RF SYSTEMS

Written Test of June 20, 2019

Surname & Name
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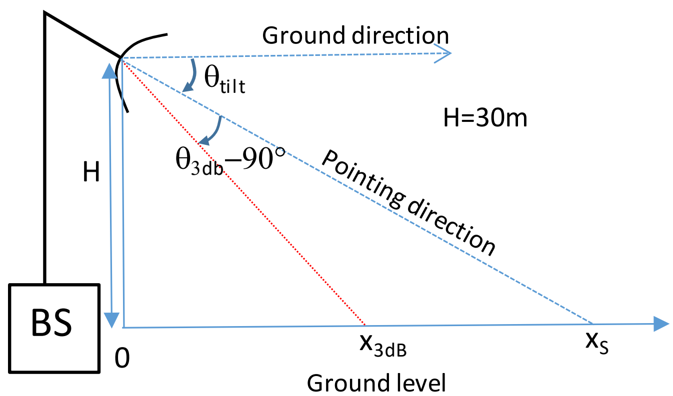
Exercise 1 (8+2)

Consider the antenna used in a base station for LTE service, placed at H=30 m above ground. The operating (downlink) frequency is f_0=2.12 GHz.



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The directivity function is expresses as:

$$f(\varphi,\theta) = f_\varphi(\varphi)\cdot f_\theta(\theta) \qquad \varphi\text{: azimuth, } \theta\text{: elevation}$$
$$f_\varphi(\varphi) = \cos(\varphi) \qquad -\pi/2 \le \varphi \le \pi/2, \quad 0 \text{ elsewhere}$$
$$f_\theta(\theta) = \sin(5\theta) \qquad 2\pi/5 \le \theta \le 3\pi/5, \quad 0 \text{ elsewhere}$$

Note that angles θ and φ refer to the pointing direction (θ =90°, φ=0°). This direction is tilted by θ_{tilt} with respect to the direction parallel to ground in order to reduce the interference among adjacent cells.

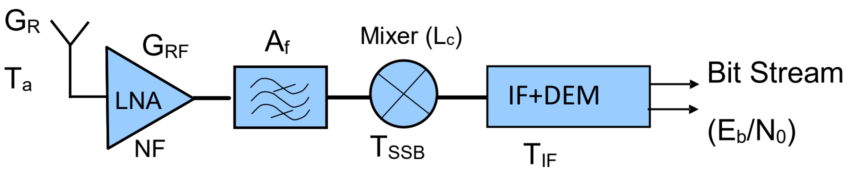
1) Specify the angles θ_{max} and φ_{max} where $f(\theta,\varphi)$ is maximum
2) Compute the antenna gain (efficiency factor η=0.85).

   Hint: $\int \sin(x)\sin(5x) = \dfrac{\sin(4x)}{8} - \dfrac{\sin(6x)}{12}$
3) x_S represents the distance from the base station at ground level determined by antenna tilting (see the figure). Imposing x_S=1.5 Km, determine the value of θ_{tilt}
4) The transmitter power level is 30 dBm. Evaluate the power density (W/m^2) at x_S (with $\varphi = \varphi_{max}$).

   (Optional)
5) Evaluate the angles θ_{3dB} and φ_{3dB} where f_φ and f_θ are reduced by -3dB with respect to the optimum pointing (Hint: there are two values for each angle)
6) Evaluate also the power density at x_{3dB} (the point at ground level determined by θ_{3dB}). Assume $\varphi = \varphi_{3dB}$

Exercise 2 (8)

The scheme in the figure represents the RF front-end of a phone receiver using the BS of the previous exercise.



- G_R=1dB
- NF= 7
- G_{RF}=10 dB
- T_a=300 °K
- A_f=2 dB
- L_c=6 dB
- T_{SSB}=500 °K
- T_{IF}=500 °K

a) Draw the scheme of the receiver referred to the equivalent noise sources. Evaluate the equivalent noise temperature (T_{eq}) at the receiver input. Compute the noise power at the receiver input (assume B=20 MHz). Use $K=1.38 \cdot 10^{-23}$ (Boltzmann Constant)

b) Write the expression of the received power (at the antenna output) as function of the distance R from the BS

c) It is requested the data rate R=300 Mbit/sec, with (E_b/N_0) =10 dB. Evaluate the corresponding SNR of the receiver.

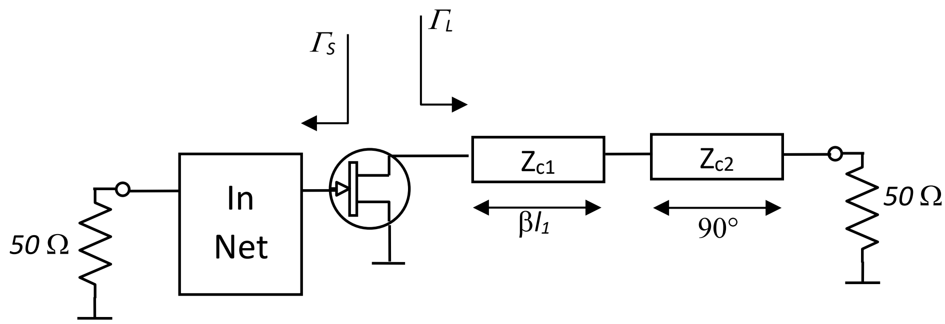
d) Evaluate the maximum distance *R* from the BS at which the data rate R is still guaranteed.

Exercise 3 (8)

The following scheme represents an amplifier operating at 12 GHz.

a) Determine the values Γ_s and Γ_L for the maximum transducer gain

b) Evaluate the unknown parameters (Z_{c1}, Z_{c2}, βl_1) of the output matching network

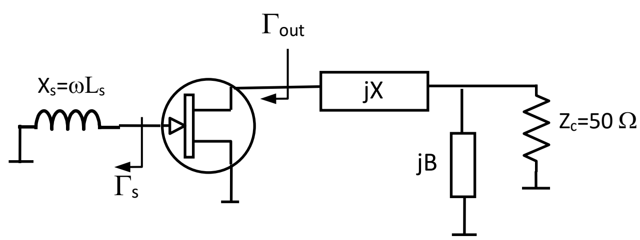


Scattering parameters of the transistor:

$S_{11}=0.66\angle 146°$ $S_{21}=2.39\angle 45°$ $S_{12}=0.088\angle 69°$ $S_{22}=0.3\angle -48°$

Exercise 4 (9)

The following scheme refers to an oscillator working at 432 MHz. The S parameters of the transistor are also reported on the figure.



$S_{11}=0.54\angle -90°$

$S_{12}=0.01\angle 66°$

$S_{21}=45.38\angle 137°$

$S_{22}=0.76\angle -26°$

1) Select a suitable value for L_S (use the mapping circles of Γ_s for obtaining $|\Gamma_{out}|=1.5$)
2) Evaluate the parameters of the output network (jX, jB) to ensure the start of oscillation and the transfer of the output power to the external load (50 Ω).
3) Find the values of the lumped elements parameters implementing X and B (select an inductor or a capacitor depending on the sign of X and B)

Solution

Exercise 1

1) θ_{max}=90°, φ_{max}=0°

2) The antenna gain is obtained from the formula:

$$G = \eta 4\pi \left[\int_0^{2\pi} d\varphi \int_0^{\pi} f(\theta) \sin\theta d\theta \right]^{-1} = \frac{4\pi\eta}{\int_{-\pi/2}^{\pi/2} \cos(\varphi) d\varphi \int_{2\pi/5}^{3\pi/5} \sin(5\theta)\sin(\theta) d\theta} = \frac{4\pi\eta}{0.7925} = 13.4773$$

3) $x_s = H \cdot \tan(90 - \theta_{tilt}) \Rightarrow \theta_{tilt} = 90 - \tan^{-1}\left(\frac{x_s}{H}\right) = 1.146°$

4)

$$R_s = \frac{x_s}{\cos(\theta_{tilt})} = 1500.3 \text{ m}$$

$$S_R = \frac{P_T G}{4\pi R_s^2} = \frac{1 \cdot 13.4773}{4\pi \cdot 1500.3^2} = 4.765 \cdot 10^{-7} \text{ W/m}^2$$

5)

$$f_\varphi(\varphi_{3dB}) = 0.5 \Rightarrow \varphi_{3dB} = \pm\pi/3$$

$$f_\theta(\theta_{3dB}) = 0.5 \Rightarrow \theta_{3dB} = \frac{1}{5}\left(\frac{\pi}{6} + 2\pi\right) = \frac{13}{30}\pi$$

$$= \frac{1}{5}\left(\frac{5\pi}{6} + 2\pi\right) = \frac{17}{30}\pi$$

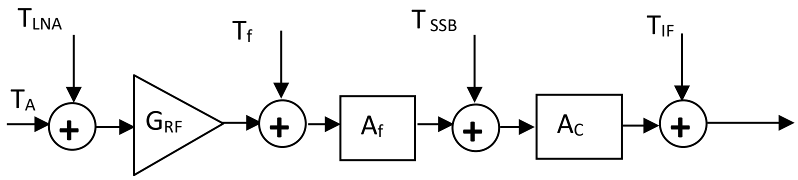
The above angles are those referred to the antenna pointing. Taking into account the tilting, the angle θ'_{3dB} with respect the absolute coordinate is given by $\theta'_{3dB} = \theta_{3dB} + \theta_{tilt}$

6) The distance R_{3dB} is given by: $R_{3dB} = H / \cos(\theta'_{3dB} - 90) = 131.9$ m. Then:

$$S_{R_{3dB}} = \frac{P_T G}{4\pi R_{3dB}^2} f(\theta_{3dB}) f(\varphi_{3dB}) = 1.548 \cdot 10^{-5} \text{ W/m}^2$$

Exercise 2

a) Equivalent noise scheme :



Where:

$$T_f = 293\left(10^{\frac{A_f}{10}} - 1\right) = 171.4\ °K\ ,\ T_{LNA} = 293\left(10^{\frac{NF}{10}} - 1\right) = 1175.5\ °K$$

The equivalent noise temperature at the input of the receiver is then given by:

$$T_{eq} = T_A + T_{LNA} + \frac{T_f}{G_{RF}} + \frac{T_{SSB}A_f}{G_{RF}} + \frac{T_{IF}A_cA_f}{G_{RF}} = 1887.3\ °K$$

$$P_N = KT_{eq}B = 5.21\cdot 10^{-13}\ \text{W}$$

b) Friis equation:

$$P_r = P_T \cdot \left(\frac{\lambda}{4\pi R}\right)^2 \cdot G_T \cdot G_R = 0.002152/R^2$$

c) Evaluation of SNR:

$$SNR = \frac{P_r}{P_N} = \left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right) = 150$$

d) Evaluation of the distance R:

$$P_r = SNR \cdot P_N = 7.815\cdot 10^{-11} = 0.002152/R^2$$

$$R = \sqrt{\frac{0.002152}{7.815\cdot 10^{-11}}} = 5.248\ \text{Km}$$

Exercise 3

The transistor is unconditionally stable with Gmax=12.1 dB.
The optimum gamma are: Γ_S=0.84 ∠-147.5°, Γ_L=0.69∠40.8°.
βl_1= 139.2/2=69.6°, Z_{c1}=50Ω.

$$Z_{in} = 50 \cdot 0.18 = 9.05$$

$$Z_{c2} = \sqrt{Z_{in} \cdot 50} = 21.27\ \Omega$$

Exercise 4

The mapping circle of Γs is drawn on the S.C with |Γout|=1.5. The two intersections with the outer circle are: Γ1=1∠120.36, Γ2=1∠81.54.
We assign Γs= Γ2, from which Zs=j1.16·50=j58 Ω. The inductance Ls is then derived as:

$$L_s = \frac{X_s}{2\pi f_0} = 21.37 \text{ nH}$$

At the transistor out we get $\frac{Z_{out}}{50} = -0.726 - j1.5$.

Imposing: $Z_L = \frac{-\mathrm{Re}(Z_{out})}{3} - \mathrm{Im}(Z_{out})$ we get Z_L=50·(0.24+j1.5).

The design of the matching network can be carried out either with the S.C. or by means of the formulas. The following results are obtained:

X=1.073·50=53.64Ω, B=-1.776/50=-0.03552 S.

The lumped components value is then computed as follows:

$$L_X = \frac{|X|}{2\pi f_0} = 19.76 \text{ nH}$$

$$L_B = \frac{1}{|B| 2\pi f_0} = 10.37 \text{ nH}$$