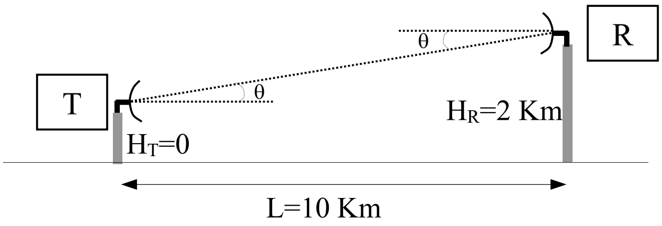
RF SYSTEMS
7th September 2020

Exercise 1



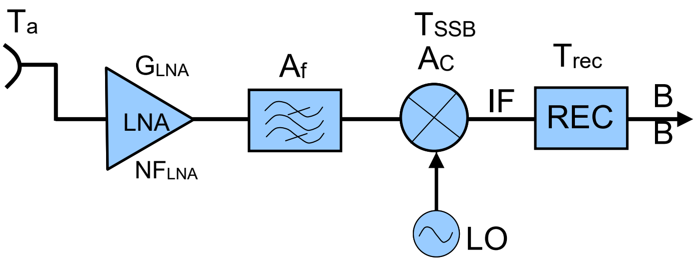
The transmitter operates at 10 GHz with a transmitted power P_T=20 dBm. The receiver is located 10 Km apart from the transmitter, placed at a height of 2000 m. The two antennas are equal, horizontally pointed and exhibit directivity function $f(\phi,\theta)=\cos^4(\theta)$ for $0<\theta<\pi/2$, =0 elsewhere. The efficiency is η=0.85.

1) Evaluate the gain G of the two antennas. Recall: $\int f^n(x) f'(x)\,dx = \dfrac{f^{n+1}(x)}{n+1}$
2) Evaluate the power density SR of the incident wave on the receiving antenna.
3) Assuming the radiation impedance of the antennas (Z_{rad}) equal to 60 Ohm and the input impedance of the receiver (Z_L) equal to 50 Ohm, evaluate the received power.

Exercise 2

Consider the following scheme of a receiver operating at 4 GHz (signal band 50 MHz). The block REC represents the demodulator at intermediate frequency (IF), producing the bit streaming in base band (BB). It is characterized by the noise temperature T_{rec} (at input).

Assume that the filter eliminates completely the image band (no noise contribution at IF).

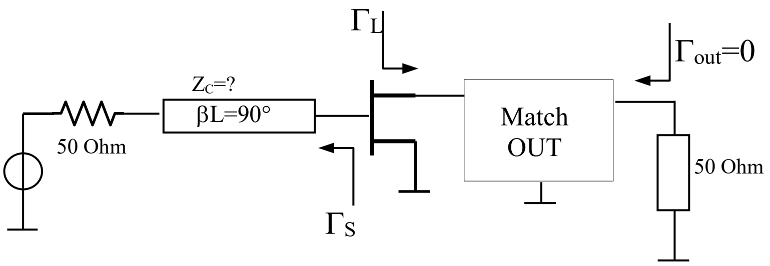


T_a=100°K
NF_{LNA}=2 dB
G_{LNA}=12 dB
A_f=1.5 dB
T_{SSB}=400 °K
A_c=6 dB
T_{rec}=150 °K

1) Evaluate the equivalent noise temperature at the input of the receiver (T_{eq}).
2) Assuming E_b/N_0=20 dB, what is the minimum signal power at IF allowing a data rate R=100 Mbit/sec?
3) What is the main drawback of connecting the antenna directly to the input of LNA? If the LNA is removed, what is the required input power to get the same data rata R?

Exercise 3

We want to design the amplifier in the figure operating at 500 MHz, with matching at output (Γ_{out}=0). It is known that the Match OUT network is lossless.
The S parameters of the active device at 500 MHz are specified in the figure.



$S_{11}=0.39\angle -137°$
$S_{12}=0.027\angle 60°$
$S_{21}=13.97\angle 99°$
$S_{22}=0.5\angle -26°$

1) Evaluate Γ_S and Γ_L to get the highest transducer gain compatible with stability and matching requirement (the selected Γ_S must be realizable with the assigned input network). Specify the value of transducer gain. Hint: the requirement of matching at output suggests using the available gain for the amplifier design.
2) Evaluate the characteristic impedance Z_c of the input transmission line (must be larger than 25 Ohm).

SOLUTION

Exercise 1

1) The gain is expressed as follows:

$$G=\frac{4\pi\eta}{\int\limits_{-\pi}^{\pi}\int\limits_{0}^{\pi}f(\theta)\sin(\theta)\,d\varphi\,d\theta}=\frac{4\pi\eta}{2\pi\int\limits_{0}^{\pi/2}\cos^4(\theta)\sin(\theta)\,d\theta}=\frac{2\cdot5\cdot0.85}{\left[-\cos(\theta_0)\right]_0^{\pi/2}}=8.5\ \ (9.29\text{ dB})$$

2) The power density at the receiving antenna position is given by $SR=\frac{P_t}{4\pi R^2}G\cdot f(\theta)$, with $R^2=10^2+2^2=104\text{ Km}^2,\ \theta=\tan^{-1}\left(\frac{2}{10}\right)=11.31°$. Replacing: SR=$6.0132\cdot10^{-10}$ W/m^2.

3) The maximum received power $P_{r,max}$ is given by $P_{r,\max}=SR\cdot A_e\cdot f(\theta)$, with $A_e=G\cdot\lambda^2/4\pi=6.0877\text{ m}^2$ and $f(\theta)=\cos^4(11.31°)=0.9246$. We get then $P_{r,\max}=3.3845\cdot10^{-13}W$.

The power developed on the load is given by $P_L=P_{r,\max}k_{mis}^2=P_{r,\max}\frac{4\,\mathrm{Re}\{Z_{rad}\}\mathrm{Re}\{Z_L\}}{\left|Z_{rad}+Z_L\right|^2}=3.3565\cdot10^{-13}W$ (-97.74 dBm)

Exercise 2

1) The equivalent noise temperature at input is given by:

$$T_{eq}=T_a+T_{LNA}+\frac{1}{G_{LNA}}\left(T_f+A_f\left(T_{SSB}+T_{rec}A_c\right)\right)$$

with:

$$T_{LNA}=293\left(10^{NF/10}-1\right)=171.37\text{ °K},\quad A_f=10^{1.5/10}=1.4125,\quad T_f=293\left(10^{A_f/10}-1\right)=120.87\text{ °K}$$

$$G_{LNA}=10^{12/10}=15.85,\quad A_c=10^{6/10}=4$$

Replacing: T_{eq}=368.11 °K.

2) The received power is determined by the required SNR:

$$SNR=\frac{P_r}{KT_{eq}B}=\frac{E_b}{N_0}\frac{R}{B}\quad\Rightarrow\quad P_r=\left(KT_{eq}B\right)\frac{E_b}{N_0}\frac{R}{B}=2.54\cdot10^{-13}\cdot100\cdot2=5.08\cdot10^{-11}\text{ W (-72.94 dBm)}$$

The power at IF is then given by:

$$P_{IF}=P_r\frac{G_{LNA}}{A_cA_f}=2.8\cdot P_r=1.425\cdot10^{-10}\ \ \text{(-68.46 dBm)}$$

3) Removing the LNA the T_{eq} becomes:

$$T_{eq}=T_a+T_f+A_f\left(T_{SSB}+T_{rec}A_c\right)=1633.4\text{ °K}$$

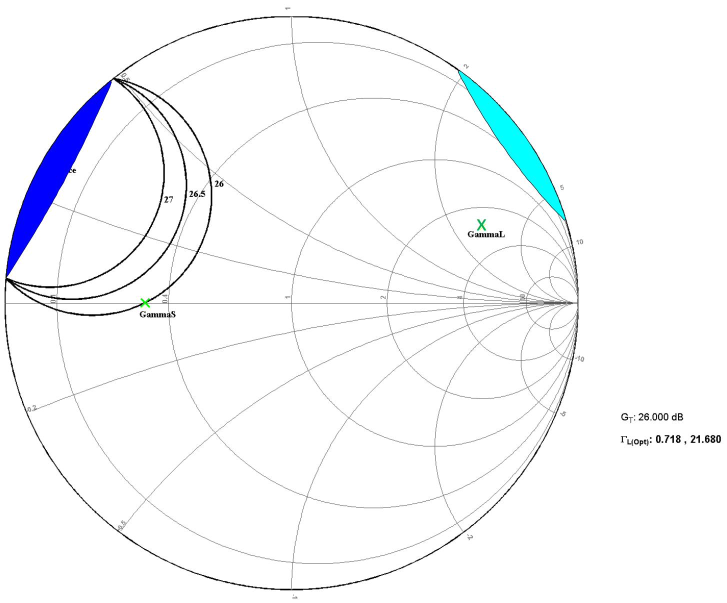
Then the received power must be increased to $P'_r=P_r$ 1633.4/368.11=$2.254\cdot10^{-10}$ W (-66.47 dBm)

Exercise 3

Active device: potentially instable (MSG=27.14 dB)

To get the output matched we chose Γ_s for the maximum available gain, compatible with the required condition (Γ_s must be real to be realized with the assigned input network). We then draw on the S. C. some circles with G_{av}=const (starting with Gav=27 dB), until the circle crosses the horizontal axis. The point to be selected for Γ_s is just the intersection with this axis (see the following figure). Finally we get: Γ_s=-0.511 with G_{av} =26 dB. Imposing conjugate matching at the transistor output we get Γ_L=0.718 $\angle$21.68°.

Note that both Γ_s and Γ_L are outside the forbidden regions (potential instability).



2) From Γ_s=-0.511 we get R_S=0.323·50=16.15 Ohm. Then the characteristic impedance of the λ/4 transformer is given by $Z_c = \sqrt{Z_0 \cdot R_s} = 28.42 \text{ Ohm } (>25 \text{ as required})$.