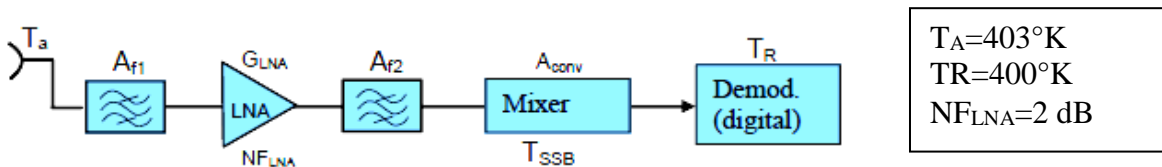


**RF SYSTEMS**  
**Written Test of 28 June 2017**

|                              |
|------------------------------|
| <b>Surname &amp; Name</b>    |
| <b>Identification Number</b> |
| <b>Signature</b>             |

Exercise 1

Consider the following receiver operating at 15 GHz with signal bandwidth  $B=70$  MHz and data rate  $R=150$  Mbit/sec.

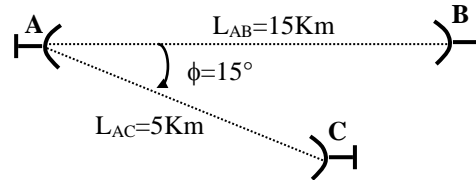


- 1) Evaluate the equivalent noise temperature at the receiver input ( $T_{eq}$ ) that determines the ratio  $E_b/N_0 = 25$  dB (with the received power at the antenna ( $P_{rec}$ ) equal to  $-63.3$  dBm).
- 2) Assuming the minimum power at the input of the demodulator  $P_{dem} = -49.3$  dBm, the attenuation of filter 1  $A_{f1} = 1$  dB, the gain  $G_{LNA} = 22$  dB and the conversion loss of the mixer  $A_{conv} = 6$  dB, find the attenuation of filter 2 ( $A_{f2}$ ) and  $T_{SSB}$  of the mixer in order to get the requested  $T_{eq}$ ,

Hint: Assume  $T_0 = 293$  °K and  $K = 1.38 \cdot 10^{-23}$  for computations.

## Exercise 2

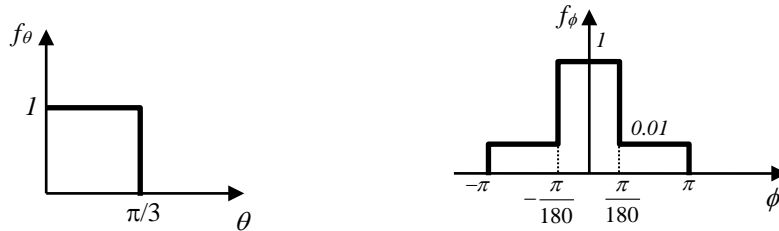
Three antennas A, B and C are placed as shown in the figure ( $x,y$  plane) and operate at 12 GHz. The power transmitted from antenna A ( $P_T=1W$ ) is received by antenna B and both antennas are directed along the maximum of directivity diagram.



Antennas A and B have the following directivity function  $f(\theta, \phi) = f_\theta(\theta) f_\phi(\phi)$ , with:

$$f_\theta(\theta) = 1 \text{ for } 0 < \theta < 60^\circ \text{ (}=0 \text{ elsewhere),}$$

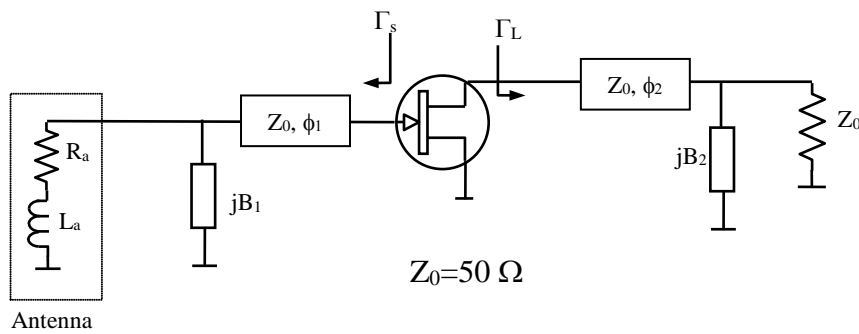
$$f_\phi(\phi) = 1 \text{ for } -1^\circ < \phi < 1^\circ \\ = 0.01 \text{ for } 1^\circ < \phi < -1^\circ$$



Note that  $\theta=0$  defines the  $(x,y)$  plane. The range of  $\phi$  is  $-\pi < \phi < \pi$

- 1) Compute the maximum directivity gain  $G$  of antennas A and B (assume the efficiency  $\eta=0.9$ ). Hint: use the integral formula relating  $f(\theta, \phi)$  to  $G$ .
- 2) Compute the power received by the antenna B
- 3) Antenna C is a dish with aperture efficiency  $e_a=0.5$ ; it is optimally pointed toward antenna A. Evaluate its diameter in order the received power is the same received by antenna B.

### Exercise 3



The scheme in the figure represents a low noise amplifier operating at 10 GHz, directly connected to an antenna whose radiation impedance is modeled by the resistance  $R=35\ \Omega$  in series with the inductance  $L=0.4\ \text{nH}$ .

The active device is characterized by the following parameters:

Scattering:  $S_{11}=0.36\angle -171^\circ$ ,  $S_{12}=0.044\angle 67^\circ$ ,  $S_{21}=7.28\angle 80^\circ$ ,  $S_{22}=0.45\angle -26^\circ$

Noise:  $r_n=0.17$ ,  $NF_{\min}=1.3\ \text{dB}$ ,  $\Gamma_{\min}=0.05\angle 28^\circ$

The design goal is to obtain  $NF=1.5\ \text{dB}$  with the corresponding maximum transducer gain  $G_T$ .

- 1) Evaluate the reflection coefficients  $\Gamma_S$  and  $\Gamma_L$  that allow the requested design goal. Specify the value of  $G_T$  obtained.
- 2) Compute the parameters of the transforming networks  $B_1$ ,  $\phi_1$ ,  $B_2$ ,  $\phi_2$ . (the computing procedure must be reported)

## Solution

### Exercise 1

1) Evaluation of the required  $T_{eq}$ :

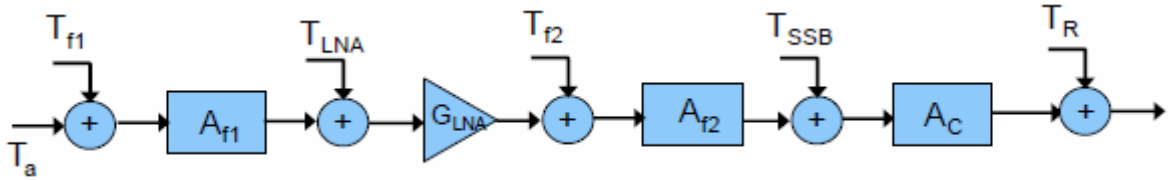
$$SNR_{dB} = P_{rec} - (KT_{eq}B)_{dBm} = \left(\frac{E_b}{N_0}\right)_{dB} + \left(\frac{R}{B}\right)_{dB} \Rightarrow (KT_{eq}B)_{dBm} = -91.61 \text{ dBm}$$

$$T_{eq} = \frac{10^{-12.161}}{70 \cdot 10^6 \cdot 1.381 \cdot 10^{-23}} = 714.02 \text{ °K}$$

2) Evaluation of  $A_{f2}$ :

$$P_{rec} - A_{f1} + G_{LNA} - A_{f2} - A_{conv} = P_{dem} \Rightarrow A_{f2} = -63.3 + 49.3 - 1 + 22 - 6 = 1 \text{ dB}$$

Teq expression:



$$T_{eq} = T_a + T_{f1} + A_{f1}T_{LNA} + \frac{A_{f1}[(T_{SSB} + A_c T_R)A_{f2} + T_{f2}]}{G_{LNA}}$$

TSSB can be derived as:

$$T_{eq} = T_a + T_{f1} + A_{f1}T_{LNA} + \frac{A_{f1}[(T_{SSB} + A_c T_R)A_{f2} + T_{f2}]}{G_{LNA}}$$

$$T_{SSB} = \frac{(T_{eq} - T_a - T_{f1} - A_{f1}T_{LNA})G_{LNA} - T_{f2}A_{f1}}{A_{f2}} - A_c T_R$$

where:

$$T_{eq} = 714.02, \quad 701T_{f1} = T_0(10^{0.1} - 1) = T_{f2} = 75.8651, \quad T_{LNA} = T_0(10^{0.2} - 1) = 171.3737$$

$$A_{f1} = 10^{0.1} = A_{f2} = 1.2589, \quad G_{LNA} = 10^{2.2} = 158.4893, \quad A_c = 10^{0.6} = 4$$

Replacing we get:  $T_{SSB} = 281.62 \text{ °K}$

## Esercizio 2

$$G = \frac{4\pi\eta}{\int_0^\pi f_\theta(\theta)\sin(\theta)d\theta \int_{-\pi}^\pi f_\varphi(\varphi)d\varphi} = \frac{4\pi \cdot 0.9}{[-\cos(2)]_0^{\pi/3} \cdot \left( \left[ \varphi \right]_{\frac{\pi}{180}}^{\frac{\pi}{180}} + 2 \cdot 0.01 \cdot \left[ \varphi \right]_{\frac{\pi}{180}}^{\frac{\pi}{180}} \right)} = \frac{4\pi \cdot 0.9}{0.5 \cdot \frac{2\pi}{180} \left( 1 + 0.01 \frac{179}{180} \right)} = 232.26 \text{ (23.66 dB)}$$

$$P_{rB,dBm} = P_{tA,dBm} - 20 \cdot \log\left(\frac{4\pi L_{AB}}{\lambda}\right) + G_{t,dB} + G_{r,dB} + f_A(0,0)|_{dB} + f_B(0,0)|_{dB} = 30 - 20 \cdot \log\left(\frac{4\pi L_{AB}}{\lambda}\right) + 2 \cdot G =$$

$$30 - 137.5472 + 2 \cdot 23.66 = -60.23 \text{ dBm}$$

$$f_C(0,15) = 0.01 \text{ | (-20 dB)}$$

$$P_{rC,dBm} = P_{tA,dBm} - 20 \cdot \log\left(\frac{4\pi L_{AC}}{\lambda}\right) + G + G_B + f_A(0,15)|_{dB} + f_C(0,0)|_{dB} = P_{rB,dBm} = -60.23$$

$$G_B = -60.23 - 30 + 128 - 23.66 + 20 = 34.11 \text{ dB}$$

$$G_B = A_e \frac{4\pi}{\lambda^2} = e_a \frac{1}{4} \pi d^2 \frac{4\pi}{\lambda^2} = e_a \left(\frac{\pi d}{\lambda}\right)^2 \Rightarrow d = \frac{\lambda}{\pi} \sqrt{\frac{G_B}{e_a}} = 0.57 \text{ m}$$

### Exercise 3

The antenna impedance is given by:  $Z_a = R_a + j2\pi f_0 L_a = 35 + j25.13 \Omega$ .

The assigned transistor is unconditionally unstable with  $G_{Tmax} = 20.4$  dB. To satisfy the goal we must find  $\Gamma_S$  which determines the imposed  $NF = 1.5$  dB; then imposing the conjugate matching at output we get  $\Gamma_L$ . We start drawing on the electronic Smith Chart the circle with  $NF = 1.5$ . Then we draw some circles with Available Gain constant until we find the one tangent to the  $NF$  circle. The tangent point gives the optimum  $\Gamma_S = 0.264 \angle 171.3$ . Selecting "Optimum Gamma Load" on the S.C. we get  $\Gamma_L = 0.541 \angle -28.67^\circ$  and  $G_T = 19.3$  dB.

The input network transforms  $Z_a$  into  $\Gamma_S$ . We start the design entering  $Z_{a,norm} = 0.7 + j0.5$  and drawing the circle  $g = \text{const.}$  passing for this point. We then enter  $\Gamma_S$ , store in memory and draw the circle  $|\Gamma| = \text{const.}$  passing for this point. We select one of the intersections of the two circles and the phase of  $\Delta\Gamma$  divided by 2 gives the parameter  $\phi_1 = 44.53^\circ$ . We store the current point in memory and select the marker representing  $Z_{a,norm}$ . The imaginary part of  $\Delta Y$  with the sign reversed gives  $B_{1,norm} = 1.2$  (the sign is reversed because in the last step the susceptance  $B_1$  must be subtracted).

The output network is a standard single stub transformer. Applying the procedure in slides we obtain:  $\phi_2 = 104.35$ ,  $B_{2,norm} = 1.28$ .