RF SYSTEMS
12 July 2018

Surname & Name
Identification Number
Signature

Exercise 1

A broadcasting transmitter operates in FM commercial band (88-108 MHz) with P_T=1KW. The antenna has the efficiency η=0.9 and the following directivity function:

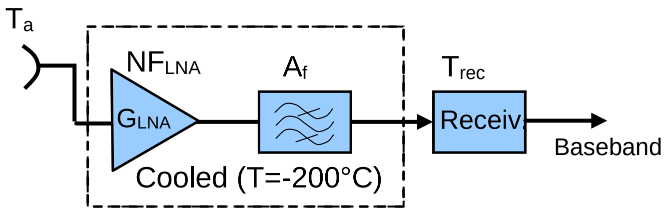
$f(\theta,\varphi)=1$ for $0<\theta<90°$, $0<\varphi<360°$, $f(\theta,\varphi)=0$ elsewhere

In the surroundings of the FM station, it is planned to build civil buildings. The current legislation however imposes severe limits for the maximum field in such buildings: $E_{max}<6$ V/m, $S_{max}<0.1$ W/m^2 (S_{max} represent the maximum power density)

1) Evaluate the gain of the antenna and the emitted ERP power
2) Evaluate the minimum distance from the antenna for which both the above limits are respected

Exercise 2

A space probe is sent in orbit around Saturn to take high-quality pictures of the planet surface. The probe has a dish antenna (4m diameter, aperture efficiency e_a=0.6) and a transmitter with P_T=30 W at 10 GHz. The receiving station on Earth employs an antenna with 70m diameter (e_a=0.65), connected to the receiver schematically depicted in the following figure.

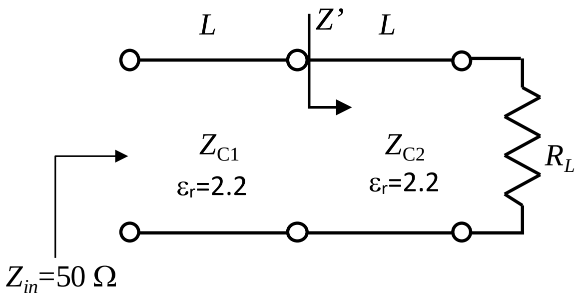


T_a=10°K
NF_{LNA}=0.5 dB
G_{LNA}=10 dB
A_f=0.5 dB
T_{rec}= 100 °K
K=$1.38 \cdot 10^{-23}$

1) Evaluate the gain of receiving and transmitting antennas
2) The distance Saturn-Earth is $1.43 \cdot 10^9$ Km. Compute the received power P_r at the antenna output
3) Evaluate the equivalent system temperature (T_{sys}) of the receiving system
4) Assuming the requested E_b/N_0=10 dB, evaluate the maximum data rate R allowed by the system
5) If the signal bandwidth *B* is 0.1 MHz, what is the SNR_{sys}?
6) The photos to be taken have a resolution of 10 Mpixels and are saved using jpeg compression. Assuming that the files so obtained are 10 Mbyte large, evaluate the time required to transmit a photo (assume 1byte=10 bit). What is the delay of the transmission?

Exercise 3

The task of the network in the following figure is to match the resistive load Z_L=125 Ω to Z_{in}=50 Ω at f_0=1250 MHz.



a) Assign the length L and compute Z_{c1}, Z_{c2} (impose Z'=79Ω)
b) Evaluate the magnitude of input reflection coefficient (in dB) at 800 MHz and 1700 MHz
Hint: this computation can be carried out analytically, using the formula of the input impedance of a transmission line terminated with an assigned Z_L, or by means the Electronic Smith Chart. We remind that the electrical length of a transmission line varies linearly with the frequency: $\beta L=\phi_0(f/f_0)$

Solutions

Ex.1

1) $$D_{MAX} = \frac{4\pi}{\int_0^{\pi/2} \sin\vartheta \, d\vartheta \cdot 2\pi} = \frac{2}{\left[-\cos\theta\right]_0^{\pi/2}} = 2$$

$$G = \eta \cdot D_{MAX} = 1.8 \quad (2.55\,dB)$$

$$ERP = P_T \cdot G = 1.8\ KW$$

2) $$d1 = \sqrt{\frac{ERP}{4\pi S_R}} = 37.85\,m$$

$$d2 = \frac{1}{E_{MAX}} \sqrt{\frac{Z_0 \cdot ERP}{2\pi}} = 54.77$$

$$d_{MAX} = d_2 = 54.77\,m$$

Ex. 2

$\lambda = 3\cdot 10^8 / 10^{10} = 0.03\,m \qquad P_T = 10\log_{10}(30) = 14.77\ dBW$

1) $G_T = e_{aT}\left(\frac{\pi d_T}{\lambda}\right)^2 = 1.053\cdot 10^5\ (50.22\ dB)$

$G_R = e_{aR}\left(\frac{\pi d_R}{\lambda}\right)^2 = 3.49\cdot 10^7\ (75.43\ dB)$

2) $P_R = P_T - 20\log\left(\frac{4\pi\cdot 1.43\cdot 10^{12}}{0.03}\right) + G_T + G_R = -155.12\ dBW$

3) $T_{sys} = T_a + T_{LNA} + \frac{T_0(a_f - 1)}{g_{lna}} + T_{Rec}\cdot a_f / g_{LNA}$

$T_0 = 273 - 200 = 73\,°K \qquad T_{LNA} = T_0\left(10^{NF/10} - 1\right) = 8.907\,°K$

$a_f = 10^{A_f/10} = 1.122 \qquad T_f = T_0(a_f - 1) = 8.907\,°K \qquad g_{lna} = 10$

$T_{sys} = 31.018\,°K$

4) $R = \frac{P_R}{K\cdot T_{sys}\cdot E_b/N_0} = 71.82\ Kbit/sec$

5) $SNR = P_R / (K\,T_{sys}\cdot B) = 8.56\ dB$

6) $S = 10\cdot 10^7\ bit \qquad T = \frac{S}{R} = 1390\ sec.$

Ex. 3

a) $L = \frac{\lambda_0}{4} = \frac{3\cdot10^8}{1.25\cdot10^6\cdot 4\sqrt{\varepsilon_2}} = 0.0405\ m$ $\qquad \beta L = \frac{\omega_0}{v} L = \frac{\pi}{2}$

$Z_{c2} = \sqrt{Z'\cdot R_L} = 99.37\Omega \qquad Z_{c1} = \sqrt{50\cdot Z'} = 62.85\Omega$

b) at $f_1 = 800\ MHz$: $\Phi_1 = \beta_1 L = \frac{2\pi f_1}{v} L = \frac{\pi}{2}\left(\frac{f_1}{f_0}\right) = 57.6^\circ$

$$Z_1 = Z_{c2}\,\frac{R_L + j Z_{c2}\tan(\Phi_1)}{Z_{c2} + j R_L \tan(\Phi_1)} = 88.33 - j\,18.5$$

$$Z_2 = Z_{c1}\,\frac{Z_1 + j Z_{c1}\tan(\Phi_1)}{Z_{c1} + j Z_1 \tan(\Phi_1)} = 43.655 - j\,11.03$$

$$|\Gamma| = \left|\frac{Z_2 - 50}{Z_2 + 50}\right| = 0.135 \quad (-17.4\ dB)$$

at $f_2 = 1700 \qquad \Phi_2 = \beta_2 L = \frac{\pi}{2}\left(\frac{f_2}{f_0}\right) = 122.4^\circ$

$$Z_1 = Z_{c2}\,\frac{R_L + j Z_{c2}\tan(\Phi_2)}{Z_{c2} + j R_L \tan(\Phi_2)} = 88.33 + j\,18.5$$

$$Z_2 = Z_{c1}\,\frac{Z_1 + j Z_{c1}\tan(\Phi_2)}{Z_{c1} + j Z_1 \tan(\Phi_1)} = 43.65 + j\,11.03 \rightarrow |\Gamma| = 0.1349 \quad (-17.4\ dB)$$