Exercise 1 (8.5)

A communication system between Earth and Mars (distance $L=2\cdot10^8$ Km) must be realized using an antenna on Earth with beamwidth $\Delta \theta=0.15^\circ$, efficiency $\eta=0.85$ and a dish antenna on Mars with diameter $d=5m$, aperture efficiency $e_a=0.6$. The uplink (Earth to Mars) operates at 10 GHz with bandwidth $B=6$ MHz; the received power at the output of Mars antenna must be $P_R=-100$ dBm.

1) Assuming constant the radiated power density inside the beamwidth and zero outside, evaluate the gain of the Earth antenna
2) Evaluate the effective area of the Mars Antenna
3) Compute the transmitted power of the transmitter connected to the Earth antenna necessary to guarantee the required received power on Mars
4) Assuming the equivalent noise temperature of the Mars receiving antenna $T_a=20^\circ$K, evaluate the equivalent noise temperature of the receiver ($T_{rec}$) connected to the antenna, allowing the system SNR equal to 10 dB

Boltzmann Constant $K=1.38\cdot10^{-23}$
Exercise 2 (8.5)

The scheme in the figure represents the receiver connected to the Mars antenna of the previous exercise. Note that filter and LNA are cooled at -200° C. The signal bandwidth is $B=6$ MHz. The equivalent noise temperature of the receiver ($T_{\text{rec}}$) must be the one previously evaluated.

- **a)** Draw the equivalent scheme for the evaluation of the receiver noise temperature
- **b)** Evaluate the value of $\text{GrF}$ determining the required $T_{\text{eq}}$ of the receiver
- **c)** Assuming $\text{Eb/N}_0=10$ dB evaluate the maximum data rate $R$ (the SNR is specified in the last point of the previous exercise).
Exercise 3 (8)

We want design a single stage amplifier at 1 GHz using the scheme in the following figure (the output network is a section of transmission line whose electrical length is 90°):

The transistor has the following parameters:

\[
\begin{align*}
S_{11} &= 0.6987 \angle -66.5^\circ \\
S_{21} &= 12.315 \angle 126.8^\circ \\
S_{12} &= 0.0474 \angle 57.3^\circ \\
S_{22} &= 0.7908 \angle -33.5^\circ 
\end{align*}
\]

The design goals are \( G_T = 24 \text{ dB}, \Gamma_{\text{in}} = 0 \)

1) What is the gain to choose for the design to have at the end the required \( G_T \) and \( \Gamma_{\text{in}} = 0 \)?
2) Evaluate \( \Gamma_L \) and \( \Gamma_S \). (Note that \( \Gamma_L \) must be compatible with the output network)
3) Design the input network, i.e. determine \( L \) (nH) and \( C \) (pF) (must be positive values!)
Exercise 4 (8)

We want design the oscillator in the following figure, operating at 1 GHz:

![Oscillator Diagram]

The scattering parameters of the transistor are given by:

\[ S_{11} = 0.832 \angle -63.466^\circ, \quad S_{21} = 7.812 \angle 130.39^\circ, \quad S_{12} = 0.071 \angle 56.027^\circ, \quad S_{22} = 0.369 \angle -58.306^\circ \]

a) Compute the minimum value of the inductor \( L \) (nH) so that \( |\Gamma_{\text{out}}| = 1.2 \)

b) Design the output network (i.e. compute \( \Phi_{\text{out}} \) and \( B \)), once the required value of \( \Gamma_L \) has been determined
Solution

Exercise 1

Using the relationship between directivity gain $D_{MAX}$ and beamwidth $\Delta \theta$ (for high directivity antennas) we get:

$$\Delta \theta = 2 \cos^{-1} \left( 1 - \frac{2}{D_{MAX}} \right) \Rightarrow D_{MAX} = \frac{2}{1 - \cos \left( \frac{\Delta \theta}{2} \right)} = 2.3344 \times 10^6$$

The gain of the Earth antenna is then given by $G_E = \eta D_{MAX} = 1.984 \times 10^6$ (63 dB).

The effective aperture of the Mars dish antenna is given by:

$$A_e = e^a \frac{1}{4} \pi d^2 = 3.75\pi = 11.78 m^2$$

The power received by the Mars antenna is given by: $P_R = \frac{G_E P_T}{4\pi L^2} A_e$

To have the received power $P_R$ equal to -100 dBm ($10^{-13}$ W) the transmitted power from Earth must be:

$$P_T = P_R \frac{4\pi L^2}{A_e G_E} = 2.15$\ KW$

The overall noise power reported at antenna terminal (due to the receiver ($T_{rec}$) and antenna ($T_a$) noise temperature) is given by $P_N = K(T_a + T_{rec})B = P_R/\text{SNR}=10^{-14}$ W. For the given $T_a$, the maximum value of $T_{rec}$ is given by:

$$T_{rec} = \frac{P_N}{KB} - T_a = 120.77 - 20 = 100.77^\circ \text{K}$$

Exercise 2

Drawing of the noise equivalent scheme of the receiver:

The equivalent noise temperature reported at the input is given by:

$$T_{rec} = T_f + 2A_f T_{LNA} + T_{SSB} \frac{A_f}{G_{RF}} + \frac{T_{IF} A_f L_c}{G_{RF}} = 100.77^\circ \text{K}$$
The reference temperature for the cooled section is $T_0=273-200=73°\ K$. Then:

$$T_f=T_0(10^{-10}-1)=8.907°\ K,\ T_{LNA}=T_0(10^{10}-1)=18.902°\ K.$$  

Then:

$$T_{rec} = \frac{838.382}{G_{RF}} = 100.77°\ K \Rightarrow G_{RF} = \frac{838.382}{49.45} = 16.95\ (12.3\ dB)$$

For the data rate it has:

$$SNR = \left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right) \Rightarrow R = B\frac{SNR}{E_b/N_0} = 6\ Mbit/sec$$

Exercise 3

Being requested the input matched, the Power gain $G_p$ must be chosen for the design. We then draw the circle $G_p=24\ dB$ on the S.C. representing $\Gamma_L$ and choose a point on this circle compatible with the output network. This latter imposes a real value for $\Gamma_L$, so the point to be selected is at the intersection of the mentioned circle with the real axis. Note that there are two intersections; the most convenient is the one closer to the center of the S.C.: $\Gamma_L=-0.175$ (it is outside the instability region of the load).

We then select the value of $\Gamma_S$ which matches the input (Optimum Gamma Source on the S.C.): $\Gamma_S=0.729\angle59.58°$ (it is outside the instability region of the source). In this way we have the required $G_T$ (equal to the imposed $G_p$), with the input matched.

The input network can be designed either with the S.C. or by means of the formulas. In both cases must be selected the solution compatible with the assumed component, i.e. $X_s$ and $B_p$ must be positive. The following result is obtained:

$$X_s=2.07850=103.9 \Rightarrow L_s=X_s/(2\pi f_0)=16.536\ nH$$

$$B_p=0.8330.02=0.01667 \Rightarrow C_p=B_p/(2\pi f_0)=2.65\ pF$$

Exercise 4

First is verified that the device is potentially instable ($k<1$). Then the mapping circle of $\Gamma_S$ is drawn imposing $|\Gamma_L|=1.2$. We have two intersection with the outer circle; the one to be selected is the one with the lower value of normalized reactance: $X_s=0.8250=41 \Rightarrow L= X_s/(2\pi f_0)=6.525\ nH$.

The required value of $\Gamma_L$ is determined from $\Gamma_{out}=1.2\angle-28.114°$, to which corresponds $z_{out}=-1.371-j3.492$; we then have $z_L=1.371/3+j3.492 \Rightarrow \Gamma_L =0.934\angle31.49°$.

The output network is a single stub network which can be designed with the S.C. with the following result: $\Phi_{out}=127.577/2=63.79°, B=0.02.(-5.23)=-0.1046\ S$. 