RF SYSTEMS
Written Test on 16 February 2022

Surname & Name
Identification Number
Signature

Exercise 1

An antenna is given with the following directivity diagram ($f(\phi)=1$):

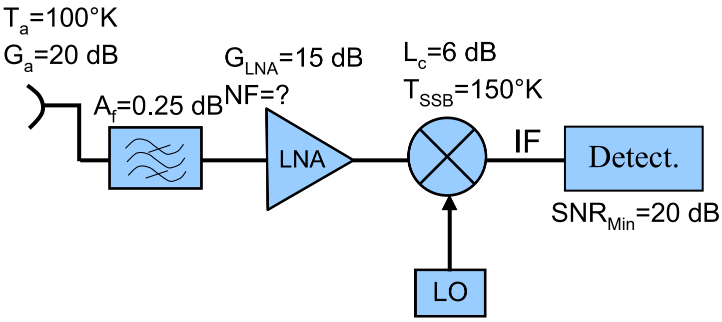
$$f(\theta)=\begin{cases}\cos\left(\dfrac{\theta}{\theta_0}\dfrac{\pi}{2}\right) & 0<\theta<\theta_0\\ \alpha & \theta_0<\theta<\pi/2\\ 0 & \pi/2<\theta<\pi\end{cases}$$

with: θ_0=10° and α=0.01. Note that the first term of $f(\theta)$ represents the main lobe of the antenna while the second term can be considered as the side lobe.

1) Evaluate the exact directivity gain D
   (hint: $\int\cos(ax)\sin(x)dx=\dfrac{\cos(x(a-1))}{2(a-1)}-\dfrac{\cos(x(a+1))}{2(a+1)}$)
2) Define the antenna beamwidth and evaluate it
3) Compute the antenna directivity gain discarding the side lobe. What is the value of the antenna beam efficiency?
4) This antenna is connected to a transmitter with available power P_T=10 W. At 100m from the antenna, in the direction of the maximum of $f(\theta)$, the measured power density is $S_R=8.5\cdot10^{-3}$ W/m^2. Evaluate the antenna efficiency η and the gain G.

Exercise 2

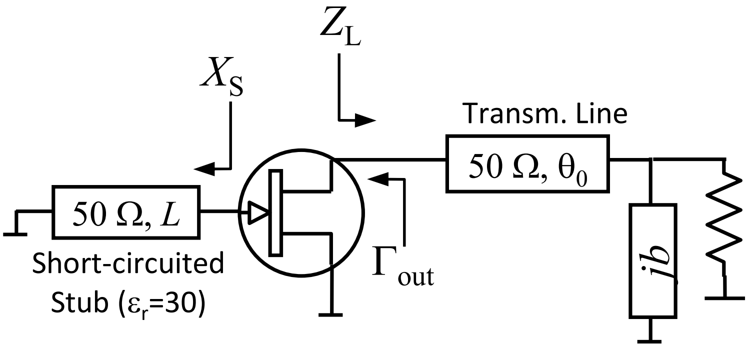
The receiver of a radar system has the RF front-end depicted in the following figure:



The radar operates at 10 GHz with the transmitted power P_T=100 W and the IF bandwidth B=0.1 MHz. Note that the IF detector requires the minimum SNR equal to 20 dB.
The radar must be able to detect a target with radar cross section σ=10m at a distance not smaller than R_{max}=3 Km.

1. Evaluate the power P_S at the antenna output produced by the echo of the target at the distance R_{max}.
2. Draw the equivalent noise scheme of the radar front-end and write the expression of the equivalent noise temperature at the input of the receiver (explain the scheme!)
3. Determine the equivalent noise power defined at the antenna input allowing the target detection (justify the answer!)
4. Evaluate the LNA noise figure determining the noise power computed at the previous point

<u>Exercise 3</u>



The scheme in the figure represents an oscillator operating at 6 GHz. The active device is characterized by the following S parameters:

$S_{11}=0.885\angle{-73.51°}$ $S_{21}=3.12\angle\ 119.76°$ $S_{12}=0.1152\ \angle 48.7°$ $S_{22}=0.5629\angle\ {-46.66°}$

The input short-circuited stub is realized with a transmission line having 50 Ohm characteristic impedance and ε_r=30.

1) Knowing that $|\Gamma_{out}|$=1.7, evaluate the normalized reactance X_s (<1) of the input stub and the length L of the stub with the constraint that L>10mm (Hint: the reactance of the stub vs. L is periodic with period $\lambda_0/2$. This means that you get the same value of X_s by adding $n{\cdot}\lambda_0/2$ to its electrical length).
2) Design the output network imposing $z_L=-r_{out}/3-jx_{out}$.
3) It is known that the oscillation may arise between 5.8 and 6.2 GHz. Starting from these limits, find the actual minimum and maximum frequencies where the start of the oscillation is actually possible (check with a step not larger than 0.05 GHz). Assume that the S parameters of the transistor and the impedance Z_L do not change in the considered range while the reactance X_s varies with the frequency as imposed by the stub. It is reminded that the condition of oscillation involves both X_S and Z_L.
4) It is known that the indirect stability coefficient (S) of an oscillator can be expressed as a function of the G_{loop} phase. Assuming this phase equal to the electrical length of the short-circuited stub (in a first approximation), compute the value of S of the oscillator.
5) Suppose that the length of the stub changes by 0.1 mm due to a temperature variation. Using the computed value of S, find the variation of the oscillation frequency.

Solutions

Exercise 1

1) The directivity gain is obtained by the general formula:

$$D = \frac{4\pi}{\int_0^{2\pi} d\varphi \int_0^{\pi} f(\vartheta,\varphi)\sin(\theta)d\theta} = \frac{2}{\int_0^{\theta_0} \cos\left(\frac{\theta}{\theta_0}\frac{\pi}{2}\right)\sin(\theta)d\theta + \alpha\int_{\theta_0}^{\pi/2} \sin(\theta)d\theta}$$

$$= \frac{2}{7.03542\cdot 10^{-3} + 9.848\cdot 10^{-3}} = 118.46 \text{ (20.73 dB)}$$

2) The beamwidth (BW) is defined as the aperture of the antenna main lobe for which $f(\theta)$=0.5. It has BW=2θ_{3dB}, where $f(\theta_{3dB})$=0.5 → $\theta_{3dB} = \theta_0 \frac{2}{\pi}\cos^{-1}(0.5) = 0.1163 \text{ rad } (6.667°)$. Then BW=13.33°.

3) In this case it has: $D = \frac{2}{7.03542\cdot 10^{-3}} = 284.27 \text{ (24.54 dB)}$. The antenna beam efficiency is the ratio between the gain discarding the side lobe and the actual gain. In this case it result: BE=118.46/284.27=0.4167

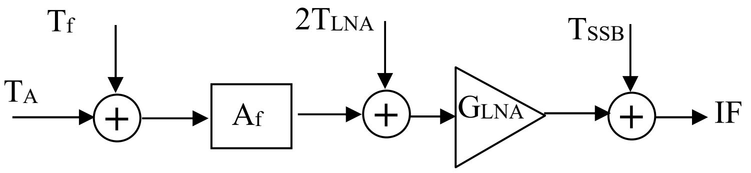
4) The power density SR at the distance R from the antenna along the maximum of the directivity diagram (θ=0) is given by: $S_R = \frac{P_{rad}}{4\pi R^2}D$. Then $P_{rad}=S_R 4\pi R^2/D$=9.017 W/m^2. The radiated power is related to the available power of the transmitter by the efficiency factor: $P_{rad}=\eta P_T$. Then $\eta=P_{rad}/P_T$=0.9017 and G= η D=106.81 (20.28 dB).

Exercise 2

1) With G_a=20 dB and λ=300/10=30mm, the effective area of the antenna results: $A_R=G_a\lambda^2/(4\pi)=7.16\cdot 10^{-3}$ m^2. The received power is then obtained from the following equation:

$$P_{R,\min} = \sigma \frac{P_T G_a A_R}{(4\pi)^2 R_{\max}^4} = 5.599\cdot 10^{-14} \text{ W (-102.52 dBm)}$$

2) This is the equivalent noise temperature scheme of the front end:



Note the term 2 in front of T_{LNA}: it is due to the contribution of the noise produced by the LNA in the image band.
The expression of the equivalent temperature of the receiver (T_{eq}) is the following:

$$T_{eq} = T_a + T_f + 2a_f T_{LNA} + \frac{T_{SSB}a_f}{g_{LNA}} = 187.23 + 2.1185\cdot T_{LNA} \text{ , with}$$

$$T_0 = 293°K,\ a_f = 10^{0.025},\ T_f = T_0(a_f - 1) = 17.36°K,$$

3) The required SNR holds true also at the antenna output. Then the overall power noise is given by $P_N = P_{R,min}/SNR = 5.599 \cdot 10^{-16}$ W (-122.52 dBm).
4) The equivalent noise temperature determines the overall noise power at the antenna terminal: $P_N = KT_{eq}B = 5.599 \cdot 10^{-16}$ W. The equivalent temperature of the receiver is then given by: $T_{eq} = 4.535 \cdot 10^{-14}/(K \cdot B) = 405.74$ °K
5) Imposing the computed value of T_{eq} in the expression derived at point 2 we get: $T_{eq} = 187.23 + 2.1185 \cdot T_{LNA} = 405.74 \rightarrow T_{LNA} = 133.75°K$. The noise figure of LNA is then: $NF = 10 \cdot \log10(1 + T_{LNA}/293) = 1.63$ dB

Exercise 3

Introducing the S parameters in the S.C. we discover that the device is potentially instable (K=0.226), so it can be used for the oscillator.

1) The mapping circle with $|\Gamma_{out}|=1.7$ is drawn and the intersection with the outer circle where $X_s/Z_0=0.975$ is selected. The imposing $X_s/Z_0=\tan(\theta_0)=0.975$ we get $\theta_0=\beta_0 L=44.275°$, with $\beta_0 = 360 f_0 \sqrt{\varepsilon_r}/c$. Replacing we get L=1.123mm, which however does not satisfy the constraint L>10mm. As suggested, we can add the length corresponding to a multiple of $n\lambda_0/2 = n \cdot 4.564$mm, determining the same X_s. Whit n=2 we get L=1.123+9.13=10.25mm.
2) Imposing $X_s/Z_0=0.975$ we get $Z_{out}/Z_0=-1.374-j1.664$. Then $Z_L/Z_0=0.457+j1.664$. The output network is a single stub transforming network. We start assigning the current point to Z_L/Z_0 and storing it in memory. Then draw the circle g=1 and $|\Gamma|$=const passing for the current point. Clicking on one of the intersections we arrive to the end of the line, where the observed admittance in the direction of the load is 1+jb. The phase variation ($\Delta\theta$=83°) gives 2 times the electrical length θ_0. Then: = $\Delta\theta/2$=41.5°. The value b=-2.59 is read from the current point (Y tab).
3) The conditions for the start of the oscillation are $|\Gamma_{in}|>1$, $|\Gamma_{out}|>1$. The first condition can be verified only once because Z_L is assumed frequency independent. Assigning $Z_L/Z_0=0.457+j1.664$ and selecting "S Param." → "Gamma In" in the Smith Chart Menu, we get $|\Gamma_{in}|=1.267$, so the first condition is always verified. The second condition requires to compute first $X_S/Z_0 = \tan\left(360 f \sqrt{\varepsilon_r}/c\right)$ for f=[5.8, 5.85, 5.9, 6.1, 6.15, 6.2]: X_s/Z_0=[0.462, 0.5696, 0.688, 1.377, 1.656, 2.028]. Now we draw the stability circle of the generator on the S.C. and enter the values of X_s/Z_0. The first and last point inside the circle define the required frequency range (in fact, for X_s/Z_0 inside the circle $|\Gamma_{out}|>1$ and the start of oscillation is assured). These points are the third (5.9 GHz) and the fourth (6.1 GHz).
4) The indirect stability coefficient is given by $S = \omega_0 \left.\frac{\partial\phi}{\partial\omega}\right|_{\omega=\omega_0}$ where ϕ is the phase of G_{loop}. Assuming this phase equal to the electrical length of the input stub ($\phi = \beta L = \omega\sqrt{\varepsilon_r}L/c$) it has: $S = \beta_0 L = \omega_0\sqrt{\varepsilon_r}L/c = 584.28°$
5) The variation Δf of the oscillation frequency for a variation $\Delta\phi$ of the phase is given by $\Delta f = f_0 \Delta\phi / S$. The considered length variation (0.1mm) produces a phase variation $\Delta\phi$ =3.94°. Replacing we get Δf=0.0405 GHz.