RF SYSTEMS
Written Test on 20 January 2022

Surname & Name
Identification Number
Signature

<u>Exercise 1</u>

A dish antenna operating at 12 GHz has the directivity function depicted in Fig. A ($f(\varphi)$=1 everywhere):

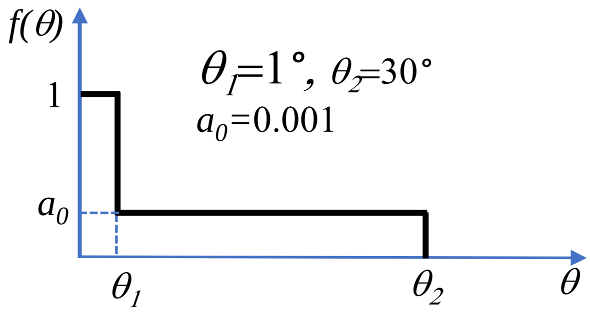


Fig. A

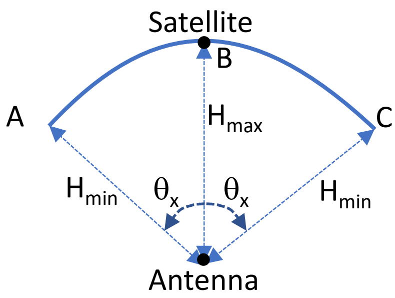
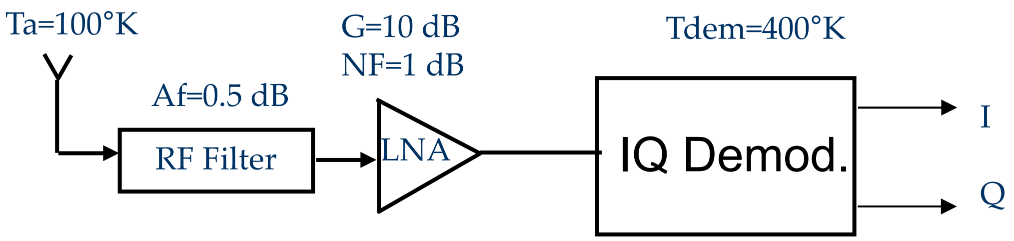


Fig. B

1) Evaluate the directivity gain D
2) The dish diameter D=1m and the efficiency factor η=0.85 are known. Evaluate the aperture efficiency (e_a) of the dish.
3) The dish is used in a receiving station of a LEO (Low Earth Orbit) satellite system. During the period the satellite is visible the antenna is pointed at the centre of the satellite path (Fig. B). The path is an elliptical arc with maximum and minimum distance from the receiving station equal to H_{max}=750 Km and H_{min}=400 Km. Note that the antenna is pointed toward the maximum distance position (point B), while the angle θ of the directivity diagram is θ_x = 1.5° at points A and C (θ=0 in B).
   Assuming 500 W the ERP (effective radiated power) from the satellite, evaluate the power at the receiving antenna output when the satellite is at the position A, B, C.

Exercise 2

The general scheme of a direct-conversion receiver for M-QAM signals at 1.8 GHz (bandwidth B=15 MHz) is shown in the following figure (the relevant parameters are also specified).

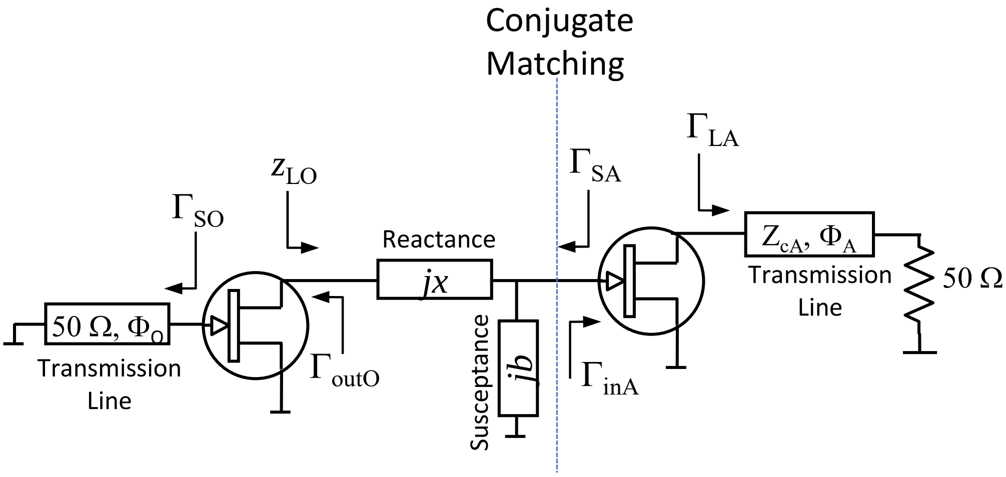


1) Data rate R is required to be greater than 100 MB/s. Assuming α=0.3 the roll-off factor of the raised-cosine filter, evaluate the parameter M of the modulation and the actual value of R (note that M must be a power of 2)
2) Evaluate the equivalent noise temperature (T_{REC}) at the input of the receiver
3) The received signal ($Pr=10^{-10}$ W) is generated with a transmitter affected by non-linear distortion. The ratio between the signal power and the distortion power is CI=40 dB. Evaluate the equivalent temperature T_{NL} producing a white noise with the same power of the distortion (in the signal band).
4) Consider the system noise temperature (Tsys) as the sum of the receiver temperature T_{REC} and the non-linear distortion temperature (T_{NL}). Evaluate the required E_b/N_0 ratio (in dB) for the assigned (R/B).

Exercise 3

The scheme in the figure represents an oscillator operating at 10 GHz whose output feeds an amplifier (used to increase the output signal power and reduce the load pulling effect).
Both oscillator and amplifier use the same transistor whose scattering parameters are the following:

$S_{11}=0.73\angle -102°$ $S_{21}=2.27\angle 96°$ $S_{12}=0.1\angle 42°$ $S_{22}=0.54\angle -49°$



1) Design the amplifier stage (i.e., evaluate Γ_{SA} and Γ_{LA}) by imposing the conjugate matching at the input of the transistor and the maximum transducer gain compatible with stability and with the condition imposed by the output network (Γ_{LA} real).
2) Evaluate the characteristic impedance Z_{cA} and the electrical length Φ_A of the output transmission line.
3) Design the oscillator by evaluating the required values of Γ_{SO} and z_{LO} (impose $|\Gamma_{outO}|=1.1$). Chose Γ_{SO} for the minimum value of Φ_O (to be specified). Verify that the oscillation conditions are satisfied.
4) Evaluate jx and jb by imposing the transformation of Γ_{inA} into z_{LO}.

Solutions

Exercise 1

1) The directivity gain is obtained by the general formula:

$$D = \frac{4\pi}{\int_0^{2\pi} d\varphi \int_0^{\pi} f(\vartheta,\varphi)\sin(\theta)d\theta} = \frac{2}{\int_0^{\theta_1}\sin(\theta)d\theta + a_0\int_{\theta_1}^{\theta_2}\sin(\theta)d\theta}$$

$$= \frac{2}{1-\cos(\theta_1)+0.001\cdot(\cos(\theta_1)-\cos(\theta_2))} = 6990 \text{ (38.4 dB)}$$

2) The gain of the antenna is given by: G=η·D=5941.4 (37.7 dB). Using the formula expressing the gain of the dish as function of the diameter it has: $G = A_e 4\pi/\lambda^2$ with $A_e = e_a \pi d^2/4$. Imposing the same value of G, we obtain the parameter e_a: $e_a = G\left(\frac{\lambda}{\pi d}\right)^2 = 0.376$ (λ=300/*f*=25mm).

3) The received power is given by the Friis equation (in dB): P_r=P_{ERP}+G_{dB}-20·log(4π·H/λ)+10·log(*f*(θ)). With the satellite at B, θ=0 → *f*=1 then Pr=57+37.7-171.53=-76.8 dBm. In the positions A and C: θ =1.5° → *f*=0.001 (-30 dB), then: Pr=57+37.7-166.1-30=-101.4 dBm.

Exercise 2

1) The relationship between bandwidth and data rate for a M-QAM signal is: $B = \frac{R}{\log_2 M}\cdot(1+\alpha)$.

   Assuming the minimum value of R equal to100 MB/s, α=0.3, with B=15 MHz we have: $\log_2 M = \frac{R}{B}\cdot(1+\alpha) = 8.6667$. Being M a power of 2, it has: M=2^9=512. The actual data rate is then $R = B\cdot\frac{\log_2 M}{(1+\alpha)} = 103.85$ MB/s

2) The equivalent noise temperature reported at input is given by:

   $T_{REC} = T_a + T_f + a_f T_{LNA} + \frac{T_{dem} a_f}{g_{LNA}}$, with $T_0 = 293°K$, $a_f = 10^{0.05}$, $T_f = T_0(a_f - 1) = 35.75°K$, $T_{LNA}=T_0(10^{0.1}-1)$=75.86°K. Replacing: T_{REC}=265.75°K.

3) The distortion power is given by P_D=Pr/CI=10^{-14} W. Imposing P_D=K·T_{NL}·B, we get T_{NL}=48.3 °K.

4) The equivalent System temperature including the contribution from non-linear distortion is T_{SYS}=T_{REC}+T_{NL}=314.05 °K. The formula for the SNR is: $SNR_{sys} = \frac{P_r}{KT_{sys}B} = \left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right)$, from which we get: $\left(\frac{E_b}{N_0}\right) = \left(\frac{B}{R}\right)\frac{P_r}{KT_{sys}B} = 222.18 \text{ (23.47 dB)}$.

Exercise 3

Introducing the S parameters in the S.C. we discover that the device is potentially instable (K=0.714, MSG=13.56 dB), so it can be used both for the oscillator and the amplifier stage.

1) We design the amplifier stage using the Power Gain (G_P), being the input matching required. The selected Γ_{LA} must be on a circle with G_P =const and also on the horizontal axis (real requirement). If we draw the circle for G_P=MSG, we see that this circle does not cross the horizontal axis. We have then to reduce G_P until the corresponding circle touches (i.e. it is tangent) to the horizontal axis. This happens for G_P=10.534, giving the tangent point at Γ_{LA}=0.135. The conjugate matching at input is obtained with $\Gamma_{SA}=0.713\angle 104.16$. Note that both Γ_{LA} and Γ_{SA} are outside the instability regions of load and source and are then admissible.
2) The impedance corresponding to Γ_{LA} is given by $Z_{LA}=50\cdot 1.313=65.65$. The transmission line at the output of the amplifier is a quarter wavelength transformer (Φ_A=90°) with characteristic impedance $Z_{cA}=\sqrt{50\cdot Z_{LA}}=57.29\ \Omega$.
3) The mapping circle of Γ_S is first drawn with $|\Gamma_{outO}|=1.1$. This circle crosses the outer circle of the S.C. in two points: the one to be selected is the closer to the short circuit (so the length of the input transmission line is the shortest). It has $\Gamma_{SO}=1\angle 121.94°$ and $\Phi_O=(180-121.94)/2=29.03°$ We then compute Gamma OUT at the transistor output: $\Gamma_{outO}=1.1\angle -55.47$, $z_{outO}=-0.218-j1.882$. The impedance z_{LO} is then assigned as $z_{LO}=-r_{outO}/3-jx_{outO}=0.073+j1.882$.
4) The reflection coefficient at the input of the amplifier is determined by the matching condition: $\Gamma_{inA}=\Gamma^*_{SA}=0.713\angle -104.16$. The input admittance results $y_{inA}=0.424+j1.192$. To design the transforming network with the S.C. we draw the circles $g=g_{inA}=0.424$ and $r=r_{LO}=0.073$. We then store y_{inA} in memory and select one of the intersections of the two circles. The value read on DeltaY tab (imaginary) represents *b*=1.185. Now store this point in memory and enter with z_{LO}. The value read on Delta Z tab represents *x*=2.29.