RF SYSTEMS
Written Test of February 11, 2021

Exercise 1

A satellite earth station operates at 6 GHz (uplink) with a geostationary satellite (assume the link length equal to 36000 Km). The earth antenna has the following directivity function: $f_T=cos(110\cdot\theta_T)$ ($\theta<\pi/200$), $f_T=0$ ($\theta>\pi/200$). The transmitted power from earth is 5 KW.

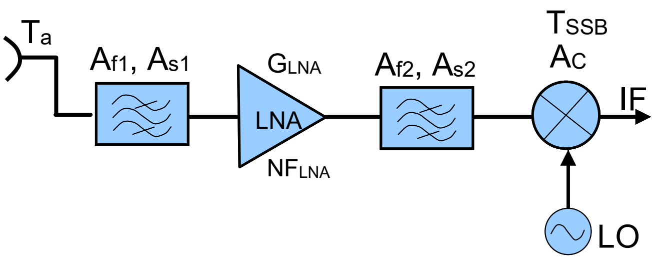
a) Evaluate the directivity gain of the earth antenna. Hint:

$$\int \sin(x)\cos(nx)\,dx = \frac{\cos((n-1)x)}{2n-2} - \frac{\cos((n+1)x)}{2n+2}$$

b) Evaluate the gain of the antenna satellite so that the received power is at least -65 dBm. Verify that the beamwidth of the antenna corresponding to the computed gain is sufficient to cover the portion of Earth seen by the satellite (assume for the satellite antenna $f(\theta)=1$ inside the beam and the efficiency of both antennas equal to 1)

Exercise 2

Consider the following scheme of a receiver front-end operating in the band 10-10.5 GHz (signal bandwidth 10 MHz). Note that the filters are characterized by a finite attenuation A_S in the stopband (which coincides with the image band of the receiver), in addition to the attenuation A_f in the passband.



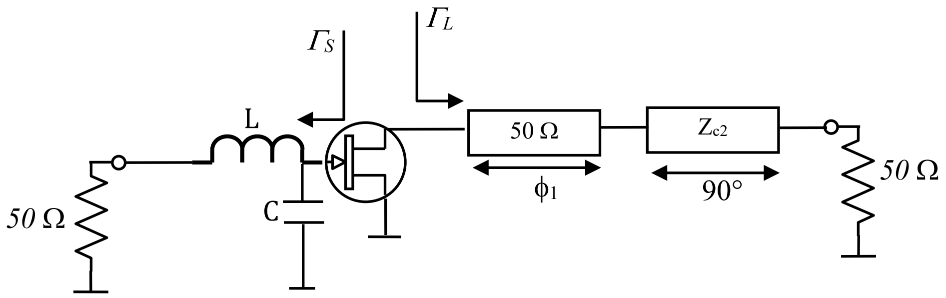
$T_a=100°K$
$A_{f1}=1$ dB, $A_{s1}=10$ dB
$NF_{LNA}=2$ dB
$G_{LNA}=10$ dB
$A_{f2}=2$ dB, $A_{s2}=15$ dB
$T_{SSB}=350$ °K
$A_c=6$ dB

1) Assuming the local oscillator tunable in the range 11-11.5 GHz, specify the limits of the image band and the value of the intermediate frequency.
2) Draw the equivalent scheme for the evaluation of the noise temperature and evaluate of the equivalent noise temperature at <u>the input</u> of the receiver (assume $T_0=293$ °K).
3) Assuming $E_b/N_0=20$ dB, what is the minimum signal power at IF (in dBm) allowing a data rate R=20 Mbit/sec?

Exercise 3

The following scheme represents an amplifier operating at 20 GHz. The active device (whose S parameters are given below) is unconditionally stable with G_{Tmax}=12.9 dB. The following constraint is imposed on the reflection coefficient at the input of the device: $|\Gamma_s| \leq 0.5$.

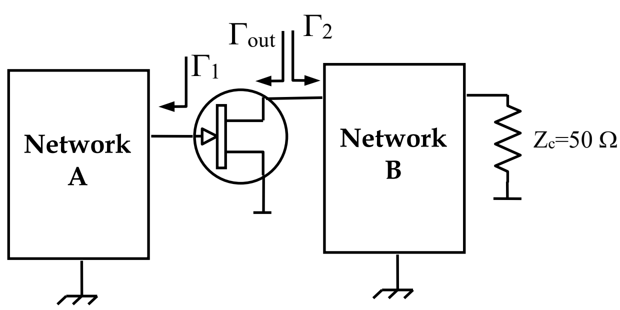
a) Determine the values Γ_s and Γ_L so that the transducer gain of the amplifier is maximum (compatibly with the imposed constraint)
b) Compute the parameters of the input and output matching networks.



Scattering and noise parameters of the transistor:
$S_{11}=0.642\angle 47.3°$ $S_{21}=2.934\angle -79.2°$ $S_{12}=0.107\angle -75.2°$ $S_{22}=0.204\angle 67.8°$

Exercise 4

The following figure represents the general configuration of a microwave oscillator. Using the reported scattering parameters of the active device, evaluate the reflection coefficients Γ_1 and Γ_2 which ensure the start of oscillation (the magnitude of Γ_1 must be imposed equal to 1. Assume $|\Gamma_{out}|=1.2$).



$S_{11}=0.69\angle -55°$
$S_{12}=0.026\angle 64.7°$
$S_{21}=27.5\angle 146.3°$
$S_{22}=0.82\angle -35.7°$

<u>Solution</u>

<u>Exercise 1</u>

1) Directivity gain:

$$D_M = 4\pi\left[\int_0^{2\pi} d\varphi \int_0^{\pi} f(\theta)\sin\theta d\theta\right]^{-1} = \frac{2}{\int_0^{\pi/200} \sin(\theta)\cos(110\cdot\theta)d\theta} = \frac{2}{\left[\frac{\cos(109\cdot\theta)}{218} - \frac{\cos(111\cdot\theta)}{222}\right]_0^{\pi/200}} =$$

$$= \frac{2}{4.5468\cdot 10^{-5}} = 4.4\cdot 10^4 \text{ (46.4 dB)}$$

2) Friis equation of the link ($\lambda = c/f = 0.05$m):

$$P_r = P_t + G_E + G_S - 20\log\left(\frac{4\pi R}{\lambda}\right)$$

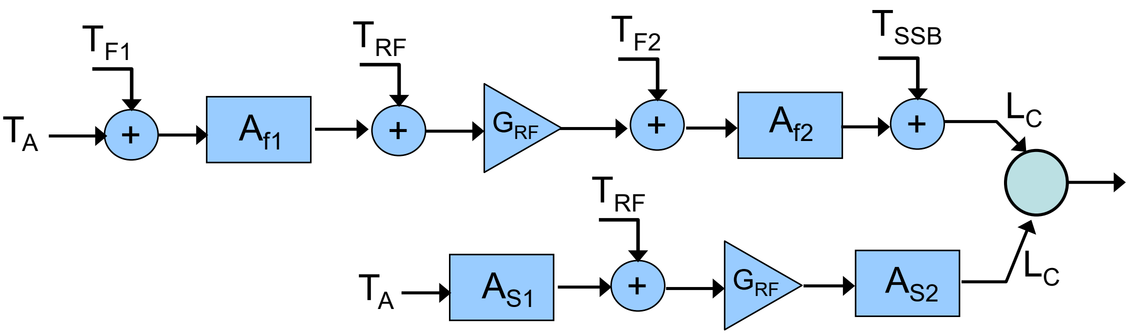
Assigning P_r=-65 dBm, we get G_S=-65-67-46.4+199.13=20.73 dB

Assuming unitary the directivity function:

$$\theta_B = \cos^{-1}\left(1 - \frac{2}{D_S}\right) = 10.55° \Rightarrow \Delta\theta = 2\theta_B = 21.1° \quad (>17.36°)$$

<u>Exercise 2</u>

1) The intermediate frequency (IF) is given by f_{RF}-f_{OL}=1 GHz. The image band is then given by f_{image}=f_{OL}+f_{IF}=12-12.5 GHz.
2) Equivalent scheme for the evaluation of the Noise Temperature:



Note that the two filters do not produce noise in the image band because the attenuation is generated by reflection in the filter stopband.

Evaluation of T_{eq}:

$$T_{F1} = 293\left(10^{\frac{A_{f1}}{10}} - 1\right) = 75.86°K, \quad T_{F2} = 293\left(10^{\frac{A_{f2}}{10}} - 1\right) = 171.37°K, \quad T_{RF} = 293\left(10^{\frac{NF}{10}} - 1\right) = 171.37°K$$

$$T_{eq} = T_a + T_{F1} + T_{RF}A_{f1} + T_{F2}\frac{A_{f1}}{G_{RF}} + T_{SSB}\frac{A_{F1}A_{F2}}{G_{RF}} + T_A\frac{A_{f1}A_{f2}}{A_{S1}A_{S2}} + T_{RF}\frac{A_{f1}A_{f2}}{A_{S2}} = 494.45\ °K$$

3) Evaluation of P_{IF}:

$$SNR_{sys} = \frac{P_r}{KT_{eq}B} = \left(\frac{E_b}{N_0}\right)\frac{R}{B} = 20 + 10\log(20/10) = 23 \text{ dB}$$

$$\mathrm{K}T_{eq}B = 1.38\cdot10^{-23}\cdot494.45\cdot10\cdot10^{6} = 6.82\cdot10^{-14} \quad (\text{-101.66 dBm})$$

$$P_r = SNR_{sys} + \mathrm{K}T_{eq}B\Big|_{dBW} = -108.65 \text{ dBW}$$

$$P_{IF} = P_r - A_{f1} + G_{LNA} - A_{f2} - A_c = -77.65 \text{ dBm}$$

Exercise 3

1) To get Γ_S we draw first the circle $|\Gamma_S|$=0.5. Then we draw some circle with G_{AV}=const. until we find the one tangent to the previous circle. We get $\Gamma_S=0.5\angle53°$ and G_{AV}=11.93 dB. To get the maximum gain we find Γ_L so that the output is matched: $\Gamma_L=0.328\angle-111.9°$. With this choice $G_T=G_{AV}$=11.93 dB.
2) For the input matching network we start from Γs as current point and draw the circle *g*=const passing for it. The we draw the circle *r*=1 and select the intersection for which Delta B is negative (C is removed from what we observe in the direction of load): b_C=-Delta B=0.922. In the intersection point what we observe is the impedance in the load direction, that is $z=1+jx_L$. The current point gives then x_L=1.213. The values of C and L are obtained be de-normalizing b_C and x_L: $C=0.02\cdot b_C/2\pi f$=0.147 pF, $L= 50\cdot x_L/2\pi f$=0.48 nH.
The output network require that the impedance seen at the input of the second transmission line is real. Then we start from Γ_L (current point) and draw the circle $|\Gamma|$=const. The first intersection with the horizontal axis in the load direction defines the length of the first line: ϕ_1=111.9/2=55.95°. The impedance in this point is also the input impedance of the second line: z_{in}=1.98. De-normalizing: $Z_{in}=50\cdot z_{in}$=99 Ohm. For the property of the $\lambda/4$ inverter: Z_{c2}=sqrt(50*99)=70.36 Ohm.

Exercise 4

We draw the mapping circle of source with $|\Gamma_{out}|$=1.2 and select one of the two points intersecting the unit circle: $\Gamma_1=1\angle143.3°$. Then we get z_{out}=-6.4-j5.39. We assign $z_L=r_{out}/3-x_{out}$=2.1+j5.39 corresponding to $\Gamma_2=.885\angle18.37°$.