RF SYSTEMS
21 February 2018

Surname & Name
Identification Number
Signature

Exercise 1

It is given an antenna operating at 10 GHz having the following directivity function (constant along the angle φ): $f\left(\vartheta\right)=\left(\frac{\sin\left(10\cdot\vartheta\right)}{10\cdot\vartheta}\right)^2$ for $0<\vartheta<\pi/2$ (angle ϑ in rad). It is reminded that the main beam is related to the range $0<\vartheta<\vartheta_{\max}$ with $f\left(\vartheta_{\max}\right)=0$ (first null of *f*).

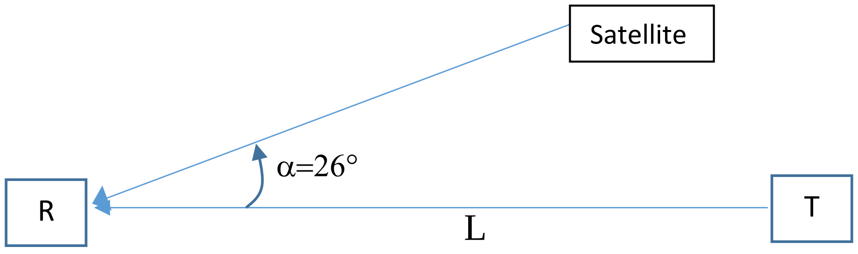
1) Evaluate $\vartheta_{\max}$, the width of the main beam Δ_{θ_B} and the directivity gain *D* of the antenna assuming *f*=0 for $\vartheta>\vartheta_{\max}$. Assume the following approximation for the gain computation:

   $$Q\left(\vartheta\right)=\int_0^{\vartheta} f\left(x\right)\sin\left(x\right)dx \cong 0.01372\cdot\vartheta^3\text{-}0.042609\cdot\vartheta^2+0.045343\cdot\vartheta \qquad (\vartheta \text{ in rad})$$

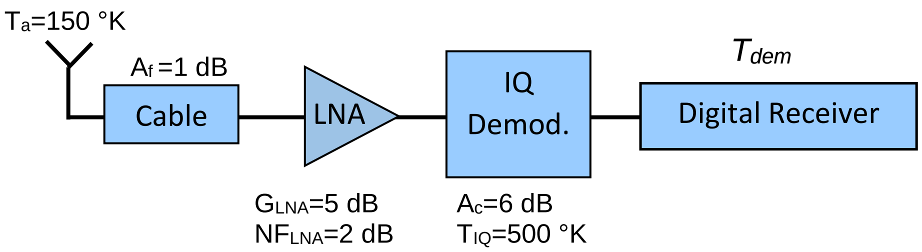
2) Evaluate the exact value of the half-power beamwidth (solve $f\left(\vartheta_{3dB}\right)=0.5$ by trial). Compute also the beamwidth ($2\theta_B$) imposing $f\left(\vartheta\right)=1$ for $0<\vartheta<\theta_B$ with the directivity gain *D* obtained at point 1).
3) Evaluate the actual directivity gain of the antenna (including the side lobes) and its beam efficiency.

Exercise 2

Consider a mobile receiving station R for a satellite communication system at 10 GHz which employs the receiving antenna of exercise 1. The antenna is optimally pointed in the satellite direction (elevation angle α=26°). The power density of the satellite signal at the receiving station is $SR=3 \cdot 10^{-7}$ W/m^2.
At distance L from this station there is the transmitter T operating on the same frequency, with P_T=1 W, connected to an omnidirectional antenna (gain G_T=2 dB). The signal from this transmitter represents an interferer for the receiving station R and must be treated as noise.



1) Compute the received power P_r from the satellite at the antenna output (consider the actual gain of the antenna computed at point 3 of exercise 1)
2) The system SNR is defined: $SNR_{sys} = \frac{P_r}{P_N + P_{int}}$, where P_r is the received power from the satellite, P_N is the equivalent noise power at the antenna output produced by the receiver ($P_N = K \cdot T_{eq} \cdot B$) and P_{int} is the power received from the interfering transmitter.
Assuming B=32 MHz and T_{eq}=3000 °K, evaluate the minimum value of the distance L of the interfering transmitter for which SNR_{sys} is at least 30 dB.
Hint: the actual directivity function of the receiving antenna must be taken into account for the evaluation of P_{int}; note that angle α represents the deviation with respect the optimum pointing (then coincides with the variable ϑ of the directivity function).
3) The receiving station employs a direct conversion receiver having the following block diagram:



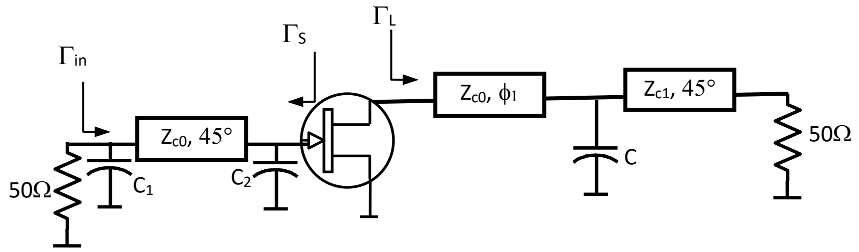
Determine the noise temperature T_{dem} of the base band demodulator producing the equivalent temperature at input of the receiver Teq=3000 °K

Exercise 3

Consider the following amplifier operating at 1 GHz. The following parameters are assigned:

Z_{c0}=50Ω, Z_{c1}=35Ω, S_{11}=0.844∠ -62.3°, S_{12}=0.06∠42.7°, S_{21}=5.273∠121.7°, S_{22}=0.52∠ -52.6°

The requirements concern the transducer gain G_T=18 dB and the conjugate matching condition at the input (Γ_{in}=0)



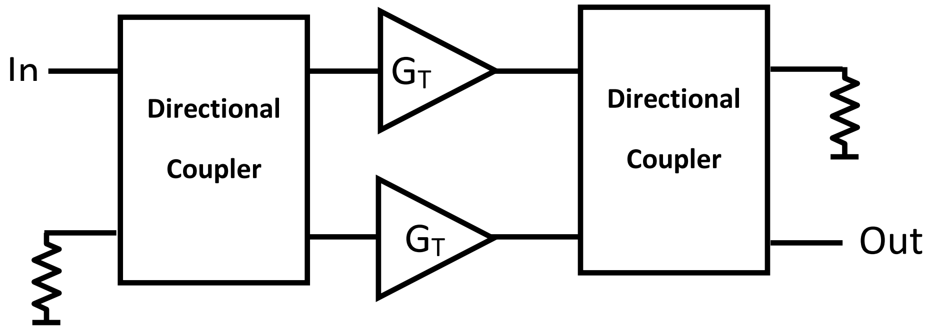
1) Evaluate Γs and Γ_L in order to satisfy the requirements (select a real, positive value for Γ_L)

2) Design the double-stub network at input (i.e. compute the value of C_1 and C_2). Note that Γ_{in} is zero when a suitable load is assigned at the opposite side of this network.

3) Design the output network, i.e. compute C and ϕ_1.

Note that the computed values of all the capacitors must be positive.

Two amplifiers identical to those above designed are then inserted in the following configuration:



4) Define the type of directional coupler to use for getting the overall transducer gain (In → Out) equal to G_T (the same of the two amplifiers). Specify the coupling parameter (magnitude and phase) of the directional couplers selected.
5) What is the value of the input (Γ_{In}) and output (Γ_{Out}) reflection coefficients in this configuration?

Solution

Exercise 1

1) $f(\theta_{max})=0 \rightarrow \theta_{max}=\pi/10$ (18°), $\Delta\theta_{MB}=2\theta_{max}=36°$

$$D=\frac{4\pi}{\int_0^{2\pi} d\varphi \int_0^{\vartheta_{\max}} f(\vartheta)\sin(\vartheta)d\vartheta}=\frac{2}{\int_0^{\vartheta_{\max}} f(\vartheta)\sin(\vartheta)d\vartheta}=191.12 \ \ (22.81 \text{ dB})$$

2) $f(\theta_{3dB})=0.5 \rightarrow \theta_{3dB}=\pi/22.58$ (7.97°), $\Delta\theta_{3dB}=2\theta_{3dB}=15.96°$

$$2\theta_B=2\cos^{-1}(1-2/D)=0.2896 \text{ rad } (16.6°)$$

3)
$$D_{act}=\frac{4\pi}{\int_0^{2\pi} d\varphi \int_0^{\pi} f(\vartheta)\sin(\vartheta)d\vartheta}=\frac{2}{\int_0^{\pi} f(\vartheta)\sin(\vartheta)d\vartheta}=103.82 \ \ (20.16 \text{ dB})$$

$$BE=\frac{D_{act}}{D}=0.5432$$

Exercise 2

1)
$$A=G\frac{\lambda^2}{4\pi}=D_{act}\frac{(c/f)^2}{4\pi}=0.0137 \text{ m}^2$$

$$P_r=SR\cdot A=2.23\cdot 10^{-9} \text{ W}$$

2)
$$P_N=KT_{eq}B=1.325\cdot 10^{-12} \text{ W}$$

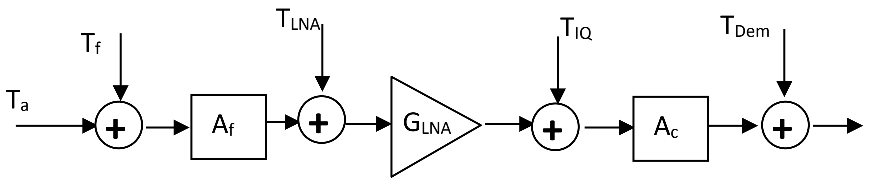
$$P_{\text{int}}=\frac{P_r}{SNR}-P_N=0.906\cdot 10^{-12} \text{ W}$$

Using the Friis Equation:

$$P_{\text{int}}=P_T\left(\frac{\lambda}{4\pi L}\right)^2 G_T G_{act} f\left(26\frac{\pi}{180}\right)$$

$$\frac{4\pi L}{\lambda}=\sqrt{\frac{P_T G_T G_{act} f\left(26\frac{\pi}{180}\right)}{P_{\text{int}}}}=2.925\cdot 10^6 \Rightarrow L=6.983 \text{ Km}$$

3)



$$T_{eq}=T_a+T_0\left(10^{A_f/10}-1\right)+T_0\left(10^{NF/10}-1\right)10^{A_f/10}+T_{IQ}10^{(A_f-G_{LNA})/10}+T_{DEM}10^{(A_f+A_c-G_{LNA})/10}=3000$$

$$640.66+T_{DEM}10^{0.2}=3000$$

$$T_{DEM}=1488.6 \text{ °K}$$

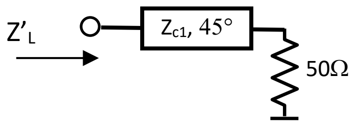
Exercise 3

1) Draw the circle G_p=const.=18 dB. Select the intersection of this circle with the real axis (there are two intersections; the one to select is the real, positive number). We have: $\Gamma_L=0.407\angle 0°$. To get $G_p=G_T$ we must impose the conjugate matching at input: Γ_s= conj(Γ_{inT})=$0.731\angle 68.9°$.
2) We assume $\Gamma_{inT}=0.731\angle -68.9°$ as the load terminating the input network. The design of the network will be carried out to get $\Gamma_{in}=0$.
    a. Draw the circle g=1 rotated by 90° toward the load
    b. Draw the circle g=const, passing for Γ_{inT}.
    c. Store Γ_{inT} and select the intersection between the circles where *DeltaB* increases. The increment is b_1=0.975
    d. Give an increment $\Delta\phi$=-90° (source direction) to the phase of gamma
    e. Take the opposite of Im(Y) of the current point. It represents the increment moving the current point to the center of the S.C. We have: b_2=3.79

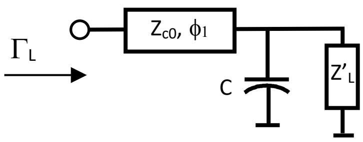
   The capacitances are the obtained as:

$$C_1 = \frac{b_1 \cdot 0.02}{2\pi f} = 3.1 \text{ pF}, \qquad C_2 = \frac{b_2 \cdot 0.02}{2\pi f} = 12.06 \text{ pF}$$

3) Let compute first the impedance Z'_L in parallel to C looking in the load direction:



   Using the S.C. (normalized to Z_{c1}=35Ω) we get Z'_L=32.9-j11.97 Ω.
   Now Z'_L must be transformed into Γ_L with a single-stub network:



    a. Draw the circle |Γ|=0.407
    b. Enter Z'_L (normalized to 50Ω), store, draw the circle *g=const* passing for the current point
    c. Select the intersection between the circles where *b* increases. *DeltaB*=*b*=0.489
    d. Store. Enter Γ= Γ_L=0.407. The variation of the gamma phase is $\Delta\phi$=131.9°
    e. Being the movement toward the source, ϕ_1 is given by: ϕ_1=(360- $\Delta\phi$)/2=114.05°.
    f. The capacitance C is given by: $C_1 = \dfrac{b \cdot 0.02}{2\pi f} = 1.56 \text{ pF}$
4) The directional couplers are 3dB hybrids, i.e. $C=0.5\angle 90°$. $\Gamma_{In}=\Gamma_{Out}=0$.