## RF SYSTEMS $-1^{\text {th }}$ Midterm test <br> 30 October 2017

## Surname \& Name

## Identification Number

## Signature

## Exercise 1




The transmitter $\mathrm{T}\left(\mathrm{P}_{\mathrm{T}}=20 \mathrm{~W}, \mathrm{f}_{0}=4 \mathrm{GHz}\right)$ is located on an aircraft flying parallel to ground at an altitude $\mathrm{H}=10 \mathrm{Km}$. The antenna on T is optimally directed toward the receiving station R on ground and has the gain $\mathrm{G}_{\mathrm{T}}=10 \mathrm{~dB}$.
The antenna at the ground station R is directed along the vertical direction and exhibits the following directivity diagram: $f(\theta, \varphi)=f^{\prime}(\theta) \cdot \cos (\varphi / 2)$ where $f^{\prime}(\theta)$ is shown on the figure ( $\mathrm{A}_{1}=$ -10 dB ). Note that the range of $\varphi$ is: $-\pi<\varphi<\pi$. The scheme on the above figure (left) refers to the plane $\varphi=0$.

1) Compute the parameter $A_{2}$ of the directivity diagram in order that the gain of the receiving antenna $\left(\mathrm{G}_{\mathrm{R}}\right)$ is equal to 17 dB . Assume the radiation impedance $\mathrm{Z}_{\mathrm{R}}=50 \Omega$ e the loss resistance $\mathrm{R}_{\mathrm{p}}=10 \Omega$.
2) Compute the 3 dB beamwidth $\Delta \varphi_{3 d B}$ of the antenna in the plane $\theta=0$.
3) Compute the power received by R when the aircraft crosses the positions where $\theta$ is equal $10^{\circ}$ and $50^{\circ}$ (with $\varphi=0$ )
4) It is known the sensitivity of the receiver $S=-90 \mathrm{dBm}$. Evaluate the maximum value of L where the signal is still detected (note that $\theta>50^{\circ}$ for this distance)

## Exercise 2

Consider the Satellite link in the figure (left) operating at 12 GHz . The transmitter on the satellite has $\mathrm{P}_{\mathrm{TS}}=200 \mathrm{~W}$ and the onboard antenna has the gain $\mathrm{Gs}_{\mathrm{s}}=35 \mathrm{~dB}$. The ground station uses an antenna with $\mathrm{G}_{\mathrm{R}}=40 \mathrm{~dB}$; the antenna noise temperature $(\mathrm{Ta})$ is $80^{\circ} \mathrm{K}$. The transmitting and receiving antennas are optimally directed.


1) Evaluate the receiver power $P_{r}$ at the ground station (antenna output)
2) Let consider the architecture of the receiving station (top figure, right). It is know that the digital receiver must operate with $\left(\mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0}\right)=14 \mathrm{~dB}$ in order to get the data rate $\mathrm{R}=150 \mathrm{Mbit} / \mathrm{sec}$.
Compute the SNR $_{\text {sys }}$ of the system for a signal bandwidth $\mathrm{B}=30 \mathrm{MHz}$.
3) Evaluate the equivalent noise temperature of the system ( $\mathrm{T}_{\text {sys }}$ ) imposing SNR $_{\text {sys }}$ and the received signal power ( $\mathrm{P}_{\mathrm{r}}$ )
4) The minimum power at the input of the digital receiver must be $P_{d}=-60 \mathrm{dBm}$. Find the value of the product $\mathrm{G}_{\text {LNA }} \cdot \mathrm{G}_{\text {IF }}$ (not in dB !) requested for satisfying this requirement
5) Draw the equivalent scheme of the receiver with the noise temperature sources. Write the expression of $\mathrm{T}_{\text {sys }}$ as function of the blocks parameters. Imposing the requested $\mathrm{T}_{\text {sys }}$ and the value of $G_{\text {LNA }} \cdot G_{\text {IF }}$ computed above, evaluate $G_{\text {LNA }}$ and $G_{I F}$ in dB .

## Exercise 1

1) First the directivity gain $D_{\max }$ is computed:

$$
\begin{aligned}
& D_{\max }=\frac{4 \pi}{\int_{-\pi}^{\pi} \int_{0}^{\pi} \sin (\varphi) \sin (\theta) f^{\prime}(\theta) d \varphi d \theta}=\frac{4 \pi}{4\left[\left(1-\cos \left(10^{\circ}\right)\right)+A_{1}\left(\cos \left(10^{\circ}\right)-\cos \left(50^{\circ}\right)\right)+A_{2}\left(1+\cos \left(50^{\circ}\right)\right)\right]}= \\
& =\frac{\pi}{0.0494+A_{2} 1.6428}=\frac{G}{\eta}=60.1425
\end{aligned}
$$

Where $\eta=\mathrm{Z}_{\mathrm{rad}} /\left(\mathrm{Z}_{\mathrm{rad}}+\mathrm{R}_{\mathrm{p}}\right)=0.8333$ and $\mathrm{G}=10 \wedge 1.7=50.1187$
From the above equation it is found $\mathrm{A}_{2}=0.00173(-27.6 \mathrm{~dB})$
2) The directivity in $\varphi$ is:

$$
f(\bar{\varphi})=\cos \left(\frac{\bar{\varphi}}{2}\right)=0.5 \Rightarrow \bar{\varphi}=120^{\circ}
$$

The 3 dB beamwidth is then $\varphi=2 \bar{\varphi}=240^{\circ}$
3) At $i$-th crossing the distance $L$ is equal to:
$\mathrm{L}_{\mathrm{i}}=\mathrm{H} / \cos \left(\theta_{\mathrm{i}}\right)$ with $\theta_{\mathrm{i}}$ equal to $10^{\circ}$ and $50^{\circ}$. The received power is obtained from the Friis eqution:

$$
P_{r}=P_{t}+G_{T}+G_{R}-L_{f, i}+A_{i}
$$

For $\theta_{\mathrm{i}}=10^{\circ}, \mathrm{L}_{1}=\mathrm{H} / \cos \left(10^{\circ}\right)=10.154 \mathrm{Km}, \mathrm{Lf}_{\mathrm{f}, \mathrm{i}}=20 \log \left(4 \pi \mathrm{~L}_{1} / \lambda\right)=124.61 \mathrm{Km}$
For $\theta_{\mathrm{i}}=50^{\circ}, \mathrm{L}_{2}=\mathrm{H} / \cos \left(50^{\circ}\right)=15.557 \mathrm{Km}, \mathrm{L}_{\mathrm{f}, \mathrm{i}}=20 \log \left(4 \pi \mathrm{~L}_{2} / \lambda\right)=128.32 \mathrm{Km}$
Replacing on the Friis equation ( $\mathrm{Pt}=43 \mathrm{dBm}, \mathrm{G}_{\mathrm{T}}=10 \mathrm{~dB}, \mathrm{G}_{\mathrm{R}}=17 \mathrm{~dB}$ ) it has:
$\operatorname{Pr} 1=-64.616 \mathrm{dBm}, \operatorname{Pr} 2=-85.94 \mathrm{dBm}$
4) From the Friis equation:

$$
P_{r}=P_{t}+G_{T}+G_{R}-L_{f}+A_{2}=-90 \Rightarrow L_{f}=132.4 \mathrm{dBm}
$$

Then replacing the expression of $L_{f}$ :
$L=\frac{\lambda}{4 \pi} 10^{L_{f} / 20}=24.88 \mathrm{Km}$

## Exercise 2

1) Applying the Friis equation we get the received power at the ground station:

$$
P_{r}=P_{t}+G_{T}+G_{R}-L_{f}=53+35+40-20 \log \left(\frac{4 \pi 36 \cdot 10^{6}}{0.025}\right)=-77.15 \mathrm{dBm}
$$

2) The SNRsys is related to $\mathrm{Eb} / \mathrm{No}$ as follows:

$$
S N R_{s y s}=\frac{E_{b}}{N_{0}} \frac{R}{B}=14+10 \log \left(\frac{150}{30}\right)=21 \mathrm{~dB}
$$

3) It has:

$$
S N R_{s y s}=\frac{P_{r}}{K T_{\text {sys }} B}=21 \mathrm{~dB} \Rightarrow \mathrm{~T}_{\text {sys }}=369.83^{\circ} \mathrm{K}
$$

4) The power Pd is given by:
$\mathrm{Pd}=\mathrm{P}_{\mathrm{r}}-\mathrm{A}_{0}+\mathrm{G}_{\mathrm{LNA}}-\mathrm{A}_{\mathrm{C}}+\mathrm{G}_{\mathrm{IF}}=-60 \mathrm{dBm}$.
Replacing we get $\mathrm{G}_{\mathrm{LNA}}+\mathrm{G}_{\mathrm{IF}}=23.25 \mathrm{~dB}$ that is, in natural units, $\mathrm{G}_{\mathrm{LNA}} \cdot \mathrm{G}_{\mathrm{IF}}=211.4$
5) Equivalent scheme:


The expression of Tsys results:

$$
T_{s y s}=T_{a}+T_{f}+2 T_{L N A} A_{0}+\frac{A_{0}}{G_{L N A}}\left(T_{S S B}+T_{I F} A_{c}\right)+\frac{T_{e q} A_{0} A_{c}}{G_{L N A} G_{I F}}
$$

Equating the above expression to the previously computed values we get the following expression for Glna:

$$
G_{L N A}=\frac{A_{0}\left(T_{S S B}+T_{I F} A_{c}\right)}{T_{s y s}-\left(T_{a}+T_{f}+2 T_{L N A} A_{0}+\frac{T_{e q} A_{0} A_{c}}{G_{L N A} G_{I F}}\right)}=12.54(11 \mathrm{~dB})
$$

Then:
$G_{I F}=\frac{G_{L N A} G_{I F}}{G_{L N A}}=18.5(12.67 \mathrm{~dB})$

