RF SYSTEMS – 1th Midterm test
4 November 2019

Surname & Name	
Identification Number	
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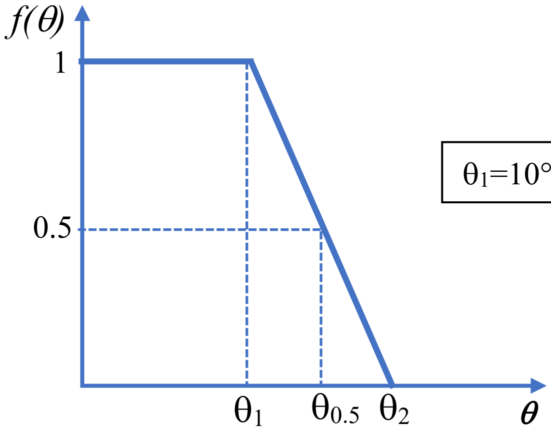
Exercise 1



Consider the antenna exhibiting the directivity function $f(\theta)$ shown in the figure (f is constant along the φ coordinate).

Analytically, f is expressed as follows: (it is zero for $\theta > \theta_2$)

$$f(\theta)=1 \quad \text{for } 0<\theta<\theta_1,$$

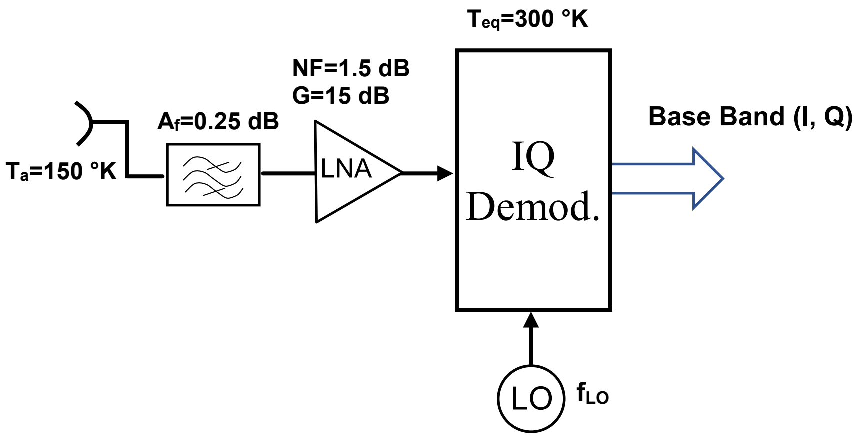
$$f(\theta)=(\theta-\theta_2)/(\theta_1-\theta_2) \quad \text{for } \theta_2>\theta>\theta_1$$

1) Evaluate the directivity gain (D_M) of the antenna.

   Hint: $\int\left(\frac{x-x_2}{x_1-x_2}\right)\cdot\sin(x)\cdot dx=\frac{\sin(x)-(x-x_2)\cdot\cos(x)}{x_1-x_2}$

2) What is D_M in the case $\theta_1=\theta_2=10°$?
3) Compute the angle $\theta_{0.5}$ where $f(\theta)=0.5$ and the beamwidth Δ_{3dB} for this angle
4) Assume that $f(\theta)$ is equal to 1 up $\theta_{0.5}$ to and zero elsewhere. What is the new value of the directivity gain?
5) An antenna with the directivity function in the above figure and efficiency $\eta=0.75$ is used in reception, producing a received power $P_r=-100$ dBm. It is known that the antenna operates at 10 GHz, is optimally directed and its radiation impedance is $Z_R=45-j10$ Ohm (the input impedance of the receiver is 50 Ohm). Evaluate the power density of the radiation incoming on the antenna.

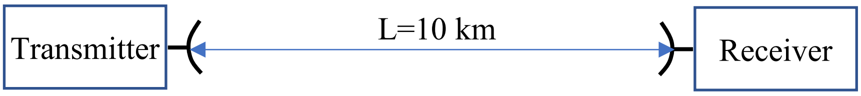
Exercise 2



The scheme in the figure refers to a direct conversion receiver operating at 12 GHz with bandwidth B=10 MHz, data rate R=100 Mbit/sec, E_b/N_0=10 dB The gain of the antenna is G_A=20 dB.

1. What is the frequency f_{LO} of the local oscillator used in the IQ demodulator?
2. Draw the equivalent scheme for the noise computation and evaluate the equivalent system temperature T_{sys} at the receiver input
3. Is the presence of the input filter necessary? Can we remove the filter without degrading the performances of the receiver? (justify the answer)
4. If we remove the LNA what is the receiver element whose noise parameter must be changed to have T_{sys} not modified? What is the new value of the parameter?
5. Assume an interfering 2-tone signal is present at the antenna output at 11.9 GHz. The filter is assumed to present a stopband attenuation A_s=20 dB. Evaluate the maximum average power of the interferer so that the intermodulation power at the amplifier output is equal to the noise power (assume IP3=10 dBm for the LNA)

The above receiver is used in the link shown below (the antennas are equal). Evaluate the minimum transmitter power P_T requested to satisfy the data rate requirement. Include the effect of the interferer assuming it increases the noise power (referred to the input of the receiver) of the same amount of the intermodulation power produced by the LNA, reported at the input of the receiver.



Solutions

Exercise 1

1) Computation of directivity gain D$_m$:

$$D_M = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} f(\theta)\sin(\theta)\,d\varphi d\theta} = \frac{2}{\int_0^{\theta_1}\sin(\theta)\,d\theta + \int_{\theta_1}^{\theta_2}\left(\frac{\theta-\theta_2}{\theta_1-\theta_2}\right)\sin(\theta)\,d\theta} =$$

$$\int_0^{\theta_1}\sin(\theta)\,d\theta = \int_0^{\theta_1} -\cos(\theta)\,d\theta = 1-\cos(10°) = 1.5192\cdot 10^{-2}$$

$$\int_{\theta_1}^{\theta_2}\left(\frac{\theta-\theta_2}{\theta_1-\theta_2}\right)\sin(\theta)\,d\theta = \frac{1}{\theta_1-\theta_2}\left[\sin(\theta)-(\theta-\theta_2)\cos(\theta)\right]_{\theta_1}^{\theta_2} = \cos(\theta_1) - \frac{\sin(\theta_1)-\sin(\theta_2)}{\theta_1-\theta_2} = 8.8215\cdot 10^{-3}$$

$$D_M = \frac{2}{2.4014\cdot 10^{-2}} = 83.2856 \;\; (19.2 \text{ dB})$$

2) $D'_M = \frac{2}{\int_0^{\theta_1}\sin(\theta)\,d\theta} = \frac{2}{1-\cos\theta_1} = 131.6461 \; (21.19 \text{ dB})$

3) $\theta_{0.5} = \frac{\theta_1+\theta_2}{2} = 12.5°, \qquad \Delta_{3dB} = 2\theta_{0.5} = 25°$

4) $D''_M = \frac{2}{\int_0^{\theta_{0.5}}\sin(\theta)\,d\theta} = \frac{2}{1-\cos\theta_{0.5}} = 84.37 \; (19.26 \text{ dB})$

5) The available power at the antenna output is given by:

$$P_{r,\max} = S_R A_e = S_R \frac{\lambda^2}{4\pi}\eta D_M = S_R 4.42\cdot 10^{-3} \qquad (\lambda = c/f = 0.03\text{m})$$

The power developed at the receiver input results:

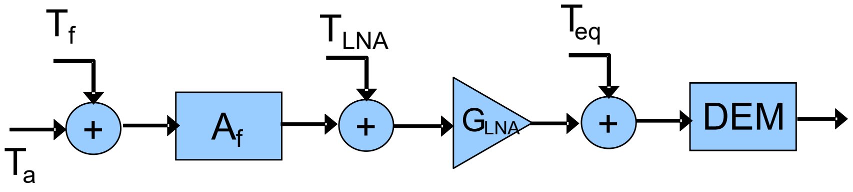
$$P_r = P_{r,\max}k_{mis}^2 = S_R 4.42\cdot 10^{-3}\frac{4\cdot 45\cdot 50}{|45+50-j10|^2} = S_R 4.42\cdot 10^{-3}\cdot 0.986 = S_R\cdot 4.359\cdot 10^{-3}$$

Imposing P$_r$=-100 dBm=10^{-13}W we get S$_R$=$2.294\cdot 10^{-11}$ W/m^2.

Exercise 2

1) The IQ demodulator require the LO with f_{LO}=f_{RF}=12 GHz

2) The equivalent scheme for the noise temperature is reported in the following figure:



The equivalent system temperature is given by:

$$T_{sys} = T_a + T_f + T_{LNA} A_f + T_{eq} \frac{A_{RA}}{G_{LNA}} = 305.45 \text{ °K}$$

$$A_f = 10^{0.025} = 1.0593,$$

$$T_f = T_0 \left(A_f - 1 \right) = 17.361 \text{ °}K, \quad T_{LNA} = T_0 \left(10^{NF/10} - 1 \right) = 120.87 \text{ °}K$$

3) The filter is required to limit the noise and reduce the effect of out-of-band blockers.

4) We can remove the LNA maintaining unchanged T_{sys} by reducing the equivalent temperature (T_{eq}) of the IQ demodulator. It has:

$$T_{sys} = T_a + T_f + T'_{eq} A_f = 305.45 \text{ °K} \quad \Rightarrow \quad T'_{eq} = \frac{T_{sys} - T_a - T_f}{A_f} = 130.36 \text{ °K}$$

5) The noise power at the LNA output is given by:

$$T_{out,LNA} = \left(T_a + T_f \right) \frac{G_{LNA}}{A_{RA}} + T_{LNA} G_{LNA} = 8818.4 \text{ °K}, \quad P_N = K T_{out,LNA} B = 1.2169 \cdot 10^{-12} \text{ (-89.15 dBm)}$$

The average power of the interferer at LNA output is given by $P_m=P_{intf}-A_f+G_{LNA}$ (in dBm), with P_{intf} the power of the interferer at antenna output. The intermodulation (average) power at LNA output is given by: $P_{int}=3P_m-2IP3-6$. Imposing $P_{int}=P_N$ we get $P_m=(P_N+2IP3+6)/3=-21.05$ dBm
The power P_{intf} will then result: $P_{intf}=P_m+A_s-G_{LNA}=-16.05$ dBm

6) In order to satisfy the requirements we must have:

$$SNR_{sys} = \frac{P_r}{P_{N,tot}} = \frac{E_b}{N_0} \frac{R}{B} = 20 \text{ dB}$$, with Pr received power and $P_{N,tot}=KT_{sys}B+P_{N,eq}$.

$P_{N,eq}$ represent the intermodulation power referred to the antenna output, i.e. $P_{N,eq}=P_N-G_{LNA}+A_f=$ -103.9 dBm → $4.074 \cdot 10^{-14}$ W.
We then have: $P_{N,tot}=4.2152 \cdot 10^{-14}+4.074 \cdot 10^{-14}=8.2892 \cdot 10^{-14}$ (-100.81 dBm)
The received power must then be given by: $P_r=SNR_{sys}-P_{N,tot}=-80.81$ dBm.
We now write the link budget equation from which P_t is derived:

$$P_r = P_t + 2G_A - 20\log\left(\frac{4\pi L}{\lambda} \right) = P_t + 40 - 134.03 = -80.81 \text{ dBm} \quad \Rightarrow \quad \text{P}_t = 13.215 \text{ dBm}$$