RF SYSTEMS

Written Test of February 11, 2019

Surname & Name	
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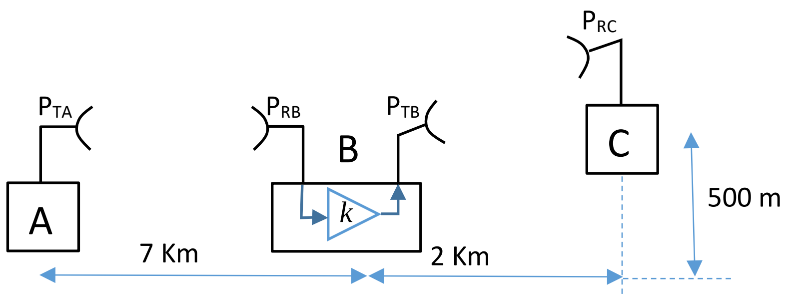
Exercise 1



The figure shows a terrestrial multi-link system operating at 6 GHz with bandwidth 150 MHz. All the antennas are identical and optimally directed, with efficiency η=0.9 and directivity function $f(\theta)=|\cos^9(\theta)|$.

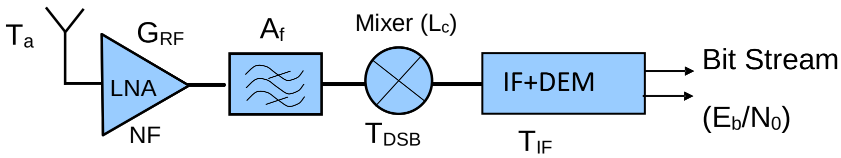
1) Evaluate the gain of the antennas (hint: $\int f^n(x)\cdot f'(x)\,dx=\dfrac{f^{n+1}(x)}{n+1}$)
2) Assume the power transmitted by A is P_{TA}=30 dBm. Assume also that the power transmitted by B is linearly related to the received power, i.e. $P_{TB}=k\cdot P_{RB}$. Evaluate the power P_{RB} and the gain k in dB in order that the power received in C is P_{RC}=-80 dBm
3) It is know that the receivers in B and C are characterized by T_{eq}=150 °K. Compute the overall SNR of the system defined as $SNR_{tot}=P_{RC}/KT_{sys}B$, with T_{sys} the overall system noise temperature (i.e. the overall noise computed at the input of receiver C).
4) If we increase the transmitted power P_{TA} by 3 dB and assume T_{eq}=300 °K, what is the new value of SNR_{tot}?

Use $K=1.38\cdot10^{-23}$ (Boltzmann Constant)

Exercise 2

The scheme in the figure represents the RF front-end of the receiver C of the previous exercise.

The goal is to get T_{eq}=150°K with P_{rec}=-80 dBm.

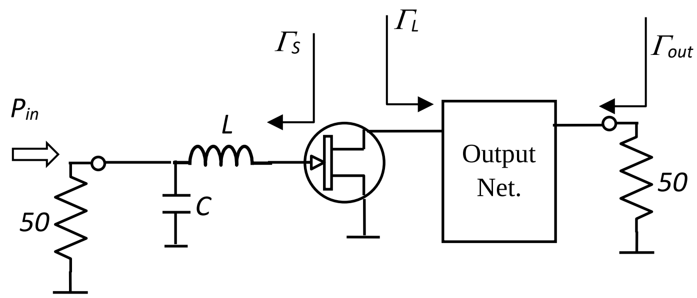


NF= ?
G_{RF}=20 dB
T_a=100 °K
A_f=0.2 dB
L_c=4 dB
T_{DSB}=100 °K
T_{IF}=150 °K

a) Evaluate the Noise Figure NF_{RF} of LNA in order to meet the goal
b) What is the maximum data rate R of the demodulated bit stream with (E_b/N_0) =10 dB?
c) If we assume T_{eq}=300 °K with P_{rec}=-77 dBm and NF=1.5 dB, what is the new requested value for G_{RF}? Does the maximum data rate R change? (Justify the answer!)

Exercise 3

We want design a single stage amplifier at 1.8 GHz using the scheme in the following figure (the output network is assumed lossless):



The transistor has the following parameters:

S_{11}=0.52∠143.8° S_{21}=4.255∠ 28.7° S_{12}=0.081∠4.9 S_{22}=0.287∠ -87.7°

Γ_{opt}=0. 19∠-170.4° F_{min}=0.85 dB R_n=0.06 (Noise parameters)

P_{1dB}: 0 dBm

The design goals are: G_T=15 dB and NF=1 dB.

1) Evaluate Γ_S and Γ_L producing the requested values of G_T and NF. Note that the chosen value of Γ_S must be compatible with the assigned input network
2) Design the input network, i.e. determine L (nH) and C (pF)
3) The dynamic range (DR) is here defined as the ratio between the output power for a required back-off (BO) and the output power determining a specified input SNR. Evaluate DR for the designed amplifier with BO=3 dB and SNR=15 dB (assume the noise bandwidth B=1 MHz)
4) What is the power at input for the maximum output power?
5) Assume that the amplifier is working at the nominal BO (3 dB) but the input signal is two-tone. Evaluate the carrier-to-intermodulation CI at output (assume ideal third order non-linearity). What is the output power in each tone?

Solution

Exercise 1

The antenna gain is obtained from the formula:

$$G = \eta 4\pi \left[\int_0^{2\pi} d\varphi \int_0^{\pi} f(\theta) \sin\theta d\theta \right]^{-1} = \frac{2\eta}{\int_0^{\pi} \left| \cos^9(\theta) \right| \sin(\theta) d\theta} = \frac{2\eta}{2\left[-\frac{\cos^{10}(\theta)}{10} \right]_0^{\frac{\pi}{2}}} = 9 \text{ (9.54 dB)}$$

We have for the two links:

L_1=7000m, $L_2=(2000^2+500^2)^{0.5}$=2.0616 Km, $\lambda_0=3\cdot10^8/6\cdot10^9$=0.05m

The Friis equation for the first link defines the received power P_{RB}:

$$P_{RB} = P_{TA} + 2G_A + 20\log_{10}\left(\frac{\lambda_0}{4\pi L_1}\right) = -75.82 \text{ dBm}$$

For the second link it has:

$$P_{RC} = P_{TB} + 2G_A + 20\log_{10}\left(\frac{\lambda_0}{4\pi L_2}\right) = -80 \text{ dBm} \quad \rightarrow \quad P_{TB} = 15.204 \text{ dBm}$$

The gain *k* is then given by: $k_{dB}=P_{TB}-P_{RB}$=91.03 dB

The overall noise temperature at input of C is given by:

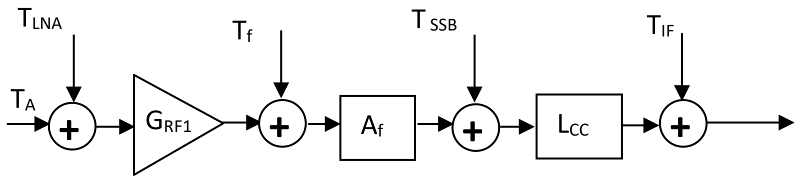
$$T_{sys} = T_C + T_B k G_A^2 \left(\frac{\lambda_0}{4\pi L_2}\right)^2 = 207.32 \text{ °K}$$

The system SNR is then given by:

$$SNR_{sys} = \frac{P_{RC}}{K T_{sys} B} = 23.3 \text{ (13.67 dB)}$$

Exercise 2

Equivalent noise scheme :



Where:

$T_f = 293\left(10^{\frac{A_f}{10}} - 1\right) = 13.809$ °K , $T_{SSB}=2\cdot T_{DSB}=200$ °K

The equivalent noise temperature at the input of the receiver is then given by:

$$T_{eq} = T_A + T_{LNA} + \frac{T_f}{G_{RF}} + \frac{T_{SSB}A_f}{G_{RF}} + \frac{T_{IF}L_cA_f}{G_{RF}}$$

$$= T_{LNA} + 106.1778$$

Imposing T_{eq}=150 we get:

$$T_{LNA} = 150 - 106.1778 = 43.8223°K \quad \rightarrow \quad NF = (1 + \frac{T_{LNA}}{T_0})_{dB} = 0.605\,\mathrm{dB}$$

The system SNR is given by:

$$SNR = \frac{P_{rec}}{K \cdot T_{eq} \cdot B} = \frac{E_B}{N_0}\left(\frac{R}{B}\right) = 15.08 \text{ dB}$$

then: $(R/B)_{dB}$=15.08-10=5.08 dB → R=3.22·B=483.16 Mbit/sec

With T_{eq}=300 °K we have :

$$G_{RF} = \frac{T_f + T_{SSB}A_f + T_{IF}L_cA_f}{T_{eq} - T_A - T_{LNA}} = 7.807 \ \ (8.93 \text{ dB})$$

The data rate R does not change (both P_{rec} and T_{eq} double so SNR remains constant)

Exercise 3

The transistor is unconditionally stable with Gmax=16.8 dB and NFmin=0.85.
Draw the circles NF=1 dB and G_{AV}=15 dB on the S.C. Γ_S is chosen between the two intersections of these circles. We must however verify that the selected value is compatible with the input network. This happens for $\Gamma_S = 0.48\angle -154.9°$. The value of Γ_L is the one which produces conjugate matching at output (in this way $G_T=G_{AV}$). With the S.C. we find Γ_L =Optimum Gamma Load=$0.48\angle 103.67°$.

The input network can be designed either with analytic formulas or with the S.C. It is found:
$X=50\cdot 0.286=14.3\ \Omega$, $B=0.02\cdot 1.33=0.0266$. From which: $L=X/\omega_0=1.2644$ nH, $C=B/\omega_0=2.352$ pF

The maximum output power for the given BO and P1dB is Pmax=-3 dBm. The minimum power is defined as:
$P_{min}=P_{in}-G_T$, with $P_{in}=P_N-SNR$, $P_N=KT_{eq}B$, $T_{eq}=T_0(10^{(NF/10)}-1)=75.865$ °K.
Replacing we get:
$P_N=1.047\ 10^{-15}$ W (-119.8 dBm), Pin=-119.8+15=-104.8 dBm, Pmin=-104.8+15=-89.8 dBm

Then:
DR=Pmax-Pmin=-3+89.8=86.8 dB

In case of two-tone signal the output average power remain -3 dBm and the power per tone is -6 dBm.
The CI is obtained with the following formula (Δ_p=10.63):
$$CI = 2\left(BO + \Delta_p + 3\right) = 33.26 \text{ dB}$$