwave filters

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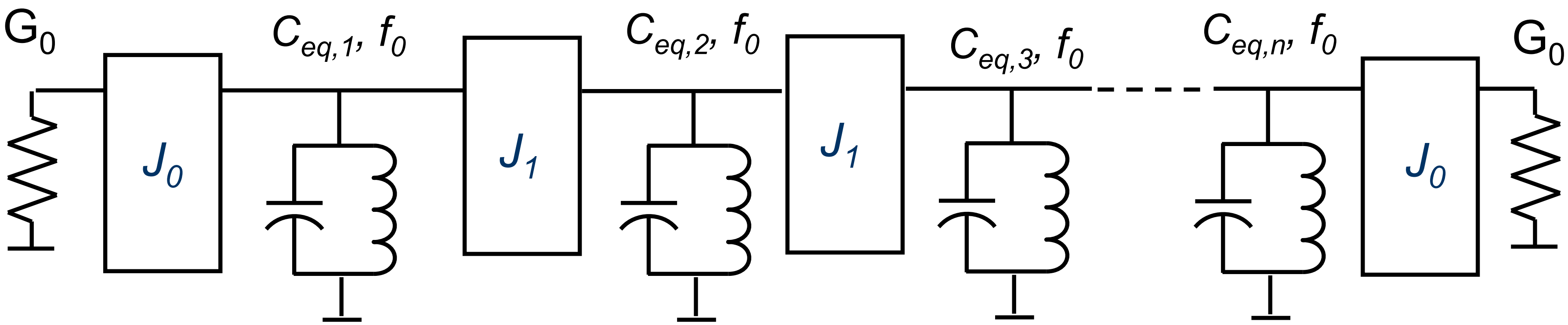
In-line filters with assigned inverters

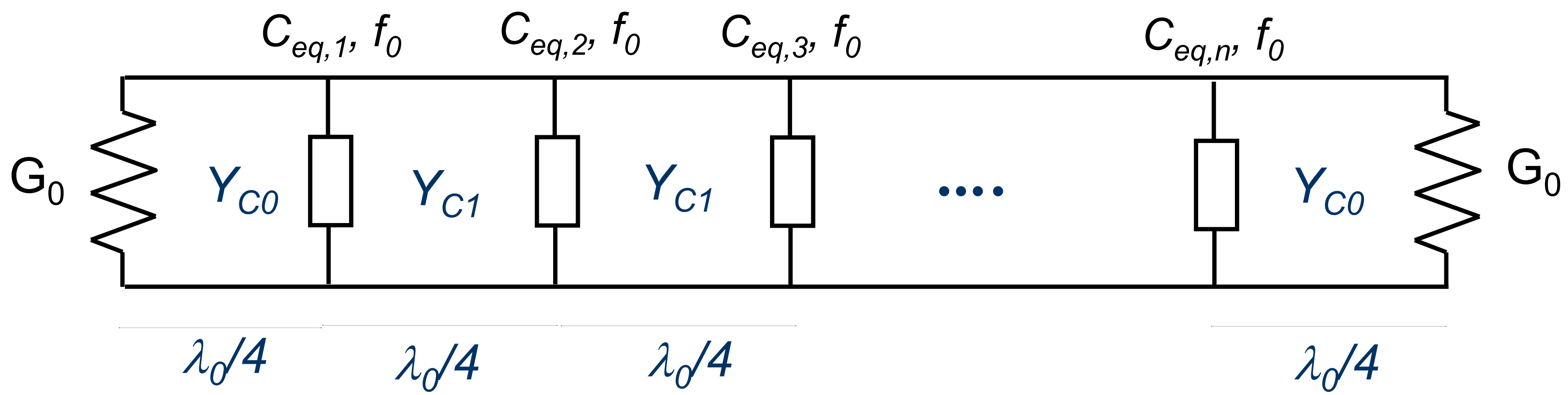


$$J_0 = \sqrt{G_0 \cdot J_1}$$

Each resonator is loaded with two conductances with value J_1.The loaded Q of *q-th* resonator is obtained from the general design equations:

$$Q_q = \frac{\omega_0 C_{eq,q}}{2J_1} = \frac{1}{2}\frac{g_q}{B/f_0} \quad \Rightarrow \quad \omega_0 C_{eq,q} = J_1 \frac{g_q}{B/f_0}$$

Inverters with $\lambda_0/4$ line sections

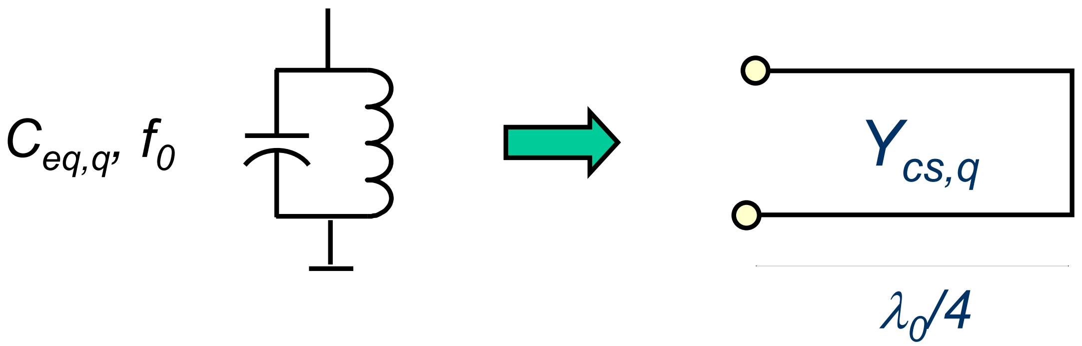


1) Assign $Y_{c1} = J_1$

2) Compute $Y_{c0} = \sqrt{G_0 \cdot Y_{c1}}$

3) Compute $\omega_0 C_{eq,q} = 2Y_{c1} \cdot Q_q = Y_{c1} \dfrac{g_q}{\left(B/f_0\right)}$

Resonators with shorted transmission lines



$$\omega_0 C_{eq,q} = \frac{\pi}{4} Y_{cs,q}$$

$$Y_{cs,q} = Y_{c1} \frac{4 g_q}{\pi \left(B / f_0 \right)}$$

<u>NOTE</u>: The choice of Y_{c1} affects the computed values of $Y_{cs,q}$. These latter must be physically realizable with the fabrication technology of the filter

Example: filter in microstrip technology

Let assume the following specifications:

f_0=1 GHz, B/f_0=0.1, RL=15 dB, n=5

Assuming the Chebycheff characteristic:

g={1.2327 1.3592 2.0599 1.3592 1.2327}

Now suppose that the range of realizable characteristic impedance is 20-150 Ohm.

Then we assume Y_{c1}=1/150=0.0067. Using the previous formula:

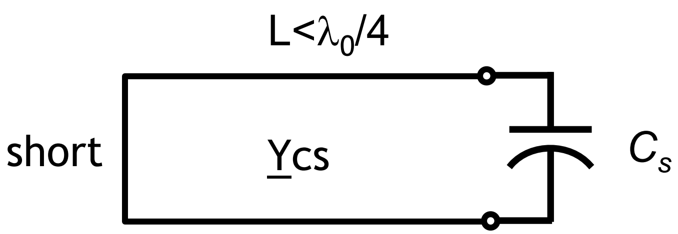
$Y_{cs,q}$={0.1046 0.1154 0.1748 0.1154 0.1046} ⇨

⇨ 1/$Y_{cs,q}$={9.5570 8.6676 5.7192 8.6676 9.5570}

The computed impedances are not realizable with microstrip technology!

Possible solution to allow the feasibility

- Increase the imposed return loss (but this reduces stopband attenuation)
- Increase the bandwidth (accuracy of design is reduced)
- Use two stubs in parallel with half $Y_{cs,q}$
- Choice a different implementation of resonators (for obtaining smaller $Y_{cs,q}$ for the same equivalent capacitance). For instance, using open-circuited, $\lambda_0/2$ stubs the characteristic impedances are doubled. Another choice is the use of capacity-loaded resonators:



$$\omega_0 C_{eq} = \frac{Y_{CS}}{4}\left[2+\pi\right] \Rightarrow$$

$$Y_{cs,q} = Y_{c1}\frac{4g_q}{\pi\left(B/f_0\right)}\left(\frac{\pi}{2+\pi}\right)$$

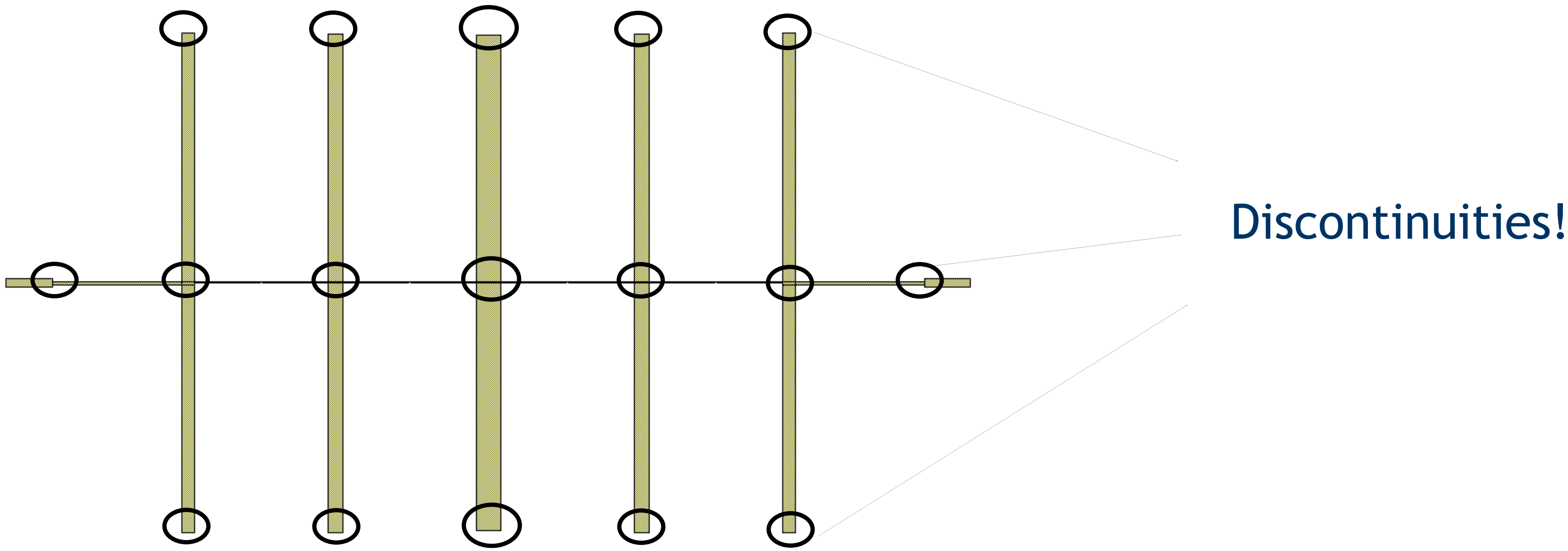
Implementation with double $\lambda_0/2$ resonators

Substrate: Duroid (ε_r=2.16, H=1.2 mm, t=50μ)

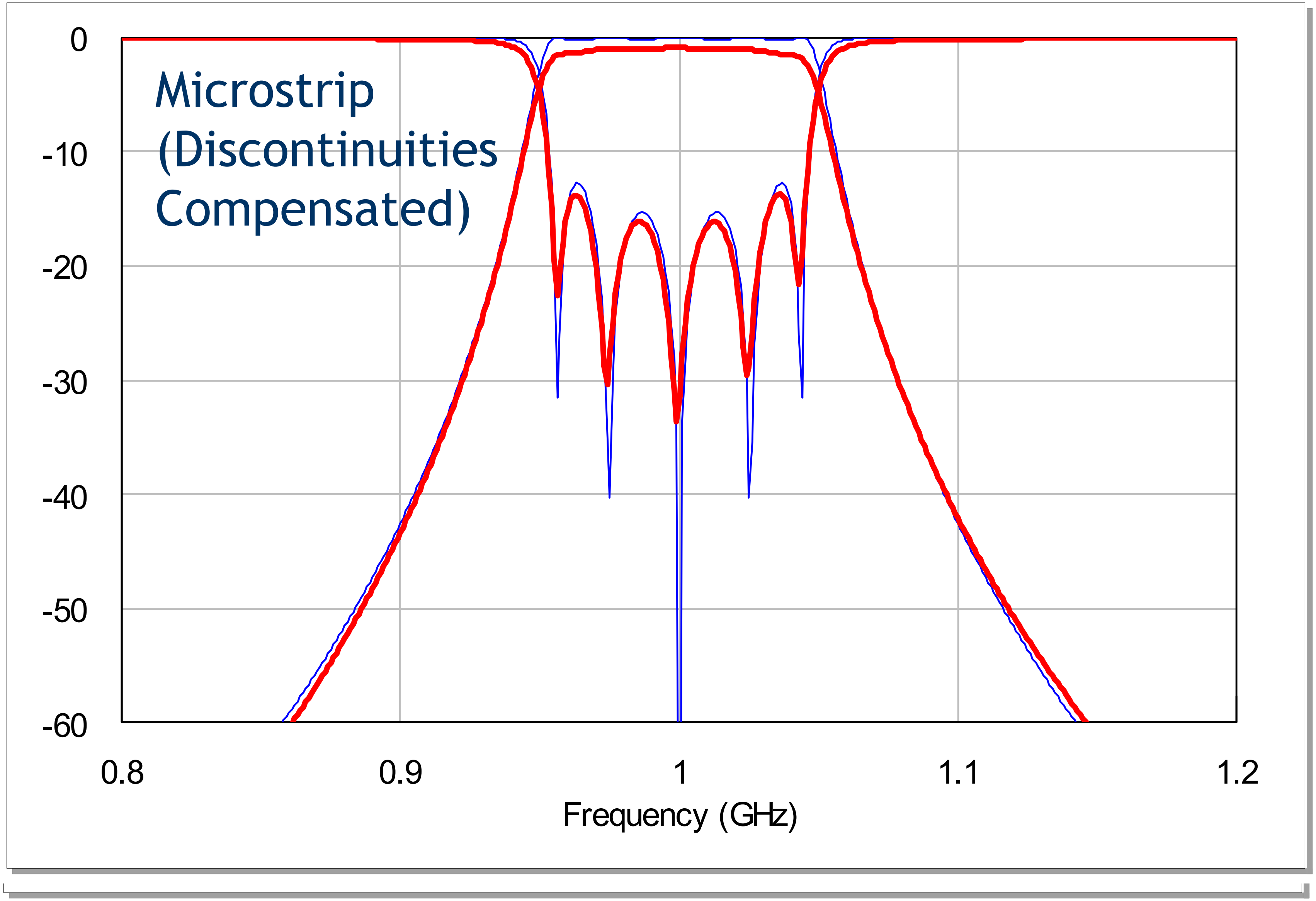
$1/Y_{cs,q}$={38.23, 34.67, 22.88, 34.67, 38.23}

$Y_{c0} = \sqrt{G_0 \cdot Y_{c1}} = 0.0115 \quad (1/Y_{c0} = 86.6)$ $\qquad$ Y_{c1}=1/150=0.0067

Layout:



Computed response



Attenuation produced by losses

An estimate of the overall attenuation at center frequency f_0 can be obtained with the following formula:

$$A_{dB} = \sum_{q=1,n} 20\log\left(1+\frac{Q_q}{Q_0}\right)$$

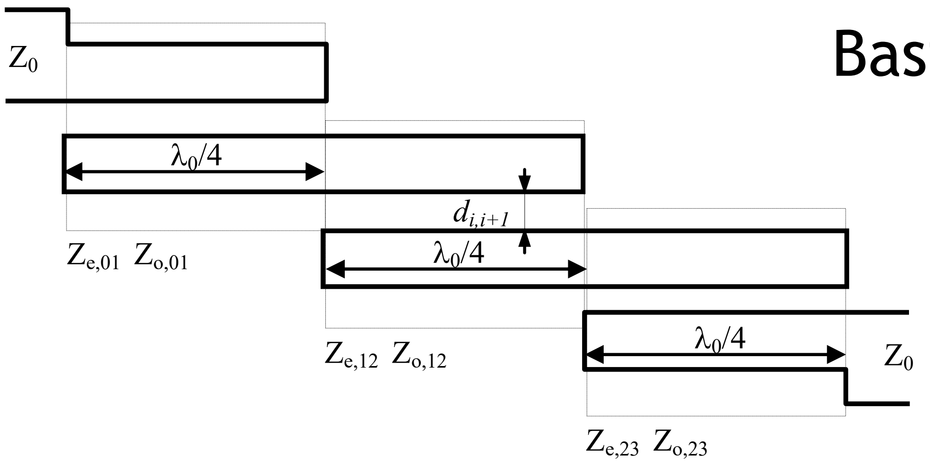
Where Q_q is expressed as function of the prototype parameters g_q:

$$Q_q = \frac{1}{2}\frac{g_q}{B/f_0}$$

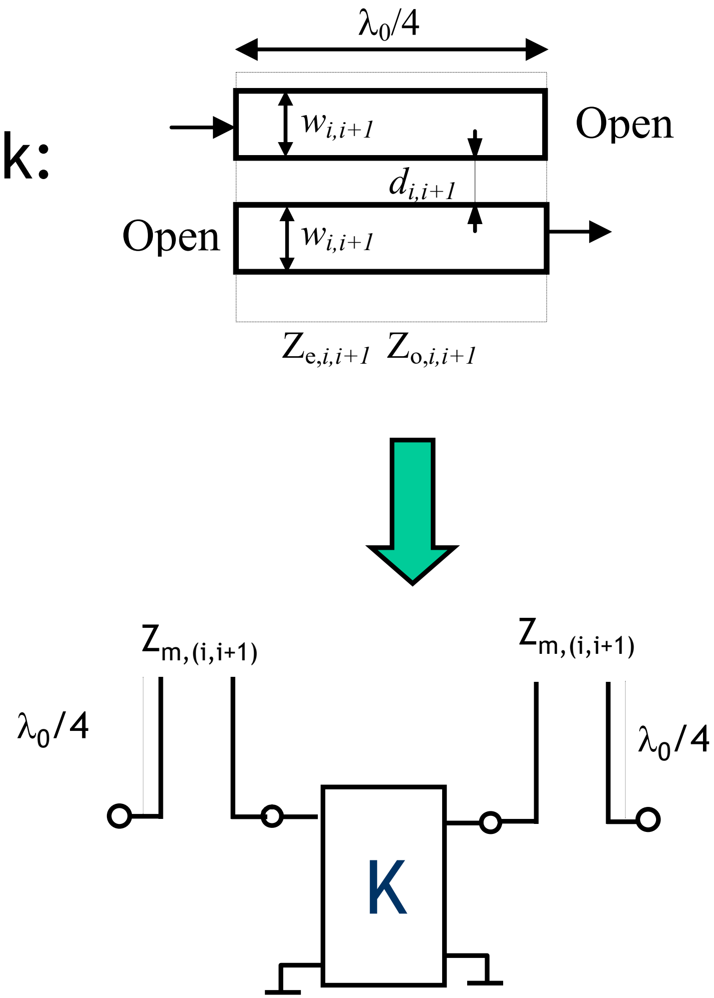
Note: Narrow band filters require high Q_0 resonators to reduce passband attenuation

Coupled-line filters

Physical structure:



Basic Block:

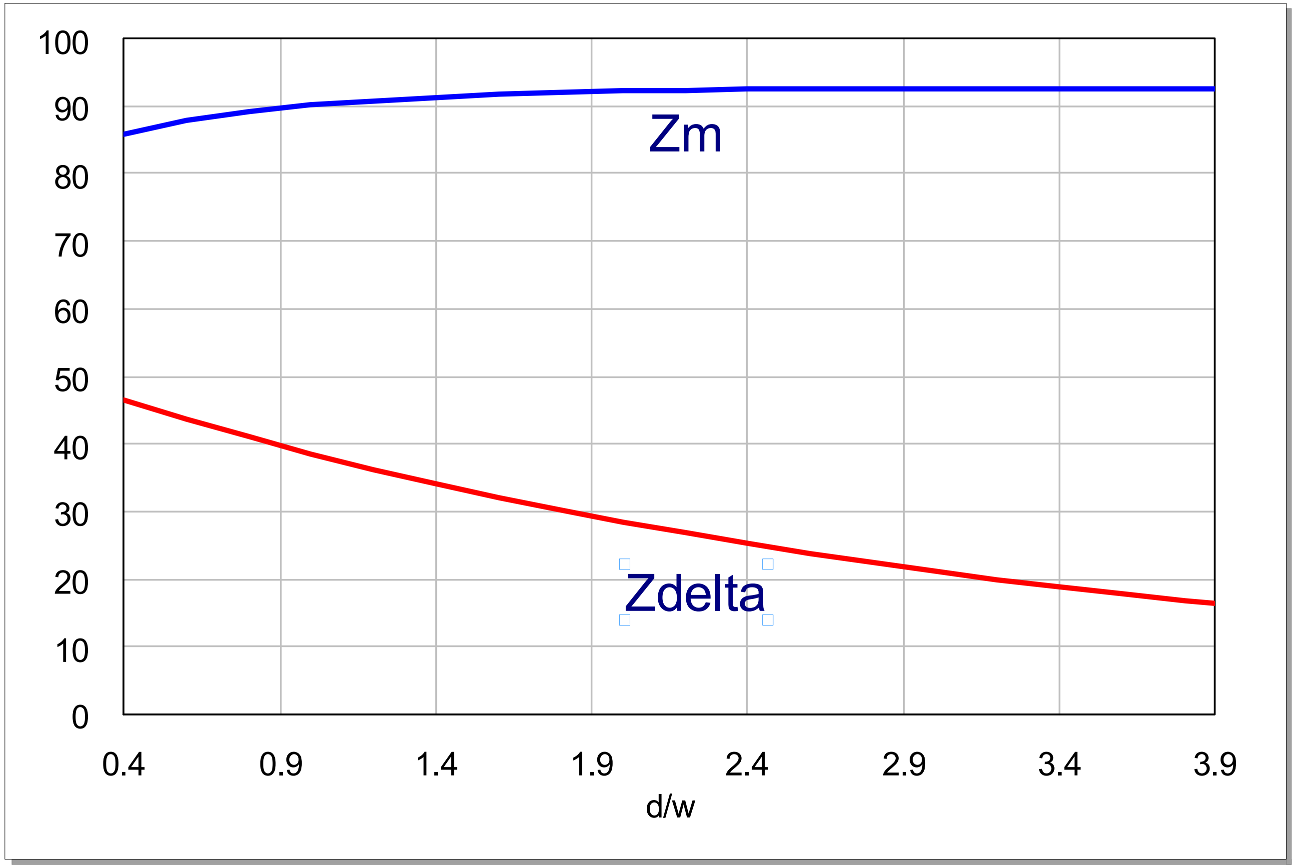


$$Z_{m,(i,i+1)} = \frac{1}{2}\left(Z_{e,(i,i+1)} + Z_{o,(i,i+1)}\right), \qquad m_{i,i+1} = \frac{Z_{e,(i,i+1)} - Z_{o,(i,i+1)}}{Z_{e,(i,i+1)} + Z_{o,(i,i+1)}}$$

$$K_{(i,i+1)} = \frac{Z_{\Delta,(i,i+1)}}{\sin\left(\beta_0 L\right)} = \frac{1}{\sin\left(\beta_0 L\right)}\left(\frac{Z_{e,(i,i+1)} - Z_{o,(i,i+1)}}{2}\right) =$$

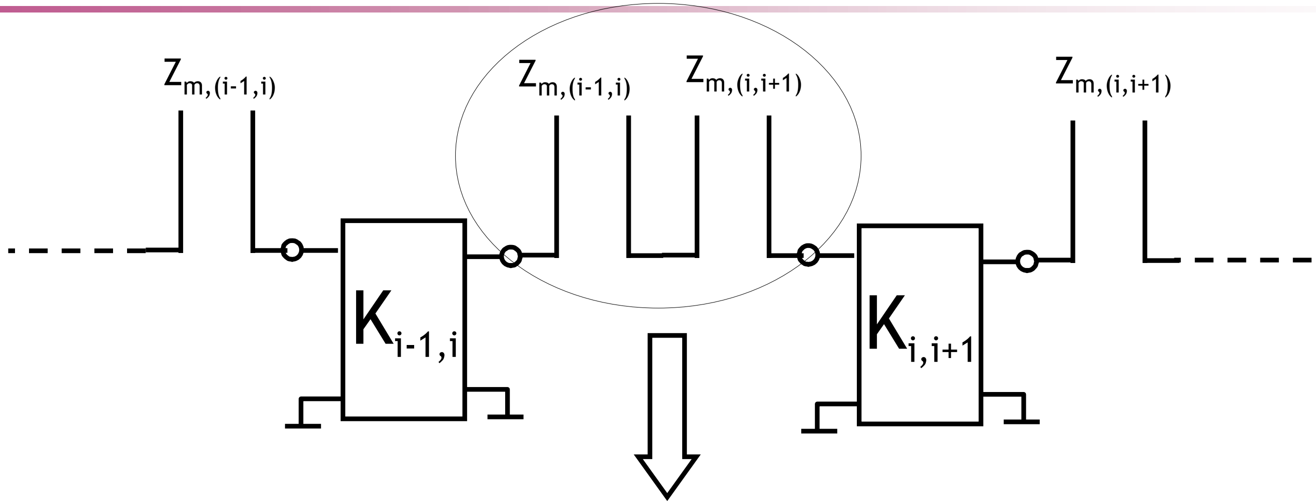
$$K_{i,i+1}\Big|_{\beta_0 L = \frac{\pi}{2}} = m_{i,i+1} Z_{m,(i,i+1)}$$

Typical dependance of Z_m and Z_Δ on *d/w*

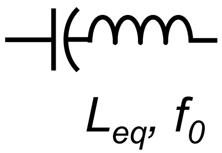


For *d/w* >1 Zm is practically independent on *d* for assigned *w* and practically coincides with the characteristic impedance of the isolated line with the same *w*.

Equivalent circuit (inner blocks)



i-th resonator

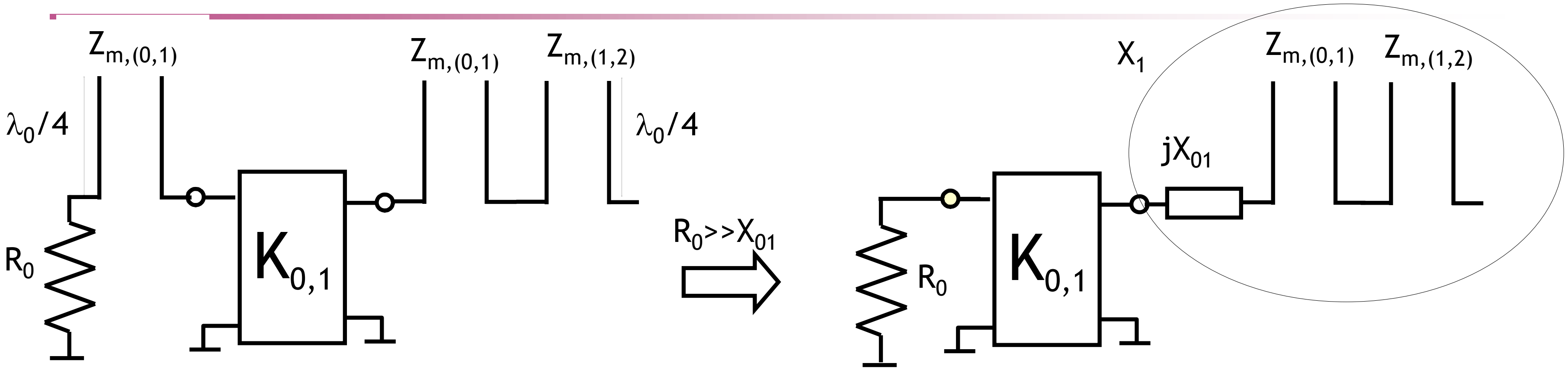


$$\Rightarrow \quad \omega_0 L_{eq,i} = \frac{\pi}{4}\left(Z_{m,(i-1,i)} + Z_{m,(i,i+1)}\right) \simeq \frac{\pi}{2} Z_c$$

Design Equations:

$$k_{q,q+1} = \frac{K_{q,q+1}}{\sqrt{\left(\omega_0 L_{eq,q}\right)\cdot\left(\omega_0 L_{eq,q+1}\right)}} = \frac{2}{\pi}\frac{Z_{\Delta,(i,i+1)}}{Z_{m,(i,i+1)}} = \frac{2}{\pi} m_{i,i+1} \Rightarrow \boxed{m_{i,i+1} = \frac{\pi}{2} k_{i,i+1}}$$

First (last) block



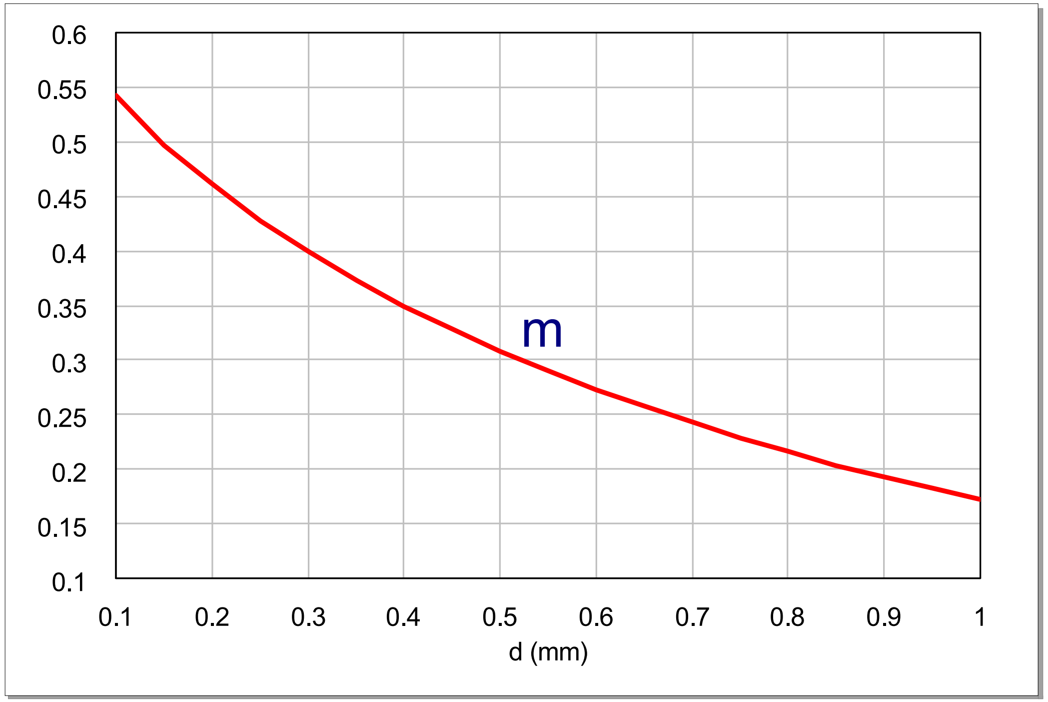
$$Q_E = \frac{\omega_0 L_{eq,1}}{K_{0,1}^2 / R_0} = \frac{\omega_0 L_{eq,1}}{m_{0,1}^2 \cdot Z_{m,(0,1)}^2} R_0 \qquad X_{01} = \left(\frac{K_{01}}{R_0}\right)^2 \left(Z_{m,(0,1)} \cot\left(\frac{\pi}{2}\frac{f}{f_0}\right)\right)$$

$$X_1 = X_{01} - \cot\left(\frac{\pi}{2}\frac{f}{f_0}\right)\left(Z_{m,(0,1)} + Z_{m,(1,2)}\right) \simeq -\left[2 - \left(\frac{K_{01}}{R_0}\right)^2\right] Z_c \cot\left(\frac{\pi}{2}\frac{f}{f_0}\right)$$

$$\omega_0 L_{eq,1} \simeq \frac{(\pi/2) Z_c}{1 + \frac{\pi}{4}\frac{Z_c}{R_0 \cdot Q_E}}, \quad m_{0,1} \simeq \frac{\sqrt{\left(\omega_0 L_{eq,1} \cdot R_0\right)/Q_E}}{Z_c}, \quad m_{1,2} \simeq \frac{(\pi/2) k_{1,2}}{\sqrt{1 + \frac{\pi}{4}\frac{Z_c}{R_0 \cdot Q_E}}}$$

Dimensioning of the filter

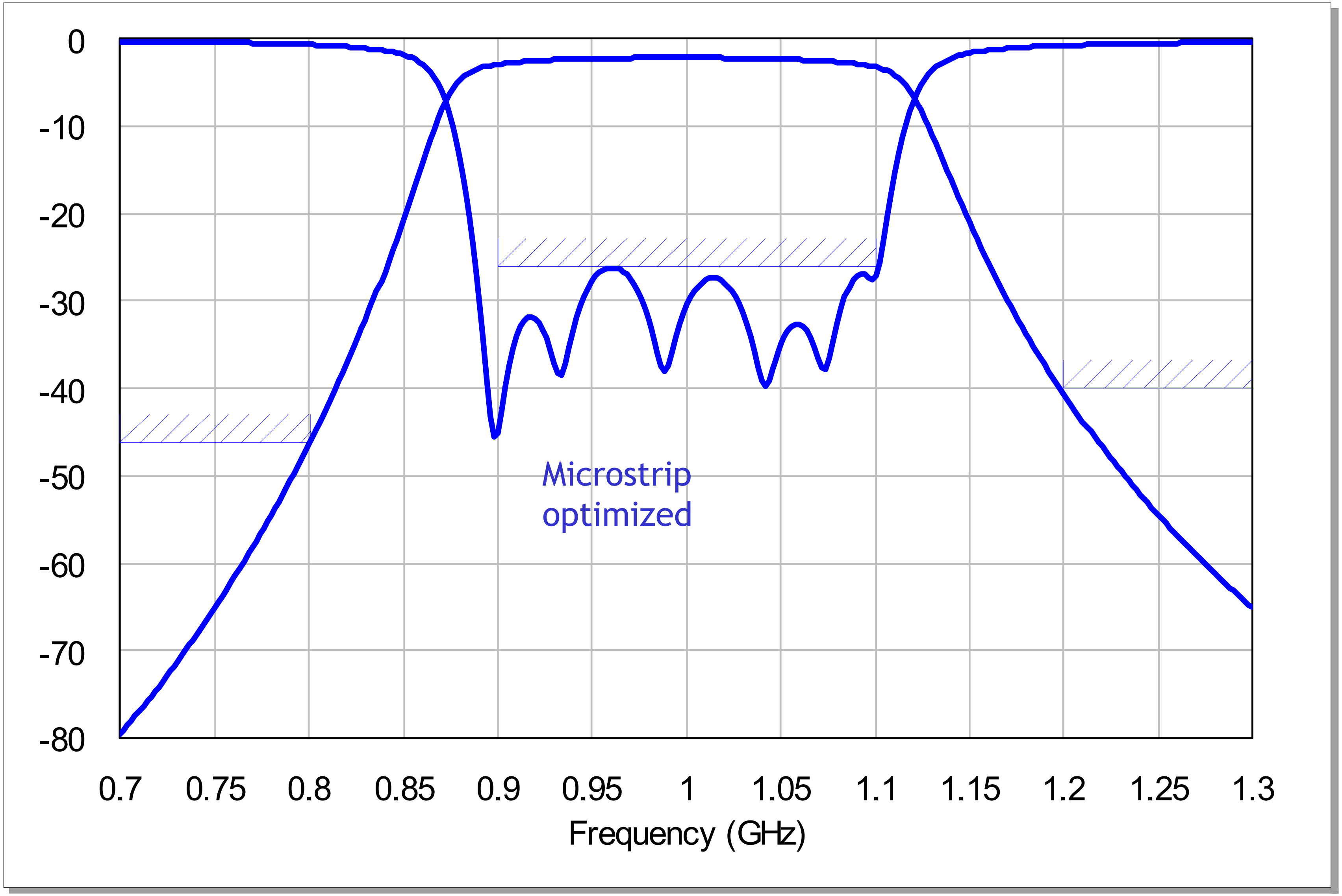
1. The dimension *w* of the lines is first assigned
2. The values of $m_{i,i+1}$ are computed with the previous formulas
3. Using a graph of *m* vs. *d* (like the one shown below) the distances $d_{i,i+1}$ are evaluated



Example of microstrip filter design

- Specifications:
  f0=1 GHz, B=200 MHz, RL=26 dB, n=7
  g={0.807, 1.397, 1.758, 1.634, 1.758, 1.397, 0.807}
- Assigned substrate parameters:
  ε_r=10, H=1.2mm, t=10μ
- Assigned width of lines (for Zc≈85 Ohm): w=0.25 mm
- Computed $m_{i,i+1}$:
  m={0.415, 0.258, 0.201, 0.186 , 0.186, 0.201, 0.258, 0.415}
- Computed distances di,i+1 (with the previous graph):
  d={0.272, 0.649, 0.861, 0.93, 0.93, 0.861, 0.649, 0.272}
- Evaluation of length of blocks (about $\lambda_0/4$ for a not coupled line): L=30.44mm

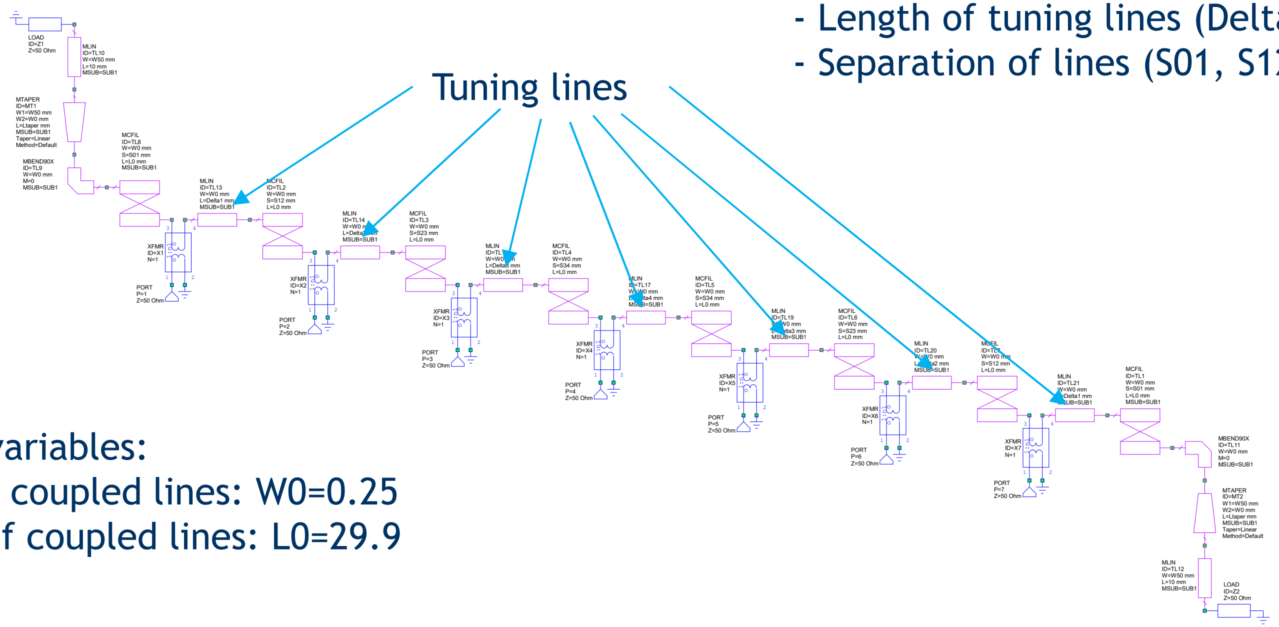
Computed filter response (circuit model)



Design of the filter with MWOffice

N-port model:

Design variables:
- Length of tuning lines (Delta1-Delta4)
- Separation of lines (S01, S12, S23, S34)



Assigned variables:
- Width of coupled lines: W0=0.25
- Length of coupled lines: L0=29.9

Note: each resonator is a half-wavelength open-circuited resonator. At center a series resonance is observed, then the required port must be connected in series (through e 1:1 transformer)

Output Equations for the dimensioning

```
Z11 = Filtro Coupled Lines N porte:Z(1,1)
Z22 = Filtro Coupled Lines N porte:Z(2,2)
Z33 = Filtro Coupled Lines N porte:Z(3,3)
Z44 = Filtro Coupled Lines N porte:Z(4,4)

Z12 = Filtro Coupled Lines N porte:Z(1,2)
Z23 = Filtro Coupled Lines N porte:Z(2,3)
Z34 = Filtro Coupled Lines N porte:Z(3,4)
Z45 = Filtro Coupled Lines N porte:Z(4,5)

X11=imag(Z11)
X22=imag(Z22)
X33=imag(Z33)
X44=imag(Z44)

K12=imag(Z12)
K23=imag(Z23)
K34=imag(Z34)
K45=imag(Z45)

F= FREQ*1e-6
I0=find_index(F,f0)

I1=find_index(X11,0)
I2=find_index(X22,0)
I3=find_index(X33,0)
I4=find_index(X44,0)
```

F[I1]: 995
F[I2]: 995
F[I3]: 995
F[I4]: 995

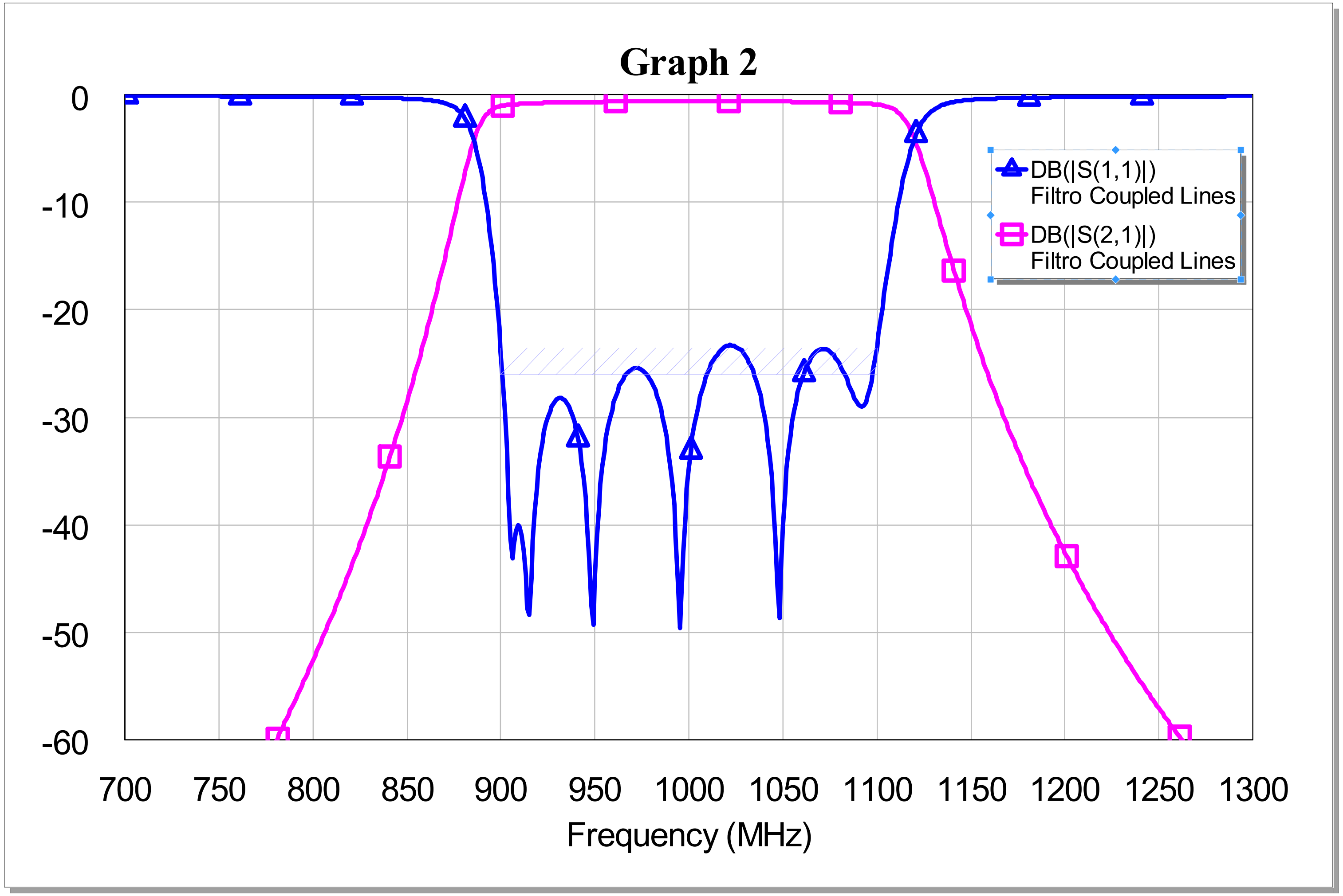
```
Xeq1=0.5*der(X11,F)
Xeq1=Xeq1[I1]*F[I1]
Xeq2=0.5*der(X22,F)
Xeq2=Xeq2[I2]*F[I2]

Xeq3=0.5*der(X33,F)
Xeq3=Xeq3[I3]*F[I3]
Xeq4=0.5*der(X44,F)
Xeq4=Xeq4[I4]*F[I4]
```

Qext=Xeq1/real(Z11[I1])	1/Qext: 0.2489	(0.2491)
k12=K12[I0]/sqrt(Xeq1*Xeq2)	k12: 0.1896	(0.1893)
k23=K23[I0]/sqrt(Xeq3*Xeq2)	k23: 0.1282	(0.1283)
k34=K34[I0]/sqrt(Xeq3*Xeq4)	k34: 0.1189	(0.1186)

Result of design

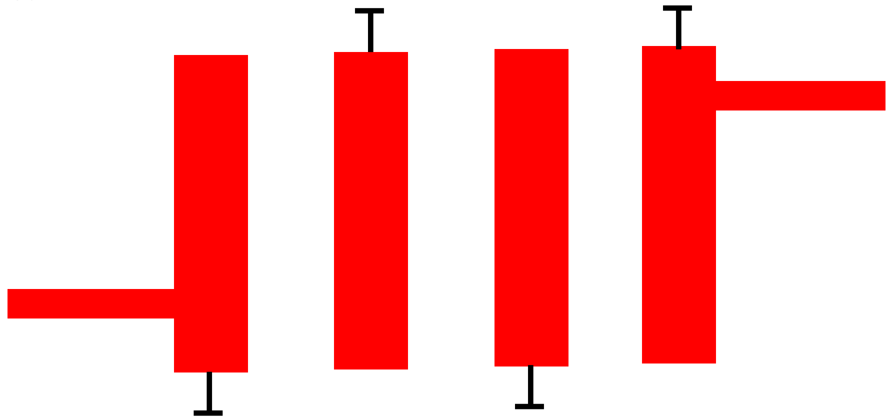
S01=0.3593	Delta1=0.523
S12=0.752	Delta2=0.1
S23=0.9882	Delta3=0.1621
S34=1.068	Delta4=0.1765

Response of the dimensioned filter

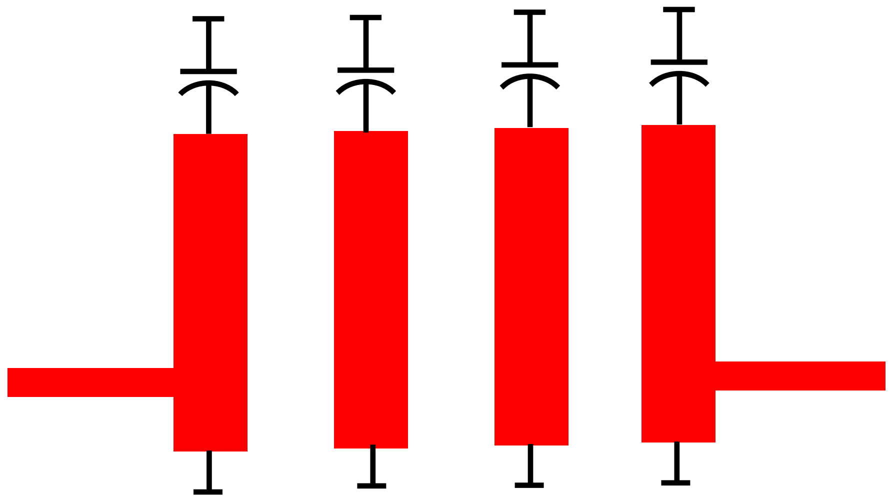


Filters with an array of coupled-lines

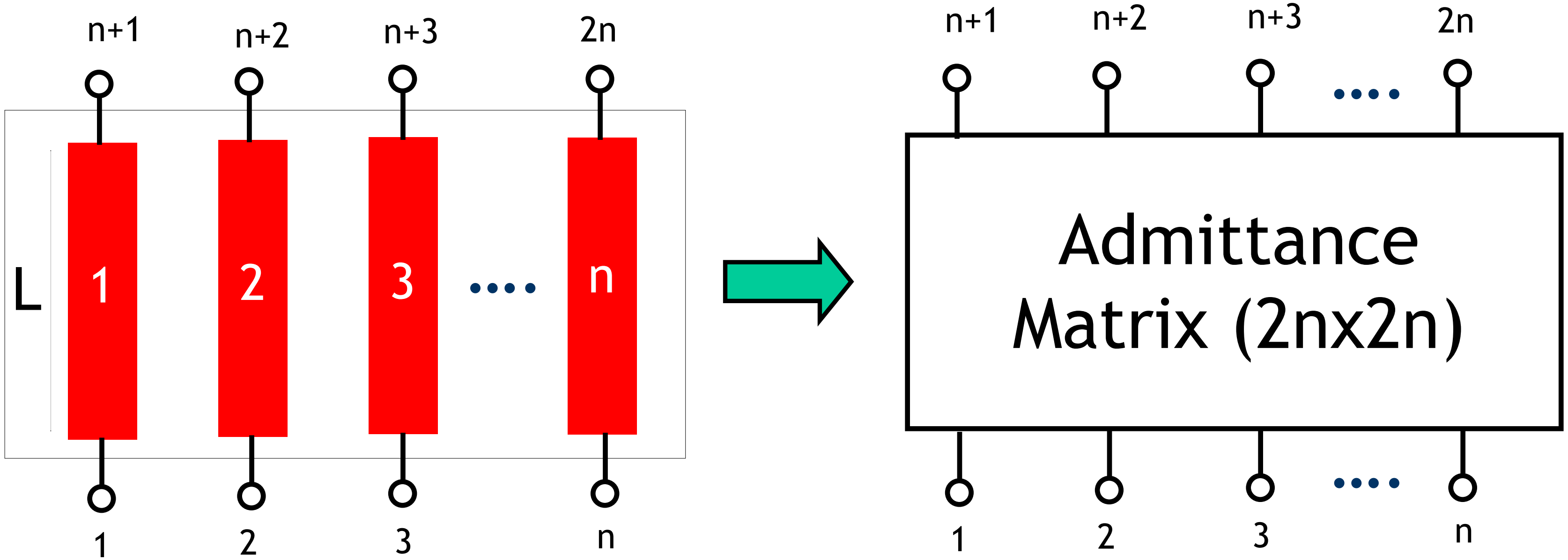
- Interdigital:



- Comb:



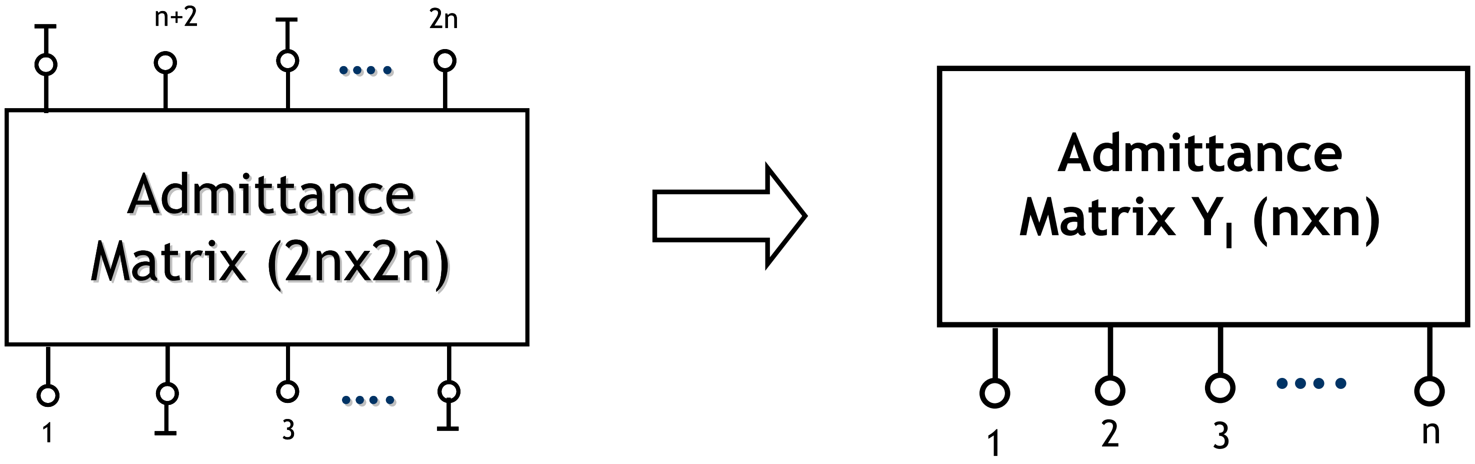
Basic block: array of coupled lines



The admittance matrix **Y** (*2nx2n*) can be evaluated from the static capacitance matrix p.u.l **C** (dimension *n* x *n*):

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}' & \mathbf{Y}'' \\ \mathbf{Y}'' & \mathbf{Y}' \end{bmatrix}, \qquad \mathbf{Y}' = \frac{v \cdot \mathbf{C}}{j\tan(\beta L)}, \qquad \mathbf{Y}'' = \frac{v \cdot \mathbf{C}}{j\sin(\beta L)}$$

Matrix Y of Interdigital Array

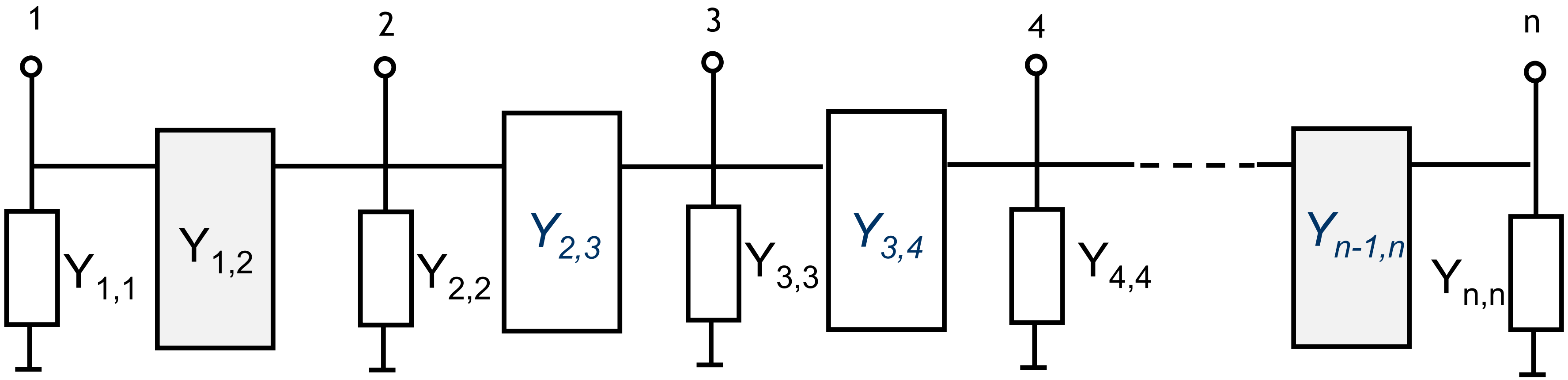


Let assume that $C_{i,j}$=0 for j>i+1 (no coupling between non-adjacent lines). Elements of matrix Y_I are then given by (ν is the propagation velocity):

$$y_I(i,j) = \begin{cases} \dfrac{v \cdot C_{i,i}}{j\tan(\beta L)} & i = j \\ \dfrac{v \cdot C_{i,i+1}}{j\sin(\beta L)} & j = i+1 \\ 0 & j > i+1 \end{cases}$$

Matrix Y_I is **tri-diagonal**

Condition on the Y matrix to represent a filter



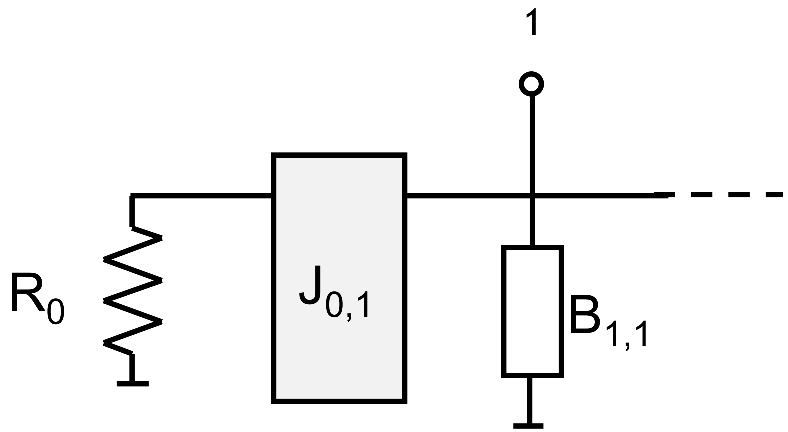
Equivalent parameters ($Y_{i,j}=jB_{i,j}$):

$$B_{i,i}(\omega_0) = -\frac{v \cdot C_{i,i}}{\tan(\beta_0 L)} = 0 \quad \Rightarrow \quad \beta_0 L = \frac{\pi}{2} \quad \Rightarrow \quad L = \frac{\lambda_0}{4}$$

$$J_{i,i+1} = B_{i,i+1}(\omega_0) = -\frac{v \cdot C_{i,i+1}}{\sin(\beta_0 L)} = -vC_{i,i+1}$$

$$\omega_0 C_{eq,i} = \omega_0 \frac{1}{2} \frac{\partial B_{i,i}(\omega)}{\partial \omega}\bigg|_{\omega=\omega_0} = \frac{\pi}{4} v C_{i,i}$$

Coupling with external loads



$$Y_{1,1} = \frac{J_{0,1}^2}{G_0} + jB_{1,1}$$

$$Q_E = \frac{\omega_0 C_{eq,1}}{J_{0,1}^2 / G_0} = \frac{\omega_0 \dfrac{1}{2} \left.\dfrac{\partial B_{1,1}(\omega)}{\partial \omega}\right|_{\omega=\omega_0}}{\mathrm{Re}\left\{Y_{1,1}\right\}_{\omega=\omega_0}} = \frac{\dfrac{\pi}{4} \nu C_{i,i}}{\mathrm{Re}\left\{Y_{1,1}\right\}_{\omega=\omega_0}}$$

Equations for interdigital filters design

$$k_{i,i+1} = k_{i,i+1} = \frac{J_{i,i+1}}{\omega_0 \sqrt{C_{eq,i} \cdot C_{eq,j}}} = \frac{4}{\pi} \frac{C_{i,i+1}}{\sqrt{C_{i,i} C_{i+1,i+1}}}, \qquad Q_E = \frac{\frac{\pi}{4} \nu C_{1,1}}{\mathrm{Re}\left\{Y_{1,1}\right\}_{\omega=\omega_0}}$$

Typically, the coupled lines are assumed of equal width (assigned a priori). The unknowns are then represented by the distances $d_{i,i+1}$ between the lines and by the dimensional parameter (d_e) of the external coupling structure.
The solution is obtained by solving the system of non-linear equations in the variables $d_{i,i+1}$ and d_e :

$$k_{i,i+1} - \frac{4}{\pi} F(d_{i,i+1}) = 0, \qquad Q_E - \frac{\pi}{4} \nu \cdot U\left(d_e, d_{1,2}\right) = 0$$

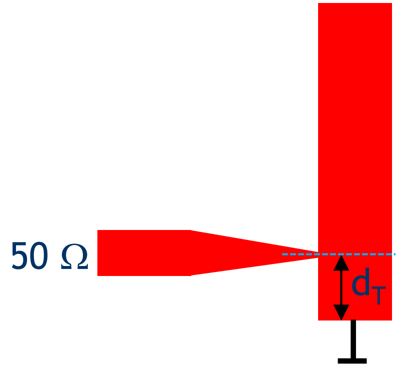
Example of interdigital filter in microstrip

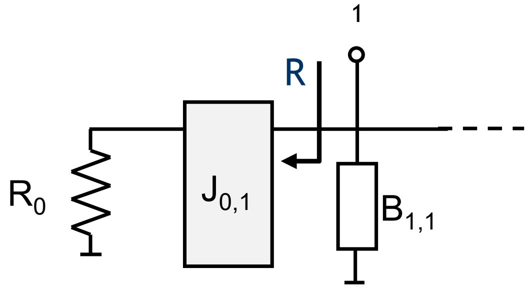
- Passband: 1800-2000 MHz, Return Loss: 18 dB
- Filter order: 5
- Result of the synthesis:
  f_{ris}=1900 MHz,
  $k_{12}=k_{45}=0.087$, $k_{23}=k_{34}=0.0653$, symmetric
  $Q_{ext}=10.1$

- Substrate parameters:
  $\varepsilon_r=2.2$, H=0.508mm, t=35μ

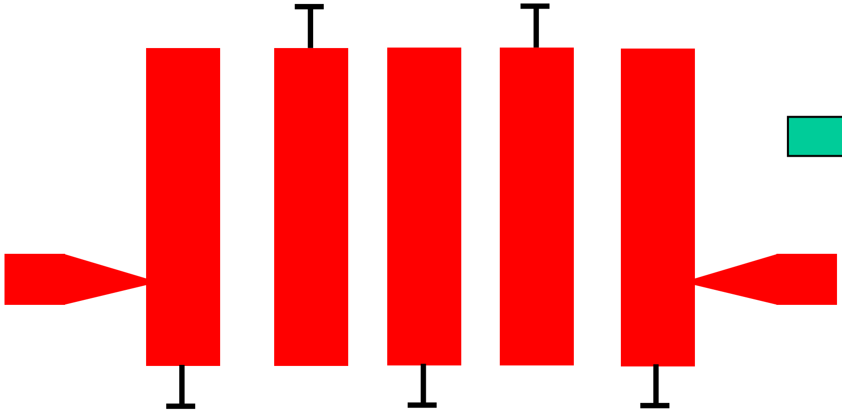
- Assigned dimensions:
  Width of coupled strips: 1mm
  Ideal length of coupled strips: 29.25mm ($\lambda_0/4$)
  Width of 50 Ohm lines: 1.52mm

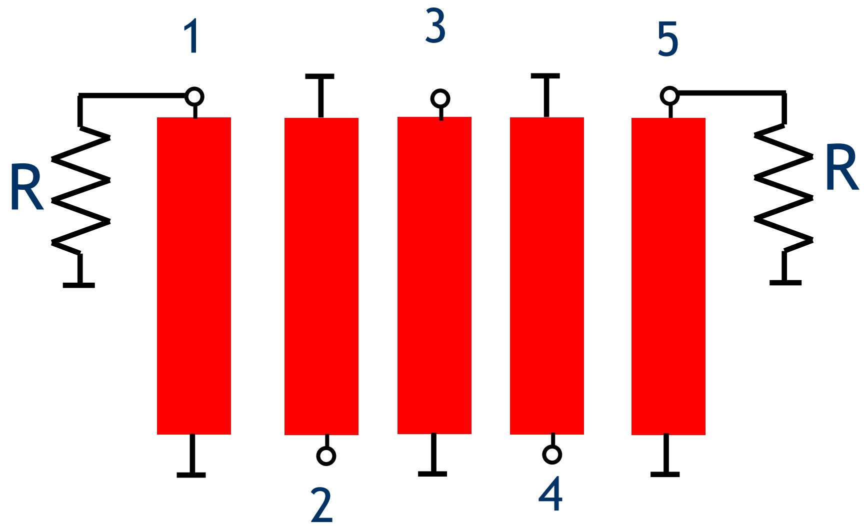
In-out coupling modelling



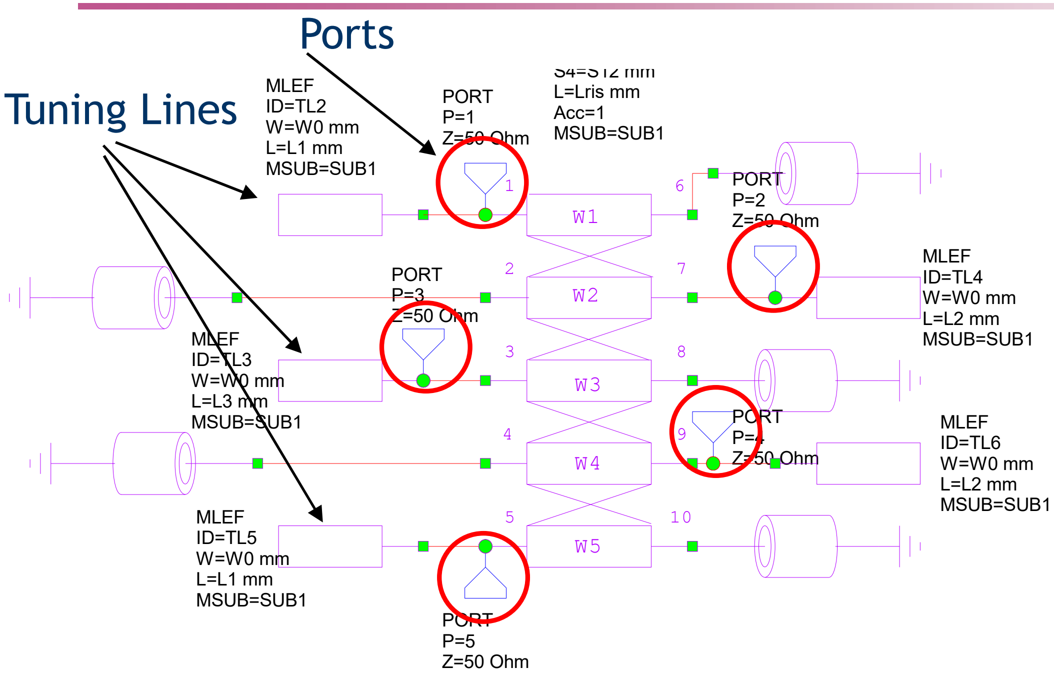


$$Q_E = \frac{B_{eq,1}}{J_{0,1}^2 R_0} \quad \Rightarrow \quad R = \frac{1}{J_{0,1}^2 R_0} = \frac{Q_E}{B_{eq,1}}$$





Extraction of universal parameters



$$B_{eq,i} = \omega_0 C_{eq,i} = \omega_0 \frac{1}{2} \frac{\partial \operatorname{Im}\left[Y_{i,i}(\omega)\right]}{\partial \omega}\bigg|_{\omega=\omega_0},$$

$$J_{i,i+1} = \operatorname{Im}\left[Y_{i,i+1}(\omega_0)\right]$$

Geometrical unknown parameters:
- separation of lines (s_{12},s_{23})
- Length of tuning lines (L_1,L_2,L_3)

Assigned dimensions:
- width of coupled lines (w0=1mm)
- Length of coupled lines (L_0=28mm)

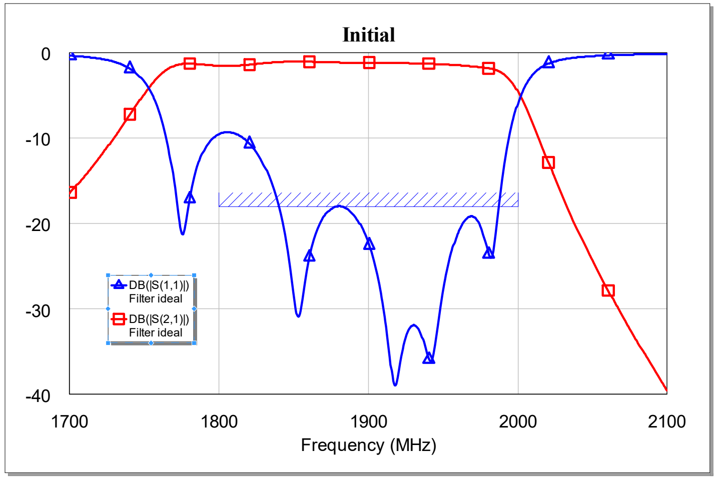
The design is carried out by means of the tuning facility of MWOffice assigning initially L_i=0 and s_{12}=s_{23}=1mm.

Result of design (circuit model)

Once the tuning of the multi-port circuit has been performed, the following values have been computed:

L1=0.238
L2=0.33
L3=0.31

S12=1.092
S23=1.342

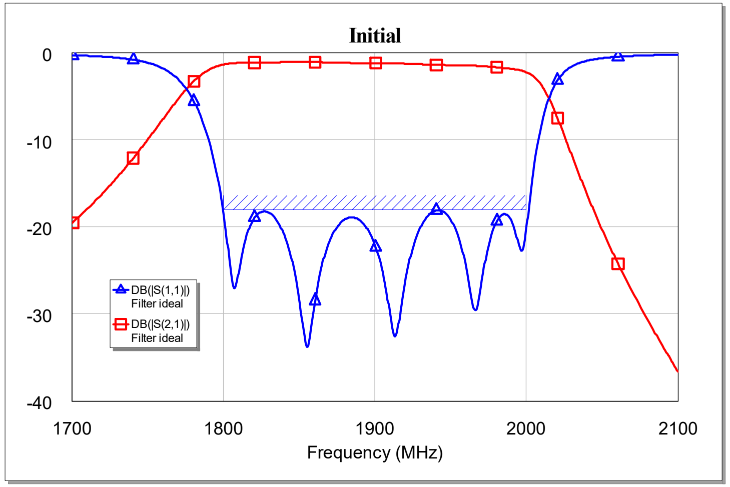


The filter response is however not satisfactory. This is due to the coupling between not adjacent lines not included in the synthesis model. The response of the filter must be improved by manual tuning.

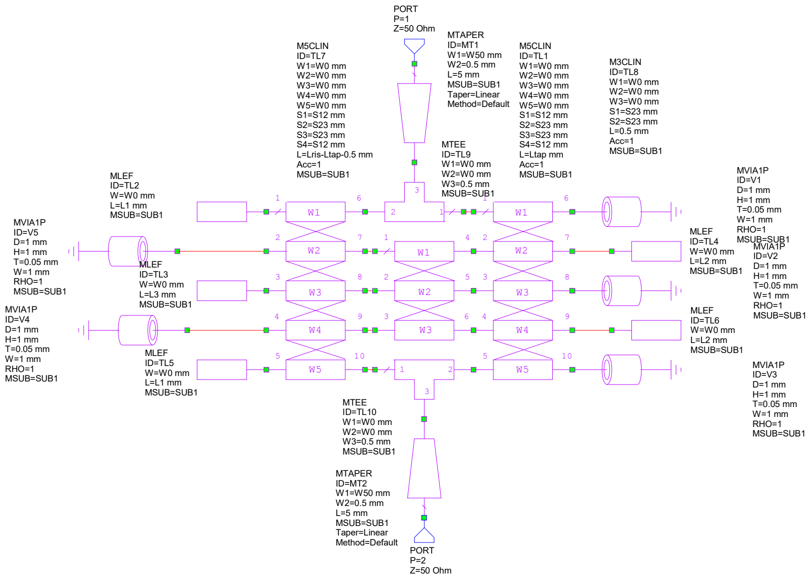
Lris=27.74

L1=0.4155
L2=0.285
L3=0.25

S12=1.1
S23=1.431



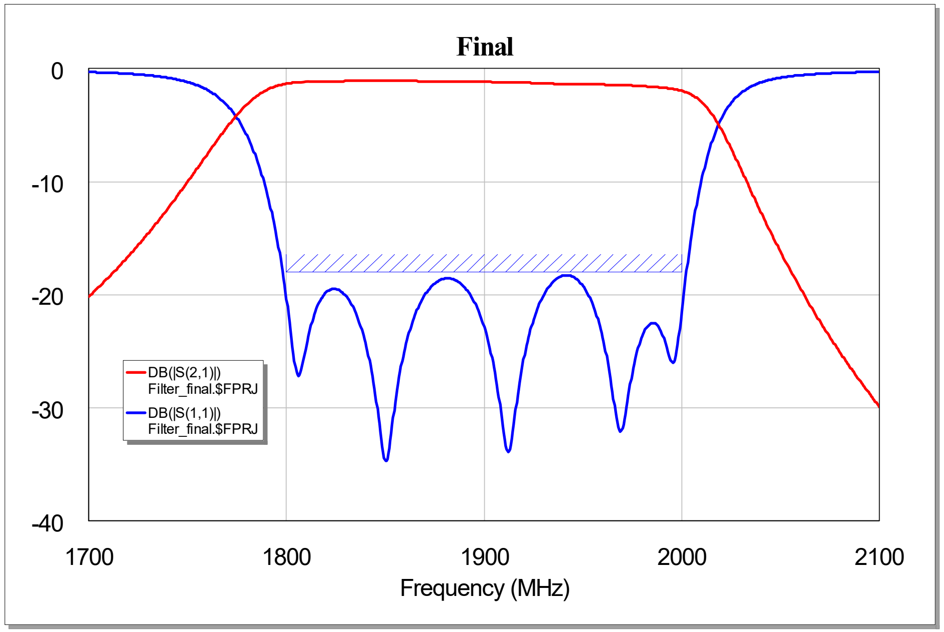
Dimensioning of in-out couplings (tap)



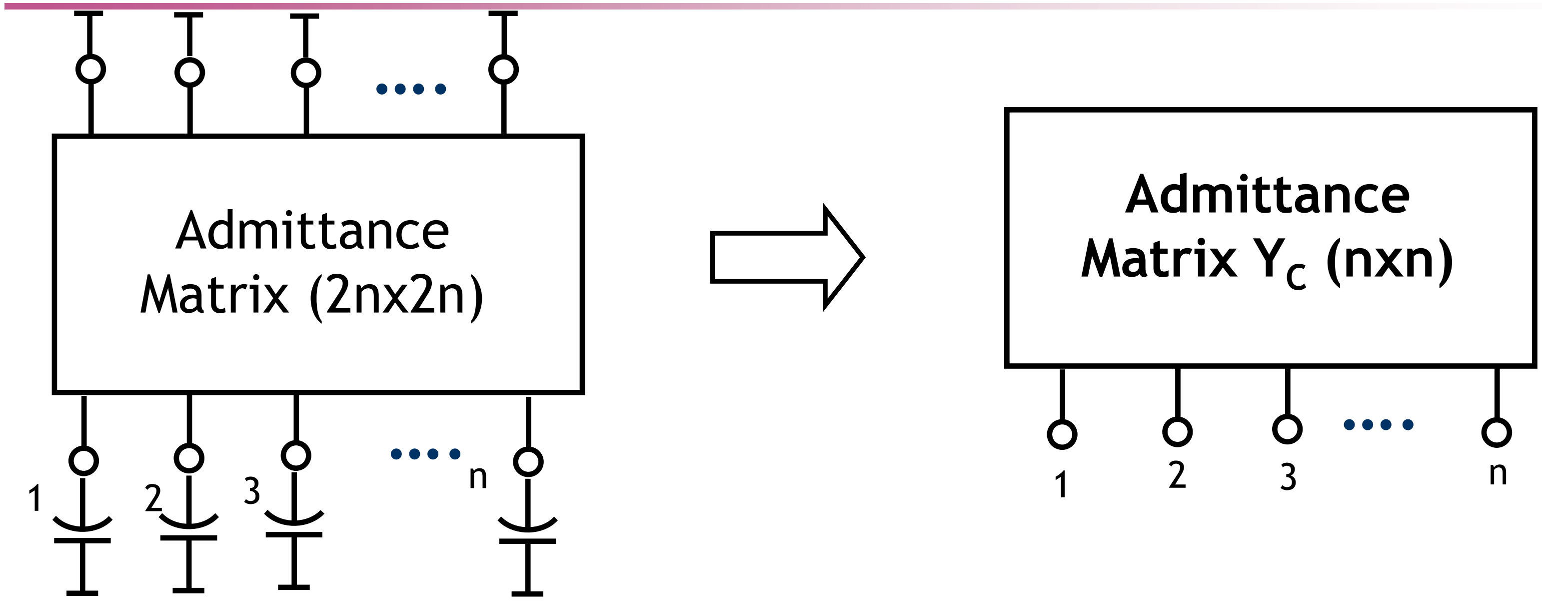
Ltap=4.16

Lris=27.94
L1=1.08
L2=0.016
L3=0

S12=1.086
S23=1.413



Matrix Y of Comb Array



With no coupling between non-adjacent lines:

$$y_C(i,j) = \begin{cases} j\left(\omega C_{s,i} - \dfrac{v \cdot C_{i,i}}{\tan(\beta L)}\right) & i = j \\ \dfrac{v \cdot C_{i,i+1}}{j\tan(\beta L)} & j = i+1 \\ 0 & j > i+1 \end{cases}$$

To get $y_c(i,i+1) \neq 0$:

$$\beta L < \frac{\pi}{2}$$

Equations for comb filters design ($\beta l=\pi/4$)

$$J_{i,i+1} = B_{i,i+1}\Big|_{\omega=\omega_0} = -\nu C_{i,i+1}, \qquad \omega_0 C_{eq,i} = \left(\frac{2+\pi}{4}\right)\nu C_{i,i}$$

$$k_{i,i+1} = \frac{4}{2+\pi}\frac{C_{i,i+1}}{\sqrt{C_{i,i}C_{i+1,i+1}}}, \qquad Q_E = \frac{\left(\frac{2+\pi}{4}\right)\nu C_{1,1}}{\mathrm{Re}\left\{Y_{1,1}\right\}_{\omega=\omega_0}}$$

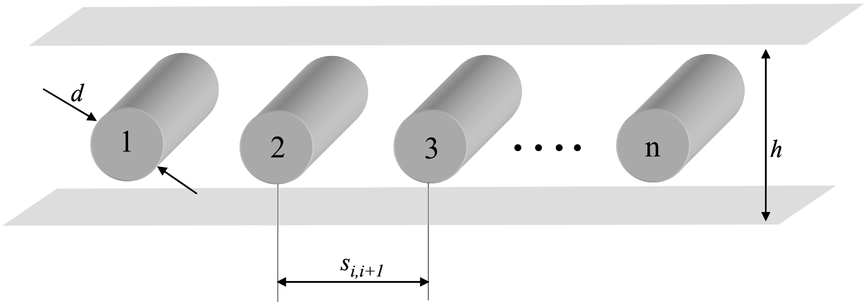
As in the case of interdigital filters, assigning all the lines with the same width, the solution is obtained by solving the system of non-linear equations in the variables $d_{i,i+1}$ and d_e :

$$k_{i,i+1} - \frac{4}{2+\pi}F(d_{i,i+1}) = 0, \qquad Q_E - \frac{2+\pi}{4}\nu \cdot U\left(d_e, d_{1,2}\right) = 0$$

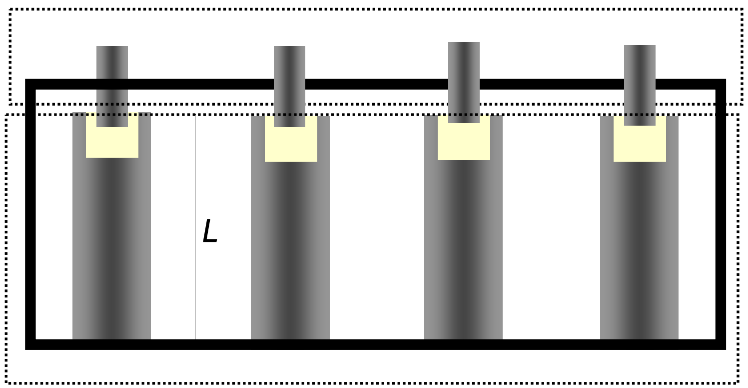
The tuning capacitances are obtained by the resonance condition:

$$\omega_0 C_{s,i} = \frac{\nu \cdot C_{i,i}}{\tan(\pi/4)} = \nu \cdot C_{i,i}$$

Example: Comb Filters in slab-line



Array of coupled rods (coupled slab-lines)



Inline comb filter configuration