RF SYSTEMS
Written Test of February 22th, 2017

Surname & Name
Identification Number
Signature

Exercise 1

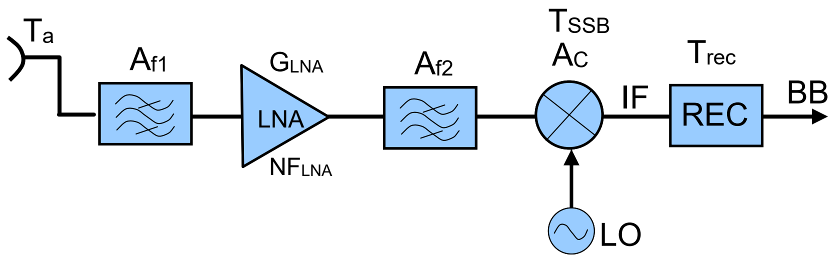
It is known the directivity function of an antenna operating at 300 MHz: $f(\theta) = \frac{25\cdot\sqrt{5}}{16}\sin^4(\theta)\cos(\theta)$ for $0<\theta<90°$, $f(\theta)=0$ elsewhere.

1) Evaluate the value of θ where f is maximum (θ_{max}). Hint: You can find θ_{max} either numerically or analytically, remembering that $(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$
2) Evaluate the directivity gain D_M. Recall: $\int f^n(x)\, f'(x)\, dx = \frac{f^{n+1}(x)}{n+1}$
3) Evaluate the minimum efficiency of the antenna which determines the gain G equal to 5 dB
4) Assuming that the antenna is used in reception, evaluate the output voltage determined by an incident wave arriving from the direction $\theta_{inc}=45°$ with power density $S_R=10^{-8}$ W/m^2. Assume that the radiation impedance of the antenna is 50 Ω and the load Z_L connected to the antenna is matched ($Z_{rad} = Z_L^*$).

Exercise 2

Consider the following scheme of a receiver operating at 6 GHz (signal band 20 MHz). The block REC represents the demodulator at intermediate frequency (IF), producing the bit streaming in base band (BB). It is characterized by the noise temperature T_{rec} (at input).

Assume that the filters eliminate completely the image band (no noise contribution at IF).

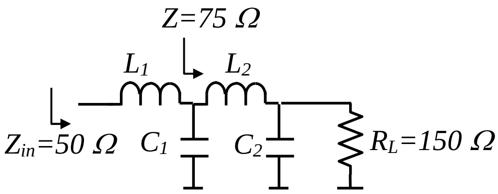


T_a=100°K
A_{f1}=1 dB
NF_{LNA}=2 dB
G_{LNA}=10 dB
A_{f2}=2 dB
T_{SSB}=350 °K
A_c=6 dB
T_{rec}=100 °K

1) Evaluate the equivalent noise temperature at the input of the receiver (T_{eq}).
2) Assuming E_b/N_0=20 dB, what is the minimum signal power at IF allowing a data rate R=40 Mbit/sec?
3) If we want remove the LNA and the second filter, we must improve the mixer in order to get the receiver performances unchanged (i.e. the same T_{eq} computed above). Assuming that the new value of the mixer losses is A'_c=3 dB, what is the new value of T_{SSB}?

Exercise 3

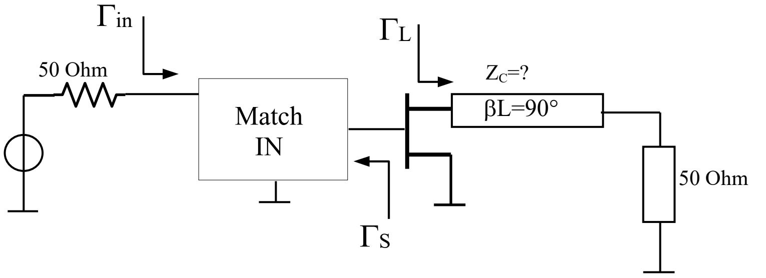
Consider the following matching network operating at 1 GHz:



Imposing the intermediate impedance Z=75 Ω, evaluate the value of L_1, L_2, C_1, C_2 that determines the input impedance Z_{in}=50 Ω. Note: explain clearly the procedure adopted for the computation. Reporting only the final values without any explanation is not accepted

Exercise 4

We want to design the amplifier in the figure operating at 2 GHz, with matching at input ($\Gamma_{in}=0$).
It is known that the Match IN network is lossless.
The S parameters of the active device at 2 GHz are given in the following table.



$S_{11}=0.828\angle -81.1°$
$S_{12}=0.076\angle 33.4°$
$S_{21}=4.390\angle 99.8°$
$S_{22}=0.539\angle -60.1°$

1) Evaluate Γ_S and Γ_L in order to get the highest transducer gain compatibly with stability and the matching requirement (note that the Γ_L selected must be realizable with the output network assigned)
2) Evaluate the characteristic impedance Z_c of the output transmission line.

<u>Solution</u>

<u>Exercise 1</u>

1) To find the angle θ_{max} we compute the derivative of $f(\theta)$ and equate to 0:

$$f'(\theta) = 4\sin^3(\theta)\cos^2(\theta) - \sin^5(\theta) = \sin^3(\theta)\left[4\cos^2(\theta) - \sin^2(\theta)\right] =$$

$$= \sin^3(\theta)\left[5\cos^2(\theta) - 1\right] = 0 \quad \Rightarrow \quad \theta_{\max} = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = 63.43°$$

2) $$D_M = 4\pi\left[\int_0^{2\pi} d\varphi \int_0^{\pi/2} f(\theta)\sin\theta d\theta\right]^{-1} = \frac{2\cdot 16}{25\cdot\sqrt{5}\int_0^{\pi/2}\sin^5(\theta)\cos(\theta)d\theta} = \frac{2\cdot 16\cdot 6}{25\cdot\sqrt{5}\sin^6\left(\frac{\pi}{2}\right)} = 3.43 \text{ (5.36 dB)}$$

3) $$G = \eta D_{\max} \Rightarrow \eta = 10^{(5.36-5)/10} = 0.921$$

4) $$A_e = \lambda^2\frac{G}{4\pi} = 0.251\ m^2,\ \ P_r = S_R A_e f(45°) = 10^{-8}\cdot 0.251\cdot 0.6176 = 1.55\cdot 10^{-9} W$$

$$P_r = \frac{1}{8}\frac{|V_{\max}|^2}{50} \quad \Rightarrow \quad |V_{\max}| = \sqrt{50\cdot 8\cdot P_r} = 0.788\cdot 10^{-3} V$$

<u>Exercise 2</u>

1) Evaluation of Teq:

$$T_{eq} = T_a + T_{f1} + T_{LNA}A_{f1} + T_{f2}\frac{A_{f1}}{G_{LNA}} + T_{SSB}\frac{A_{f1}A_{f2}}{G_{LNA}} + T_{rec}\frac{A_{f1}A_{f2}A_c}{G_{LNA}} = 559.25\ °K$$

$$T_{f1} = 290\left(10^{\frac{A_{f1}}{10}} - 1\right) = 75.088°K,\ \ T_{f2} = 290\left(10^{\frac{A_{f2}}{10}} - 1\right) = 169.62°K$$

$$T_{LNA} = 290\left(10^{\frac{NF}{10}} - 1\right) = 169.62°K$$

2) Evaluation of P_{IF}:

$$SNR_{sys} = \frac{P_r}{KT_{eq}B} = \left(\frac{E_b}{N_0}\right)\frac{R}{B} = 20 + 10\log(40/20) = 23 \text{ dB}$$

$$\mathrm{K}T_{eq}B = 1.38\cdot 10^{-23}\cdot 559.25\cdot 20\cdot 10^6 = 1.543\cdot 10^{-13} \quad \text{(-128.1 dBW)}$$

$$P_r = SNR_{sys} + \mathrm{K}T_{eq}B\Big|_{dBW} = -105.1 \text{ dBW}$$

$$P_{IF} = P_r - A_{f1} - A_{f2} - A_c + G_{LNA} = -104.1 \text{ dBW}$$

3) Evaluation of T'_{SSB}:

$$T'_{eq} = T_a + T_{f1} + T'_{SSB}A_{f1} + T_{rec}A_cA_{f1} = T_{eq}$$

$$T'_{SSB} = \frac{T_{eq} - \left(T_a + T_{f1} + T_{rec}A_cA_{f1}\right)}{A_{f1}} = 105.62\ °\mathrm{K}$$

Exercise 3

For the second network (the closed to the load):

$$B_{p2} = \sqrt{\frac{1}{R_L}\left(\frac{1}{Z} - \frac{1}{R_L}\right)} = \frac{1}{150}\left[\sqrt{\frac{150}{75}} - 1\right] = 6.666 \cdot 10^{-3} S, \ \ X_{s2} = Z\left[\sqrt{\frac{R_L}{Z}} - 1\right] = 75\ \Omega$$

$$C_2 = \frac{B_{p2}}{2\pi f_0} = 1.061 \text{ pF}, \ \ L_2 = \frac{X_{s2}}{2\pi f_0} = 11.94 \text{ nH}$$

For the first network (the closed to the source):

$$B_{p1} = \sqrt{\frac{1}{Z}\left(\frac{1}{Z_{in}} - \frac{1}{Z}\right)} = \frac{1}{75}\left[\sqrt{\frac{75}{50}} - 1\right] = 9.428 \cdot 10^{-3} S, \ \ X_{s1} = Z_{in}\left[\sqrt{\frac{Z}{Z_{in}}} - 1\right] = 35.35\ \Omega$$

$$C_1 = \frac{B_{p1}}{2\pi f_0} = 1.5 \text{ pF}, \ \ L_1 = \frac{X_{s1}}{2\pi f_0} = 5.63 \text{ nH}$$

Exercise 4

We draw the circle Gp=17 dB (<MSG=17.62 dB). Then select one of the points on this circle crossing the horizontal axis: Γ_L=0.195, to which corresponds $\Gamma_S=(\Gamma_{in})^*=0.77\angle 83.56°$. Being the output matched: $G_T=G_P$=17 dB.

The impedance Z_L corresponding to Γ_L is Zs=50·1.485=72.25 Ω. Then Z_c=sqrt(50·72.25)=60.93 Ω.