RF SYSTEMS
Written Test of June 23, 2021

Surname & Name	
Identification Number	
Signature	

Exercise 1

A vehicle moving in the *x* direction is initially connected to the mobile network through the base station BS$_1$. At the position x_A the connection switch to the base station BS$_2$ (*handover*). The two base stations have equal antennas with gain G_{BS}=5.5 dB and the following directivity function (φ is the azimuthal angle and θ is the elevation angle):

$$f(\theta,\varphi) = \cos^2(\varphi) \quad \text{for} \quad 0<\theta<\pi/2,\ -\pi/2<\varphi<\pi/2$$

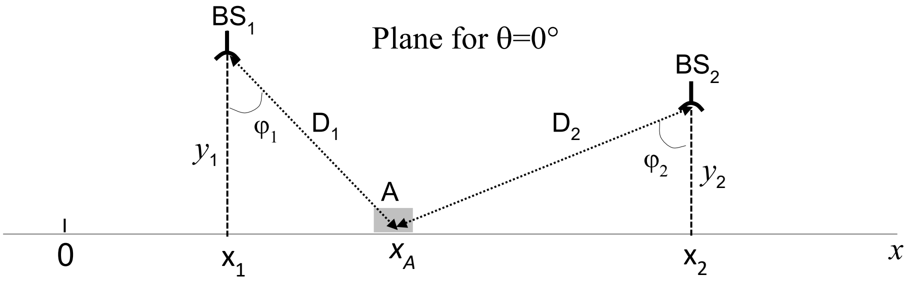
$$f(\theta,\varphi) = 0 \quad \text{elsewhere}$$

Note that the directivity is constant along θ (the plane in which the vehicle is moving is θ=0).

The transmitted power from BS$_1$ is P_{T1}=50W and the one from BS$_2$ is P_{T2}=20W. The downlink frequencies are 1.81 GHz (BS$_1$) and 1.87 GHz (BS$_2$).

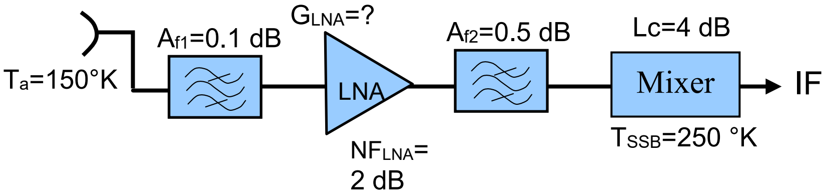
It is known that the handover happens when the signals received by the vehicle from the two base stations have the same power (P_{RH}).

1) (Mandatory, 7p) Evaluate the position x_A and the power P_{RH} at the handover. Assume the antenna on the vehicle omni-directional ($f(\varphi)=1$), with gain G_A=2dB. Hint: the solution is found by equating at x_A the power P_{r1} received from BS$_1$ to the power P_{r2} from BS$_2$. P_{r1} and P_{r2} are computed by means of the Friis equation.
2) (4p) Evaluate the directivity gain *D* and the efficiency η of the antennas used by the base stations. Hint: $\int \cos^2(x)dx = x/2 + \sin(2x)/4$



y_1=5Km
y_2=3Km
x_1=5Km
x_2=7Km

Exercise 2

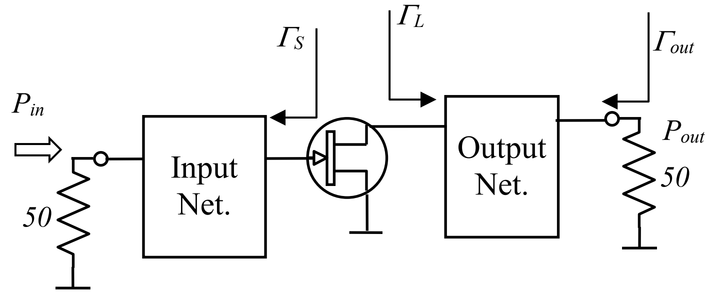


Consider the scheme in the figure, which refers to RF front-end of a digital communication system operating at 6.4 GHz with a signal bandwidth B=32 MHz. All the relevant parameters of the system are reported in the scheme (except G_{LNA} to be assessed).

1) (Mandatory, 6.5p) Assuming the received power P_r=-70 dBm and the required SNR=28 dB, evaluate the equivalent noise temperature T_{eq} of the RF front-end and the value of G_{LNA}
2) (1p) Evaluate the required E_b/N_0 to achieve 100 Mbit/sec at baseband
3) (3.5p) If the LNA and the second filter are removed, what is the new value of T_{SSB} for maintaining T_{eq} unchanged?

Exercise 3

We want design a single stage amplifier at 12 GHz using the scheme in the following figure (input and output networks are lossless):



$S_{11}=0.57\angle 166°$
$S_{12}=0.144\angle -50°$
$S_{21}=2.59\angle -21°$
$S_{22}=0.39\angle -148°$
$\Gamma_{opt}=0.435\angle 168°$
$F_{min}=1.15$ dB
$R_n=0.095$
IP3=20 dBm

The amplifier must exhibit the transducer gain as high as possible compatibly with the stability requirement and the noise figure not larger than 1.5 dB.

1) (Mandatory, 6p) Select a proper value for Γ_S and Γ_L in order to satisfy the above requirements. Specify the value of G_T and the value of Γ_{out}.
2) (3p) Assuming a single stub matching network for the output network, draw the scheme of the network and evaluate its parameters.
3) (2p) Assuming at input a 2-tone signal, evaluate the maximum input power (Pin) determining at output the level of intermodulation lines 30 dB smaller than the main lines.

Solution

Exercise 1

Using the assigned frequencies: λ_1=16.57 cm, λ_2=16.04 cm.
The powers received by the vehicle from the two BS are given by:

$$P_{R1} = P_{T_1} \frac{\lambda_1^2}{(4\pi D_1)^2} G_{BS} G_{VH} \cos^2(\vartheta_1), \quad P_{R2} = P_{T_2} \frac{\lambda_2^2}{(4\pi D_2)^2} G_{BS} G_{VH} \cos^2(\vartheta_2)$$

Then, imposing P_{R1}=P_{R2} and replacing $\cos(\theta_1) = y_1/D_1$, $\cos(\theta_2) = y_2/D_2$:

$$\left(\frac{D_2}{D_1}\right)^4 = \left(\frac{y_2}{y_1}\right)^2 \frac{\lambda_2^2}{\lambda_1^2} \frac{P_{T_2}}{P_{T_1}} = K^2 \Rightarrow K = 0.3673$$

The lengths D_1 and D_2 can be expressed as follows:

$$D_1 = \sqrt{(y_1)^2 + (x_A - x_1)^2}, \quad D_2 = \sqrt{(y_2)^2 + (x_2 - x_A)^2}$$

Replacing in the previous equation:

$$\frac{(y_2)^2 + (x_2 - x_A)^2}{(y_1)^2 + (x_A - x_1)^2} = K = 0.3673$$

Finally, the following 2nd degree equation in x_A is obtained:

$$x_A^2 (1-K) + 2x_A (K \cdot x_1 - x_2) + y_2^2 - K \cdot x_1^2 + x_2^2 - K \cdot y_1^2 = 0$$

$$0.6327 x_A^2 - 10.327 x_A + 39.635 = 0$$

Solving, we get the two following solutions: x_{A1}=10.1506 Km, x_{A2}=6.1715 Km.
The first solution must be discarded because > x_2,
The received power at the handover finally results:
P_{r1}=P_{r2}= 1.7582e-09 (-57.55 dBm)

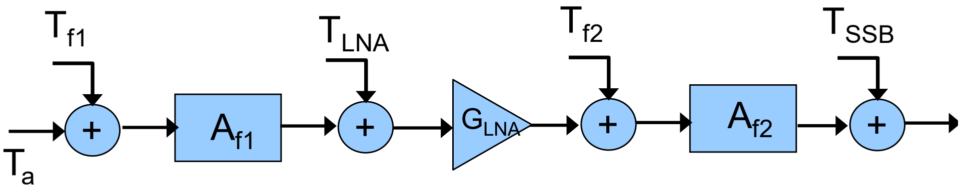
The directivity gain D is expressed by:

$$D = \frac{4\pi}{\int_0^{\pi/2} \int_{-\pi/2}^{\pi/2} f(\vartheta,\varphi) \sin(\theta) d\theta d\varphi} = \frac{4\pi}{\int_0^{\pi/2} \sin(\theta) d\theta \int_{-\pi/2}^{\pi/2} \cos^2(\varphi) d\varphi} = \frac{4\pi}{\left[1 \cdot \frac{\pi}{2}\right]} = 8$$

From the expression $\eta D = 10^{(5.5/10)}$, we get η=0.4435

Exercise 2

$$SNR = P_r - 10\log\left(KT_{eq}B\right) = 28 \Rightarrow T_{eq} = 358.8979\ \text{°K}$$



A_{f1}=1.023, A_{f2}=1.122 $T_{f1} = T_0\left(10^{A_{f1}/10} - 1\right) = 6.75°K$, $T_{f2} = T_0\left(10^{A_{f2}/10} - 1\right) = 35.38°K$

$T_{LNA} = T_0\left(10^{NF/10} - 1\right) = 171.3737°K$,

$$T_{eq} = T_a + T_{f1} + A_{f1}T_{LNA} + \frac{A_{f1}\left[T_{SSB}A_{f2} + T_{f2}\right]}{G_{LNA}} = 358.8979°K$$

$$G_{LNA} = \frac{A_{f1}\left[T_{SSB}A_{f2} + T_{f2}\right]}{T_{eq} - \left(T_a + T_{f1} + A_{f1}T_{LNA}\right)} = 12.1173 \quad (10.834\ \text{dB})$$

To get R=100 Mbit/sec we must have:

$E_b/N_0 = SNR - 10\log(R/B) = 28 - 4.95 = 23.05$ dB.

Removing the LNA and second filter:

$$T_{eq} = T_a + T_{f1} + A_{f1}T_{SSB} = 358.8979°K$$

$$T_{SSB} = \frac{1}{A_{f1}}\left\{358.8979 - \left[T_a + T_{f1}\right]\right\} = 197.4733°K$$

Exercise 3

1. Enter the S parameters on the S. C. → Device potentially instable with MSG=12.2 dB
2. Draw the circle NF=1.5 dB and several circles with Gav<MSG until the one tangent to the noise circle is found: Gav=11.24 dB
3. Select Γs on the tangent point: Γs=0.55∠-164.4
4. Imposing the matching at the transistor output the transducer gain is made equal to the available gain. Select on the S.C.: S Param→ Optimum Gamma → Load: G_T=11.24 dB, Γ_L=0.505∠-175.8, NF=1.5 dB
5. Check that both Γs and Γ_L are outside the instability regions (source and load)
6. Being the output of transistor matched, if a lossless output network is used, Γ_{out} =0.
7. Using the S.C. the single-stub network parameters are found: θ=27.45°, b=1.186.