RF SYSTEMS –
Written Test of July 2, 2020

Exercise 1

It is given the following antenna directivity function:

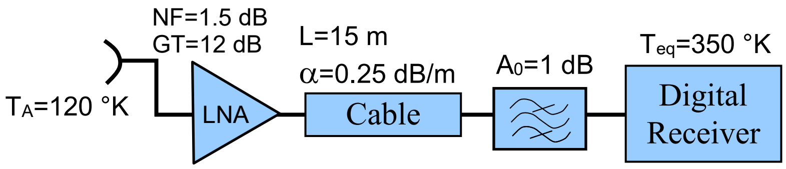
$$f(\theta,\varphi)=\left[\frac{\sin(50\cdot\theta)}{(50\cdot\theta)}\right]^2 \text{ for } 0<\theta<\pi/2,\quad f(\theta,\varphi)=0 \text{ for } \pi/2<\theta<\pi$$

The operating frequency is 10 GHz.

1) What is the direction θ_{MAX} for which $f=f_{max}=1$? Evaluate the 3dB beamwidth (θ_{3dB}) around this direction (Hint: (sin(x)/x)^2=0.5 for x= 1.3916)
2) Assuming that the emitted power is uniformly distributed on the spherical cap closing the cone of aperture $2\theta_B$ equal to θ_{3dB}, evaluate the directivity gain D_{MAX}.
3) It is known that $\frac{1}{r^2}\iint_\Sigma f(\theta,\varphi)\cdot d\Sigma=0.00505929$ where Σ is the surface of the sphere of radius r centered on the antenna. Evaluate the exact value of D_{MAX}.
4) Assuming the antenna efficiency η=0.85, evaluate its effective area.

Exercise 2

The RF front-end of a receiving station operating a 10 GH is shown in the following figure. Note that the LNA output is connected to the filter through a cable with length L and attenuation per unit length α.

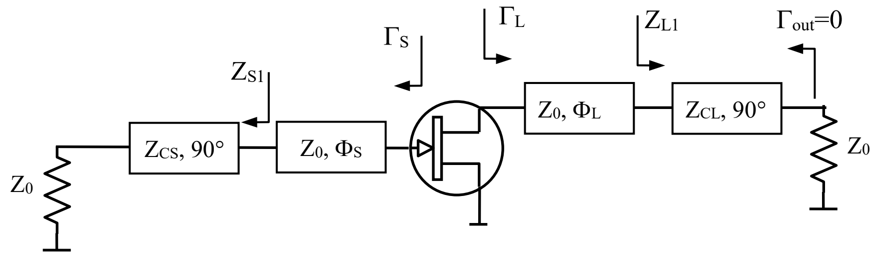


The communication system operates at DR=100 Mbit/sec and the bandwidth is B=20 MHz.

1. Evaluate the system noise temperature (T_{sys}) of the receiver ($K=1.38\cdot10^{-23}$, T_0=293 °K)
2. The power density of the incident wave on the antenna is $S_R=4\cdot10^{-9}$ W/m^2. Assuming the antenna gain G=15 dB, compute the SNR of the receiver (assume the antenna output terminals matched). Hint: SNR=(Received Signal Power)/(Noise power)
3. Evaluate E_b/N_0 of the digital receiver to allow the required data rate (DR)

Exercise 3

The following scheme shows an amplifier operating at 2 GHz (Z_0=50 Ohm)



The transistor is characterized by the following parameters (Z_0=50 Ω):
$S_{11}=0.814\angle -144.78°$, $S_{12}=0.075\angle -15.38°$ $S_{21}=2.612\angle 45.62°$ $S_{22}=0.55\angle -108.91°$
NF_{min}=1 dB, $\Gamma_{min}=0.8\angle 122$, $r_n=0.26$

The requested Transducer Gain is G_T=15 dB. Moreover, the amplifier output must be matched (Γ_{out}=0).

1) Choose Γs to get the smallest possible value of the Noise Figure NF with the assigned available power pain Gp=15 dB. Specify the value of NF
2) Compute Γ_L to get the requirement on G_T satisfied.

The input and output networks are constituted by two transmission lines, the first with assigned characteristic impedance $Z_c=Z_0$=50 Ohm and the second with assigned electrical length (=90°).

3) Evaluate Φ_S and Φ_L to get Z_{S1} and Z_{L1} real and lower than 1
4) Evaluate Z_{CS} and Z_{CL} to obtain the computed values of Z_{S1} and Z_{L1}.

Solutions

Exercise 1

1. Direction for maximum $f(\theta)$: $f(\theta_{MAX})=1$ for $\theta_{MAX}=0$.
$f(\theta_{3dB})=0.5$ —> $50\cdot\theta_{3dB}=1.3916$ —> $\theta_{3dB}=0.0278$ rad
$\Delta_{3dB}=2\theta_{3dB}=0.0557$rad (3.1893°)

2. Evaluation of D_{MAX} (approximated formula):
$\cos(\theta_{3dB})=(1-2/D_{MAX})$ → $D_{MAX}=2/(1-\cos(\theta_{3dB}))=5164.2$ (37.13 dB)

3. Exact value of D_{MAX}:

$$D_{MAX}=\frac{4\pi}{\frac{1}{r^2}\iint_\Sigma f(\theta,\varphi)\cdot d\Sigma}=\frac{4\pi}{0.00505929}= 2483.8 \text{ (33.95 dB)}$$

4. Evaluation of the effective area:
$G=\eta D_{MAX}=2111.2$ (33.245 dB)
$Ae=G\cdot\lambda^2/(4\pi)= G\cdot 0.03^2/(4\pi)=0.15$ m^2.

Exercise 2

1. Evaluation of the system noise temperature:

$$T_{sys}=T_a+T_{LNA}+\frac{T_f}{G_T}+\frac{T_{eq}A_f}{G_T}$$

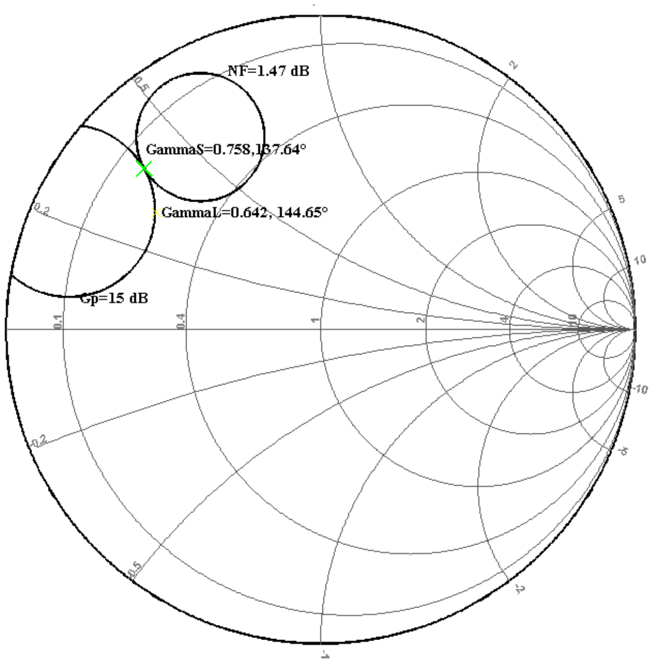
with:
$A_{f,dB}=A_0+\alpha L=1+0.25\cdot 15=4.75$ dB, $A_f=10^{0.475}=2.9854$, G_T=10^1.2=15.8489
$T_f=T_0(A_f-1)=293\cdot 1.9854= 581.7171$ °K
$T_{LNA}=T_0(10^{(NF/10)}-1)= 120.8735$ °K
Replacing:
$T_{sys}= 343.5051$ °K

2. Evaluation of SNR:
$SNR=Pr/(KT_{sys}B)$
The received power Pr is evaluated from the power density of the incident wave:
$P_r=A_eS_R$, $A_e=G\cdot\lambda^2/(4\pi)$, $\lambda=300/f_0=0.03$m → $A_e=0.0023$ m^2, $P_r= 9.0593\cdot 10^{-12}$ W
$KT_{sys}B= 9.4807\cdot 10^{-14}$ W
SNR=95.5548 (19.8 dB)

3. Evaluation of E_b/N_0:
We know that $SNR=(E_b/N_0)(R/B)$ → $(E_b/N_0)=SNR/(R/B)= 19.111$ (12.813 dB)

Exercise 3

1. Draw the circle Gp=15 dB. Draw circles at NF=const > 1dB until you find the one tangent in a point to the previous Gp=const circle. This circle is NF=1.47 dB. The tangent point is $\Gamma_S=0.758\angle 137.64°$.
2. Select S Param → Optimum Gamma → Load. You get $\Gamma_L=0.642\angle 144.65°$.



3. To get Φ_S you draw the circle $\Gamma=|\Gamma_S|$, store Γ_S and move from Γ_S toward the load until intersect the horizontal axis. The length is the phase of DeltaGamma divided by 2: Φ_S = 42.485/2=21.24°. The intersection point is r_{s1}= 0.138. We then get the impedance $Z_{S1}=r_{s1}*50=6.9\ \Omega$.
The same must be repeated for Φ_L, using Γ_L as starting point. You get: Φ_S = 35.5/2=17.75°, $Z_{L1}=50*0.218=10.9\ \Omega$.

4. The lines 90° long act as impedance inverters, i.e. $Z_{in}=Zc^2/Z_L$. Then $Zc=\text{sqrt}(Z_{in}*Z_L)$. Appling this formula with Zin given by Z_{s1} and Z_{L1} and $Z_L=50\ \Omega$, we get: $Z_{CS}=18.57\ \Omega$, $Z_{CL}=23.3452\ \Omega$.