## RF SYSTEMS

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## Surname \& Name <br> Identification Number

## Signature

## Exercise 1



The figure shows a terrestrial link operating at 12 GHz with bandwidth $\mathrm{B}=100 \mathrm{MHz}$. The two stations are located at the maximum distance allowed by their elevation $(1000 \mathrm{~m}$ and 15 m respectively). The antennas are identical and exhibit the efficiency $\eta=0.85$ and the equivalent beamwidth $\Delta \theta=10^{\circ}$.

1) Evaluate $L_{\text {max }}$
2) Evaluate the gain $G$ of the antennas

The following figure shows the scheme of the receiving station R with the relevant parameters.

3) Evaluate the received power $P_{r}$ assuming the transmitted power $P_{t}=10 \mathrm{~W}$
4) Imposing $\mathrm{SNR}=30 \mathrm{~dB}$ at the receiving antenna, compute the required equivalent temperature $\mathrm{T}_{\mathrm{eq}}$ of the receiver (at the input, including the antenna)
5) Evaluate $N F_{L N A}$ in order to get the requested $T_{\text {eq }}$
6) If the LNA and the second filter were removed, what would be the new value of $T_{\text {SSB }}$ keep $\mathrm{T}_{\text {eq }}$ unchanged?
7) Determine the minimum value of $\mathrm{P}_{1 \mathrm{~dB}}$ of the transmitter in order to obtain the mean power of intermodulation at the receiver input equal to the system noise power (use a 2-tone signal with 10 W mean power).

## Exercise 2

The following figure represents the scheme of a microwave oscillator that is required to oscillate at 2.5 GHz . The scattering parameters of the active device are reported on the figure


$$
\begin{aligned}
& \mathrm{S}_{11}=0.772 \angle 6.17^{\circ} \\
& \mathrm{S}_{12}=0.2 \angle-82.08^{\circ} \\
& \mathrm{S}_{21}=2.224 \angle-137.15^{\circ} \\
& \mathrm{S}_{22}=0.508 \angle 37.07^{\circ}
\end{aligned}
$$

1) Evaluate the reflection coefficients $\Gamma_{1}$ and $\Gamma_{2}$ ensuring the start of oscillation and the best power transfer to the load (assign the magnitude of $\Gamma_{1}$ equal to 1 ). Hint: draw the mapping circle of the source with $\left|\Gamma_{\text {out }}\right|=1.3$ for determining $\Gamma_{1}$.
2) Assign the resonance frequency of the shunt resonator in the input network equal to the oscillation frequency and evaluate the electrical length $\phi_{L}$ so that $\Gamma_{1}$ has the value computed previously
3) Design the network OUT, using the scheme in the figure (evaluate the susceptances $B_{1}$ and $B_{2}$ ).
4) Verify if oscillation can occur at 2.51 GHz

## Solution

## Exercise 1

$\left(L_{\max }\right)_{K m}=L_{1}+L_{2} \simeq 3.57\left(\sqrt{\left(H_{1}\right)_{\text {meter }}}+\sqrt{\left(H_{2}\right)_{\text {meter }}}\right)=126.71 \mathrm{Km}$
$\Delta \theta=2 \cos ^{-1}\left(1-\frac{2 \eta}{G_{R}}\right) \Rightarrow \quad G_{R}=\frac{2 \eta}{1-\cos (\Delta \theta / 2)}=446.74(26.5 \mathrm{~dB})$
Wavelength: $\lambda=300 / \mathrm{f}_{0}=25 \mathrm{~mm}$
Link equation (in dBW):

$$
P_{r}=P_{t}+20 \log \left(\frac{\lambda}{4 \pi L_{\max }}\right)+G_{T}+G_{R}=-63 \mathrm{dBm}
$$

Evaluation of $\mathrm{T}_{\mathrm{eq}}$ :

$$
P_{r}-10 \log \left(K T_{e q} B\right)=30 \Rightarrow T_{e q}=10^{\left(P_{r}-10 \log (K B)-30\right) / 10}=356.55^{\circ} \mathrm{K}
$$

Evaluation of $\mathrm{NF}_{\mathrm{LNA}}$ :
$A_{f 1}=1.023, A_{f 2}=1.122, A_{c}=2.51, T_{f 1}=T_{0}\left(10^{A_{11} / 10}-1\right)=6.825, \quad T_{f 2}=35.75$


$$
\begin{aligned}
& T_{e q}=T_{a}+T_{f 1}+A_{f 1} T_{L N A}+\frac{A_{f 1}\left[T_{S S B} A_{f 2}+T_{f 2}\right]}{G_{L N A}} \\
& T_{L N A}=\frac{T_{e q}-T_{a}-T_{f 1}-\frac{A_{f 1}\left[T_{S S B} A_{f 2}+T_{f 2}\right]}{G_{L N A}}}{A_{f 1}}=157.94{ }^{\circ} \mathrm{K} \\
& N F=10 \log \left(10^{N F / 293}-1\right)=1.87 \mathrm{~dB}
\end{aligned}
$$

Removing the second filter:
$T_{e q}=T_{a}+T_{f 1}+A_{f 1} T_{S S B} \Rightarrow T_{S S B}=\frac{T_{e q}-T_{a}-T_{f 1}}{A_{f 1}}=195.33{ }^{\circ} \mathrm{K}$
Evaluation of $\mathrm{P}_{1 \mathrm{~dB}}$ :
Note CI is unchanged at the receiver. So, with $\mathrm{P}_{\mathrm{t}, \mathrm{int}}$ the transmitted intermodulation power, $\mathrm{P}_{\mathrm{r}, \text { int }}$ the received intermodulation power we have:
$C I=P_{t}-P_{\mathrm{t}, \text { int }}=P_{r}-P_{\mathrm{r}, \text { int }}=P_{r}-K T_{e q} B=S N R=30 \mathrm{~dB}$
From the expression of CI we derive $\mathrm{IP}_{3}$ and $\mathrm{P}_{1 \mathrm{~dB}}$ :
$I P_{3}=\frac{C I+2 P_{t}-6}{2}=52 \mathrm{dBm}, P_{1 d B}=I P_{3}-10.63=41.37 \mathrm{dBm}$

## Exercise 2

The assigned transistor is potentially instable ( $\mathrm{k}=0.61$ ), so it can be used for realizing an oscillator. Using the electronic Smith Chart, the mapping circle with $\left|\Gamma_{\text {out }}\right|=1.3$ is drawn. One of the two intersection with the outer circle is then selected: $\Gamma_{1}=1 \angle-39.5^{\circ}$.
Selecting "S Param." $\rightarrow$ "Gamma OUT" the reflection coefficient at port 2 is obtained: $\Gamma_{\text {out }}=1.3 \angle 45.72^{\circ}$. The S. chart reports also the normalized impedance $Z_{\text {out }}=-0.793+\mathrm{j} 2.125$. Imposing the condition starting the oscillation, the values of $Z_{2}$ and $\Gamma_{2}$ are then obtained: $\mathrm{Z}_{2}=0.2643-\mathrm{j} 2.125 \rightarrow \Gamma_{2}=0.91 \angle-49.85^{\circ}$.

At the oscillation frequency, the shunt resonator is an open circuit, so the length $\phi_{\mathrm{L}}$ is given by: $\phi_{\mathrm{L}}=-\angle\left(\Gamma_{1}\right) / 2=19.75^{\circ}$.

The double-stub matching network is designed according the following procedure:


1) Read the normalized admittance at $\Gamma_{2}$ from the S. Chart: $y_{2}=0.058+j 0.463$
2) Draw the circle with constant conductance $g=g_{2}=0.058$ rotated of $270^{\circ}$ toward the load
3) Draw the circle $g=1$
4) Select one of the two intersections between the above circles: $\Gamma_{A}=0.922 \angle-157.29^{\circ}$. The value $\mathrm{y}_{\mathrm{A}}$ must be in the form $y_{A}=1+\mathrm{jb}_{2}=1+\mathrm{j} 4.77$. Then $\mathrm{b}_{2}=4.77$.
5) Rotate $\Gamma_{\mathrm{A}}$ toward the source of $-270^{\circ}$, arriving at $\Gamma_{\mathrm{A}}=0.922 \angle-67.29^{\circ}$. The normalized admittance results $y_{B}=0.058+j 0.664$
 the imaginary part of $y_{B}$ from the imaginary part of $y_{2}: b_{1}=0.664-0.463=0.201$

At $\mathrm{f}=2.51 \mathrm{GHz}$ the susceptance of the shunt resonator results $B=2 \pi f \cdot C_{e q}\left(\frac{f}{f_{0}}-\frac{f_{0}}{f}\right)=0.05036 \mathrm{~S}$
$\left(\mathrm{B} / \mathrm{Y}_{\mathrm{c}}=0.2518\right)$. With the S.C. we can compute the value of $\Gamma_{1}$, resulting $1 \angle-67.77^{\circ}$, which produces $\left|\Gamma_{\text {out }}\right|=0.983$. The start of the oscillation in then impossible.

