**RF SYSTEMS**
**Midterm Test - November 10th, 2021**

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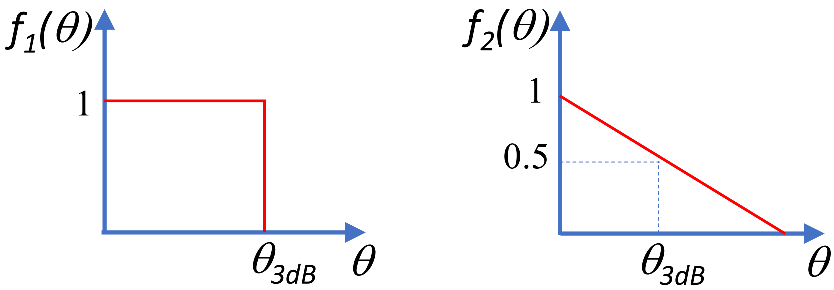
| **Surname & Name** | |
| **Identification Number** | |
| **Signature** | |

Exercise 1

An antenna is given with gain G=23 dB, efficiency $\eta$=0.85 and directivity function $f(\theta) = \cos^4(5\theta)$ for $0 \leq \theta \leq \pi/10$ ($f$=0 elsewhere) .

1) Evaluate $\theta_{3dB}$ ($f(\theta_{3dB})$=0.5) from the directivity function
2) Evaluate the directivity gain D.
3) Assume the following approximations for the directivity function:

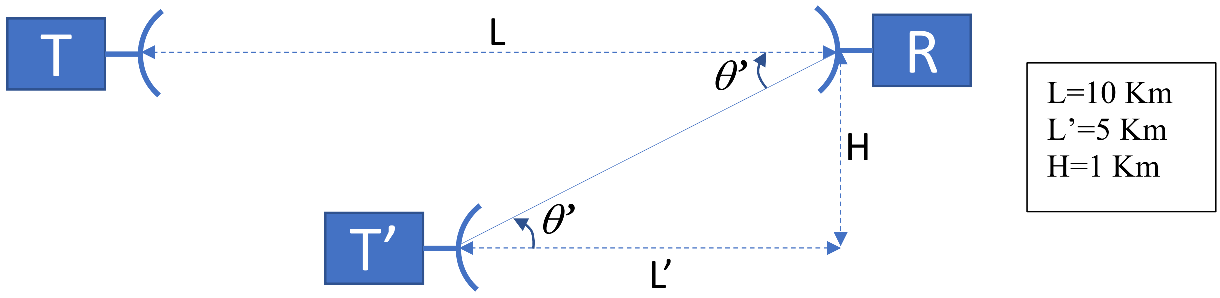



Evaluate the directivity gain in the two cases imposing $\theta_{3dB}$ the same of the actual antenna (previously computed). Specify which approximation gives the best approximation of D.
Hint: for computing D determined by $f_2(\theta)$ this integral is necessary:

$$\int x \cdot \sin(x) \cdot dx = \sin(x) - x \cdot \cos(x)$$

Exercise 2

A terrestrial link T → R is shown in the following figure, where there is another transmitter T’ representing the source of an interferer for the main link. R is a conversion receiver employing a preselector filter at antenna output to reject the image band.




T and R are aligned (antennas optimally pointed) while T’ sees R under the angle θ’. All antennas are equal and are the same of the previous exercise ($G_A$=23 dB, $f(\theta)=\cos^4(5\theta)$). An M-QAM modulated signal with carrier frequency $f_0$=20 GHz and roll-off factor α=0.35 is adopted. The data rate is DR=100 Mbit/sec.

1) Evaluate the received power ($P_R$) so that, at the receiver, $E_b/N_0$=20 dB is obtained with an equivalent system temperature $T_{sys}$=200 °K.
2) Evaluate the required transmitted power $P_T$ by T
3) Determine the value M of the M-QAM modulation imposing SNR≥ 29 dB at the output of the receiving antenna. Specify the bandwidth B of the signal (Note: the value of M must be a power of 2)
4) Consider now the interfering transmitter T’. It is known the transmitted power $P_{T'}$=20 dBm, the transmitting frequency $f_{T'}$ (equal to the image frequency ($f_{im}$) of the receiver R) and the intermediate frequency of the receiver (IF=140 MHz).
   a. Assuming $f_{T'}$>$f_0$ specify the value of $f_{T'}$ and the frequency $f_{LO}$ of the local oscillator used in the receiver
   b. Assume not ideal the preselector filter at the receiving antenna output. So, it does not completely reject the interferer signal in the image band. Defined $A_S$ the attenuation introduced by the mentioned filter in the image band, evaluate $A_S$ so that the power $P_{R'}$ of the received interfering signal at the filter output is equal to the noise power ($KT_{sys}B$). Note: Assume attenuation zero in the passband when computing the noise power at the filter output.
   c. It is possible to further attenuate the interfering signal by filtering at intermediate frequency? Justify the answer!

**Solution**

Exercise 1

1) Imposing $f(\theta_{3dB})=0.5$ we get:

$$\cos^4\left(5\theta_{3dB}\right)=\frac{1}{2} \quad \Rightarrow \quad \theta_{3dB}=\frac{1}{5}\cos^{-1}\left(0.5^{\frac{1}{4}}\right)=0.1144 \text{ rad } (6.553°)$$

2) It has G=ηD → D=G/η=$10^{2.3}$/0.85=234.7 (23.7 dB)

3) Case 1:

$$D_1=\frac{2}{\left(1-\cos\left(\theta_{3dB}\right)\right)}=306.122 \quad (24.86 \text{ dB})$$

Case 2:

The function $f_2(\theta)$ can be written as: $f_2(\theta)=\left(1-\theta/\theta_{MAX}\right)$. Imposing $f_2(\theta_{3dB})$ = 0.5 we get $\theta_{MAX}=2\theta_{3dB}$=0.2288. To evaluate $D_2$, the general expression of the directivity gain must be used:

$$D_2=\frac{4\pi}{\int_0^{2\pi}d\phi\int_0^{\theta_{MAX}}\left(1-\frac{\theta}{\theta_{MAX}}\right)\sin(\theta)d\theta}=\frac{2}{\int_0^{\theta_{MAX}}\sin(\theta)d\theta-\int_0^{\theta_{MAX}}\frac{\theta}{\theta_{MAX}}\sin(\theta)d\theta}=\frac{2}{\left[-\cos(\theta)-\frac{\sin(\theta)}{\theta_{MAX}}+\frac{\theta\cos(\theta)}{\theta_{MAX}}\right]_0^{\theta_{MAX}}}=$$

$$=\frac{2}{1-\cos(\theta_{MAX})-\frac{\sin(\theta_{MAX})}{\theta_{MAX}}+\cos(\theta_{MAX})}=\frac{2}{1-\frac{\sin(\theta_{MAX})}{\theta_{MAX}}}=229.94 \quad (23.6 \text{ dB})$$

It can be observed that the second approximating function determines the directivity gain closer to D.

Exercise 2

The SNR at the antenna output can be written as $P_R/P_N$, with $P_R$ received power and $P_N$ noise power. Expressing $P_N$ as $KT_{sys}B$, it has:

$$SNR=\frac{P_R}{KT_{sys}B}=\frac{DR}{B}\frac{E_b}{N_0} \quad \Rightarrow \quad P_R=DR\cdot K\cdot T_{sys}\frac{E_b}{N_0}=2.76\cdot10^{-11} \quad (\text{-75.6 dBm})$$

The Friis equation allows the evaluation of the transmitted power $P_T$:

$$P_R=P_T-20\cdot\log\left(\frac{4\pi L}{\lambda}\right)+2G_A \quad \Rightarrow \quad P_T=P_R+20\cdot\log\left(\frac{4\pi L}{\lambda}\right)-2G_A$$

Replacing $\lambda=c/f_0$=0.015m, L=10Km and $G_A$=23 dB we get $P_T$=16.9 dBm.

The bandwidth B of a M-QAM modulation is given by: $B=\frac{DR}{\log_2 M}\cdot(1+\alpha)$. Replacing on the expression of SNR:

$$SNR=\frac{P_R}{KT_{sys}B}=\frac{P_R\cdot\log_2 M}{KT_{sys}DR(1+\alpha)}\geq10^{2.9} \quad \Rightarrow \quad \log_2 M\geq10^{2.9}\frac{DR\cdot K\cdot T_{sys}(1+\alpha)}{P_R}=10.7234$$

Since M must be a power of 2: M=$2^{11}$=2048.

The bandwidth B can then be computed: $B=\frac{DR}{\log_2 M}\cdot(1+\alpha)=\frac{10^8\cdot1.35}{11}=12.27\text{ MHz}$.

The image frequency ($f_{im}$) of the receiver is $f_0 \pm 2 \cdot IF$. Being $f_{im} > f_0$ the plus sign must be selected, then: $f'_T = f_{im} = 20 + 2 \cdot 0.14 = 20.28$ GHz. Also the local oscillator frequency must be greater than $f_0$, then $f_{OL} = f_0 + f_{im} = 20.14$ GHz.
The received power $P_{R'}$ from the interfering transmitter is obtained from the Friis equation:

$$P_{R'} = P_{T'} - 20 \cdot \log\left(\frac{4\pi L_I}{\lambda}\right) + 2G_A + 20 \cdot \log\left(f\left(\theta'\right)\right)$$

where: $L_I = \sqrt{L'^2 + H^2} = 5.099$ Km (separation T'-R); $\theta' = \tan^{-1}\left(\frac{H}{L'}\right) = 0.1974$ rad;

$\lambda_{im} = \frac{c}{f_{im}} = 0.0148\text{m}$; $f\left(\theta'\right) = \cos^4\left(5\theta'\right) = 0.0923$. Replacing into the Friis equation we get:

$P_{R'} = -87.42$ dBm. The noise power at the filter output is given by $P_N = KT_{sys}B = 3.387\ 10^{-14}$ W (-104.7 dBm). To have the power of the interfering signal equal to the noise power, the filter attenuation must be $A_S = P_{R'} - P_N = 17.28$ dB.
The interfering signal frequency is equal to the image frequency of the receiver, then it is converted at the same IF frequency of the RF signal (20 GHz). There is no way to distinguish the interferer from the converted RF signal at IF, so filtering is useless.