## RF SYSTEMS

## Written Test of June 12, 2020

## Exercise 1



The communication system in the above figure is composed by the transmitter $\mathrm{T}\left(\mathrm{P}_{\mathrm{T}}=1 \mathrm{~W}, \mathrm{f}_{0}=60\right.$ GHz ) and the receiver R located at distance D and altitude H from T . The antennas of T and R are directed horizontally and the line connecting the antennas forms the angle $\theta$ with respect the horizontal line. The antennas are equal and their directivity function is shown in the above figure (assume $f(\varphi)=$ cost for $0<\varphi<2 \pi$ )

1) Compute the parameter $a_{0}$ of the directivity diagram to have the gain $G$ of the antennas equal to 10 dB . Assume the radiation impedance $\mathrm{Z}_{\mathrm{R}}=50 \Omega$ e the loss resistance $\mathrm{R}_{\mathrm{p}}=5 \Omega$.
2) Evaluate the received power in R for $\mathrm{D}=1 \mathrm{Km}$ and $\mathrm{H}=150 \mathrm{~m}$.

## Exercise 2

Consider the link in the previous exercise operating at 60 GHz and the received power $P_{r}=-78 \mathrm{dBm}$. The architecture of the receiving station is shown in the following figure:


1) Assuming the local oscillator frequency $\left(f_{L O}\right)$ smaller than the frequency of the received signal, determine $f_{L O}$ ) and the image frequency of the receiver.
2) It is know that the digital receiver must operate with $\left(\mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0}\right)=10 \mathrm{~dB}$ in order to get the data rate $\mathrm{R}=200 \mathrm{Mbit} / \mathrm{sec}$. Compute the $\mathrm{SNR}_{\text {sys }}$ of the system for a signal bandwidth $\mathrm{B}=100 \mathrm{MHz}$.
3) Evaluate the equivalent noise temperature of the system ( $\mathrm{T}_{\text {sys }}$ ) imposing $\mathrm{SNR}_{\text {sys }}$ and $P_{r}$ (received power). Hint: Boltzmann Constant $\mathrm{K}=1.38 \cdot 10^{-23}$.
4) Draw the equivalent scheme of the receiver with the noise temperature sources. Write the expression of $\mathrm{T}_{\text {sys }}$ as function of the blocks parameters. Imposing the required $\mathrm{T}_{\text {sys }}$ evaluate the gain GLNA (assume the equivalent noise temperature of the receiving antenna equal to $150{ }^{\circ} \mathrm{K}$ )

## Exercise 3

The following scheme refers to an oscillator working at 1 GHz . The S parameters of the transistor are also reported on the figure. The input line $\left(Z_{c}=50 \Omega\right.$, electrical length $\left.\phi_{1}\right)$ is short-circuited.


1) Evaluate $\Gamma_{\mathrm{L}}$ and $\Gamma_{\mathrm{S}}$ so that the starting of the oscillation is guaranteed (Hint: impose $\left|\Gamma_{\text {out }}\right|=1.4$ and chose for $\Gamma_{\mathrm{S}}$ the closest point to the short circuit)
2) Evaluate the length $\phi_{1}$ determining the required $\Gamma_{\mathrm{s}}$.
3) Design the output network, i.e. compute $\phi_{2}$ and the inductance $L$.

## SOLUTION

## Exercise 1

1) The gain is expressed as follows $(\eta=50 / 55=0.9091)$ :
$G=\frac{4 \pi \eta}{\int_{-\pi}^{\pi} \int_{0}^{\pi} f(\theta) \sin (\theta) d \varphi d \theta}=\frac{4 \pi \cdot 0.9091}{2 \pi\left[\left(1-\cos \left(\theta_{0}\right)\right)+a_{0} \cos \left(\theta_{0}\right)\right]}=10$
$a_{0}=0.1716$
2) From the Friis equation:
$P_{r}=P_{t}+G_{T}+G_{R}-L_{f}+f_{d B}=50-20 \log \left(\frac{4 \pi D}{\lambda_{0}}\right)+f_{d B}$
Being $\theta=\sin ^{-1}(\mathrm{H} / \mathrm{D})=0.1507<\pi / 20 \rightarrow f_{\mathrm{dB}}=0$. Then: $P_{r}=50-128=-78 \mathrm{dBm}$.

## Exercise 2

1) $f_{L O}=f_{R F}-f_{I F}=58 \mathrm{GHz}, f_{I M}=f_{L O-} f_{I F}=56 \mathrm{GHz}$.
2) The SNRsys is related to $\mathrm{Eb} / \mathrm{No}$ as follows:

$$
S N R_{s y s}=\frac{E_{b}}{N_{0}} \frac{R}{B}=10+10 \log \left(\frac{200}{100}\right)=13 \mathrm{~dB}
$$

3) It has:

$$
S N R_{s y s}=\frac{P_{r}}{K T_{s y s} B}=13 \mathrm{~dB} \Rightarrow \mathrm{~T}_{s y s}=575.6^{\circ} \mathrm{K}
$$

4) Equivalent scheme (the factor 2 in front of $T_{\text {LNA }}$ is due to the contribution from the image band):


The expression of Tsys results:

$$
T_{s y s}=T_{a}+T_{f}+2 T_{L N A} A_{0}+\frac{A_{0}}{G_{L N A}}\left(T_{S S B}+T_{e q} A_{c}\right)=575.6
$$

We get the following expression for Glna:

$$
G_{L N A}=\frac{A_{0}\left(T_{S S B}+T_{e q} A_{c}\right)}{T_{s y s}-\left(T_{a}+T_{f}+2 T_{L N A} A_{0}\right)}=35.3(15.5 \mathrm{~dB})
$$

## Exercise 3

- Draw the mapping circle of $\Gamma_{\mathrm{S}}\left(\left|\Gamma_{\text {out }}\right|=1.4\right)$. Choose the intersection with the outer circle closest to the c.c: $\Gamma_{\mathrm{s}}=1 \angle 173^{\circ}$.
- Evaluate $\mathrm{Z}_{\text {out }}=-1.988-\mathrm{j} 2.7$. Assign $\mathrm{Z}_{\mathrm{L}}=1.988 / 3+\mathrm{j} 2.7=0.663+\mathrm{j} 2.7$
- Evaluate $\phi_{\mathrm{s}}=\left(180-\angle \Gamma_{\mathrm{s}}\right) / 2=3.5^{\circ}$
- Design the output network (single stub):

Draw the circle $|\Gamma|=$ const passing for $\Gamma_{\mathrm{L}}$. Store $\Gamma_{\mathrm{L}}$. Draw the circle $\mathrm{g}=1$. Select the intersection where $\mathrm{b}<0$.

- Read the phase variation of $\Gamma\left(\Delta \Phi=110.18^{\circ}\right)$. Then $\phi_{2}=\Delta \Phi / 2=55.08$.
- Read the susceptance of the current point: $b=-3.316 . X=-50 / b=15.08=\omega \mathrm{L} \rightarrow \mathrm{L}=2.4 \mathrm{nH}$.

