## RF SYSTEMS

## Written Test of February 10, 2020

| Surname \& Name |
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| Identification Number |
| Signature |

## Exercise 1 (8.5)

A communication system between Earth and Mars (distance $L=2 \cdot 10^{8} \mathrm{Km}$ ) must be realized using an antenna on Earth with beamwidth $\Delta \theta=0.15^{\circ}$, efficiency $\eta=0.85$ and a dish antenna on Mars with diameter $d=5 \mathrm{~m}$, aperture efficiency $e_{a}=0.6$. The uplink (Earth to Mars) operates at 10 GHz with bandwidth $\mathrm{B}=6 \mathrm{MHz}$; the received power at the output of Mars antenna must be $\mathrm{P}_{\mathrm{R}}=-100 \mathrm{dBm}$.

1) Assuming constant the radiated power density inside the beamwidth and zero outside, evaluate the gain of the Earth antenna
2) Evaluate the effective area of the Mars Antenna
3) Compute the transmitted power of the transmitter connected to the Earth antenna necessary to guarantee the required received power on Mars
4) Assuming the equivalent noise temperature of the Mars receiving antenna $\mathrm{Ta}=20^{\circ} \mathrm{K}$, evaluate the equivalent noise temperature of the receiver ( $\mathrm{T}_{\text {rec }}$ ) connected to the antenna, allowing the system SNR equal to 10 dB

Boltzmann Constant $\mathrm{K}=1.38 \cdot 10^{-23}$

## Exercise 2 (8.5)

The scheme in the figure represents the receiver connected to the Mars antenna of the previous exercise. Note that filter and LNA are cooled at $-200^{\circ} \mathrm{C}$. The signal bandwidth is $\mathrm{B}=6 \mathrm{MHz}$. The equivalent noise temperature of the receiver ( $\mathrm{T}_{\mathrm{rec}}$ ) must be the one previously evaluated.

a) Draw the equivalent scheme for the evaluation of the receiver noise temperature
b) Evaluate the value of $\mathrm{G}_{\mathrm{RF}}$ determining the required $\mathrm{T}_{\mathrm{eq}}$ of the receiver
c) Assuming $\mathrm{Eb} / \mathrm{N}_{0}=10 \mathrm{~dB}$ evaluate the maximum data rate R (the SNR is specified in the last point of the previous exercise).

## Exercise 3 (8)

We want design a single stage amplifier at 1 GHz using the scheme in the following figure (the output network is a section of transmission line whose electrical length is $90^{\circ}$ ):


The transistor has the following parameters:
$\mathrm{S}_{11}=0.6987 \angle-66.5^{\circ} \quad \mathrm{S}_{21}=12.315 \angle 126.8^{\circ} \quad \mathrm{S}_{12}=0.0474 \angle 57.3^{\circ} \quad \mathrm{S}_{22}=0.7908 \angle-33.5^{\circ}$

The design goals are GT $=24 \mathrm{~dB}, \Gamma_{\mathrm{in}}=0$

1) What is the gain to choose for the design to have at the end the required $\mathrm{G}_{\mathrm{T}}$ and $\Gamma_{\mathrm{in}}=0$ ?
2) Evaluate $\Gamma_{L}$ and $\Gamma_{S}$. (Note that $\Gamma_{L}$ must be compatible with the output network)
3) Design the input network, i.e. determine $\mathrm{L}(\mathrm{nH})$ and $\mathrm{C}(\mathrm{pF})$ (must be positive values!)

## Exercise 4 (8)

We want design the oscillator in the following figure, operating at 1 GHz :


The scattering parameters of the transistor are given by:
$\mathrm{S}_{11}=0.832 \angle-63.466^{\circ}, \mathrm{S}_{21}=7.812 \angle 130.39^{\circ}, \mathrm{S}_{12}=0.071 \angle 56.027^{\circ}, \mathrm{S}_{22}=0.369 \angle-58.306^{\circ}$
a) Compute the minimum value of the inductor $L(n H)$ so that $\left|\Gamma_{\text {out }}\right|=1.2$
b) Design the output network (i.e. compute $\Phi_{\text {out }}$ and $B$ ), once the required value of $\Gamma_{\mathrm{L}}$ has been determined

## Solution

## Exercise 1

Using the relationship between directivity gain $D_{M A X}$ and beamwidth $\Delta \theta$ (for high directivity antennas) we get:

$$
\Delta \theta=2 \cos ^{-1}\left(1-\frac{2}{D_{M A X}}\right) \Rightarrow D_{M A X}=\frac{2}{1-\cos \left(\frac{\Delta \theta}{2}\right)}=2.3344 \cdot 10^{6}
$$

The gain of the Earth antenna is then given by $\mathrm{Ge}_{\mathrm{E}}=\eta D_{M A X}=1.984 \cdot 10^{6}(63 \mathrm{~dB})$.
The effective aperture of the Mars dish antenna is given by:

$$
A_{e}=e_{a} \frac{1}{4} \pi d^{2}=3.75 \pi=11.78 \mathrm{~m}^{2}
$$

The power received by the Mars antenna is given by: $P_{R}=\frac{G_{E} P_{T}}{4 \pi L^{2}} A_{e}$
To have the received power PR equal to $-100 \mathrm{dBm}\left(10^{-13} \mathrm{~W}\right)$ the transmitted power from Earth must be: $P_{T}=P_{R} \frac{4 \pi L^{2}}{A_{e} G_{E}}=2.15 \mathrm{KW}$
The overall noise power reported at antenna terminal (due to the receiver ( $\mathrm{T}_{\text {rec }}$ ) and antenna ( $\mathrm{T}_{\mathrm{a}}$ ) noise temperature) is given by $P_{N}=K\left(T_{a}+T_{\text {rec }}\right) B=P_{R} / S N R=10^{-14} \mathrm{~W}$. For the given $T_{a}$, the maximum value of $\mathrm{T}_{\text {rec }}$ is given by:

$$
T_{\text {rec }}=\frac{P_{N}}{K B}-T_{a}=120.77-20=100.77^{\circ} \mathrm{K}
$$

## Exercise 2

Drawing of the noise equivalent scheme of the receiver:


The equivalent noise temperature reported at the input is given by:
$T_{\text {rec }}=T_{f}+2 A_{f} T_{L N A}+T_{S S B} \frac{A_{f}}{G_{R F}}+\frac{T_{I F} A_{f} L_{c}}{G_{R F}}=100.77^{\circ} \mathrm{K}$

The reference temperature for the cooled section is $\mathrm{T}_{0}=273-200=73^{\circ} \mathrm{K}$. Then:
$\mathrm{T}_{\mathrm{f}}=\mathrm{T}_{0}\left(10^{\mathrm{Af} / 10}-1\right)=8.907^{\circ} \mathrm{K}, \mathrm{T}_{\mathrm{LNA}}=\mathrm{T}_{0}\left(10^{\mathrm{NF} / 10}-1\right)=18.902^{\circ} \mathrm{K}$.
$\mathrm{T}_{\text {rec }}$ can then be expressed as:

$$
T_{\text {rec }}=51.32+\frac{838.382}{G_{R F}}=100.77^{\circ} \mathrm{K} \Rightarrow G_{R F}=\frac{838.382}{49.45}=16.95(12.3 \mathrm{~dB})
$$

For the data rate it has:
$S N R=\left(\frac{E_{b}}{N_{0}}\right)\left(\frac{R}{B}\right) \Rightarrow R=B \frac{S N R}{E_{b} / N_{0}}=6 \mathrm{Mbit} / \mathrm{sec}$

## Exercise 3

Being requested the input matched, the Power gain Gp must be chosen for the design. We then draw the circle $\mathrm{Gp}=24 \mathrm{~dB}$ on the S.C. representing $\Gamma_{\mathrm{L}}$ and choose a point on this circle compatible with the output network. This latter imposes a real value for $\Gamma_{\mathrm{L}}$, so the point to be selected is at the intersection of the mentioned circle with the real axis. Note that there are two intersections; the most convenient is the one closer to the center of the S.C.: $\Gamma_{\mathrm{L}}=-0.175$ (it is outside the instability region of the load). We then select the value of $\Gamma_{\mathrm{s}}$ which matches the input (Optimum Gamma Source on the S.C.): $\Gamma_{\mathrm{s}}=0.729 \angle 59.58^{\circ}$ (it is outside the instability region of the source). In this way we have the required $\mathrm{G}_{\mathrm{T}}$ (equal to the imposed Gp ), with the input matched.
The input network can be designed either with the S.C. or by means of the formulas. In both cases must be selected the solution compatible with the assumed component, i.e. $X_{s}$ and $B_{p}$ must be positive. The following result is obtained:
$\mathrm{Xs}=2.078 .50=103.9 \rightarrow \mathrm{Ls}=\mathrm{Xs} /\left(2 \pi \mathrm{f}_{0}\right)=16.536 \mathrm{nH}$
$\mathrm{Bp}=0.833 \cdot 0.02=0.01667 \rightarrow \mathrm{Cp}=\mathrm{Bp} /\left(2 \pi \mathrm{f}_{0}\right)=2.65 \mathrm{pF}$

## Exercise 4

First is verified that the device is potentially instable $(\mathrm{k}<1)$. Then the mapping circle of $\Gamma_{\mathrm{s}}$ is drawn imposing $\left|\Gamma_{\mathrm{L}}\right|=1.2$. We have two intersection with the outer circle; the one to be selected is the one with the lower value of normalized reactance: $\mathrm{Xs}=0.82 \cdot 50=41 \rightarrow \mathrm{~L}=\mathrm{Xs} /\left(2 \pi \mathrm{f}_{0}\right)=6.525 \mathrm{nH}$. The required value of $\Gamma_{\mathrm{L}}$ is determined from $\Gamma_{\text {out }}=1.2 \angle-28.114^{\circ}$, to which corresponds $\mathrm{Z}_{\mathrm{out}}=-1.371-$ j 3.492 ; we then have $\mathrm{z}_{\mathrm{L}}=1.371 / 3+\mathrm{j} 3.492 \rightarrow \Gamma_{\mathrm{L}}=0.934 \angle 31.49^{\circ}$.
The output network is a single stub network which can be designed with the S.C. with the following result: $\Phi_{\text {out }}=127.577 / 2=63.79^{\circ}, B=0.02 .(-5.23)=-0.1046 \mathrm{~S}$.

