Microwave Amplifiers Design I

RF SYSTEM

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Microwave amplifiers with semiconductor devices

- Today amplifiers are mostly realized also at microwave frequencies with semiconductor technology
- Independently on the specific application, the active devices employed **must be biased**, i.e. specific DC voltages and currents are needed in order to guarantee the proper operation
- The biasing must be suitably separated by the RF signal by means of decoupling networks
- The biased device, including the biasing network, can be then characterized by means suitable models which are employed during the design of the amplifiers
- In case of small signal operations, the most convenient model is constituted by the **scattering matrix** of the two-port biased device

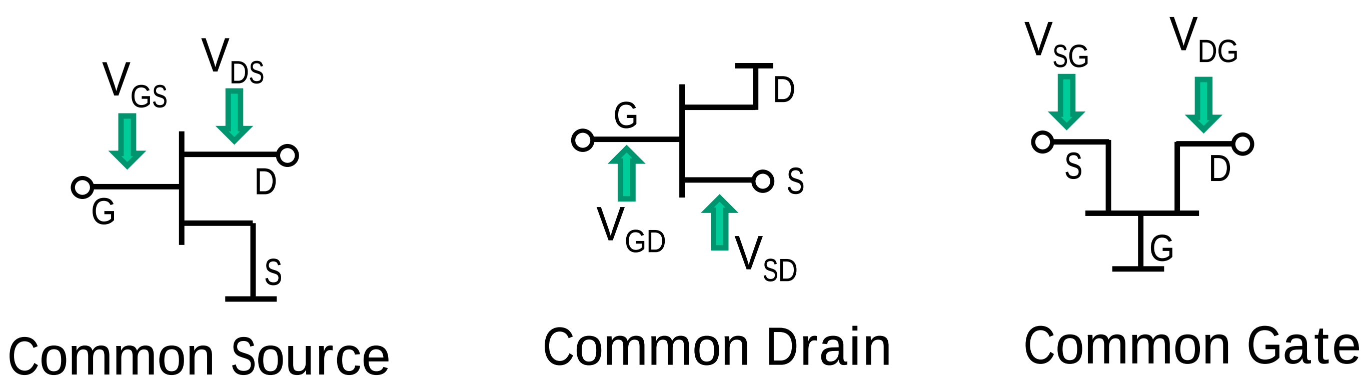
Classification of RF Amplifiers

- Small signal (general purpose)
  The design goal is to get a specific Transducer Gain (G_T) with possible requirement also on the matching at input and output. No requirement on the output power level (assumed negligible)
- Low Noise (LNA)
  In addition to the previous ones, also a requirement on the noise figure is assigned.
- Power Amplifiers (PA)
  Here the first requirement is the power level delivered to the load. Also gain, linearity and efficiency represent goals to met.

In all cases, also a bandwidth associated to the goals is given

Biasing a microwave active device

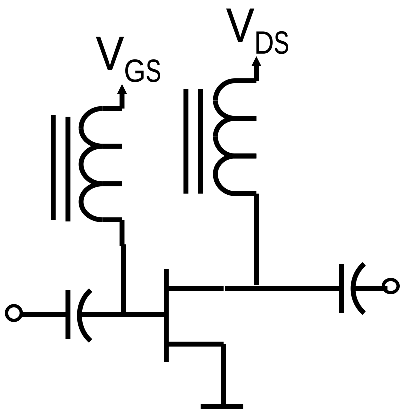
- The biasing of a general 3-terminal device consists in imposing DC voltages to two terminals with respect the remaining one. The terminal selected as reference is connected to ground.



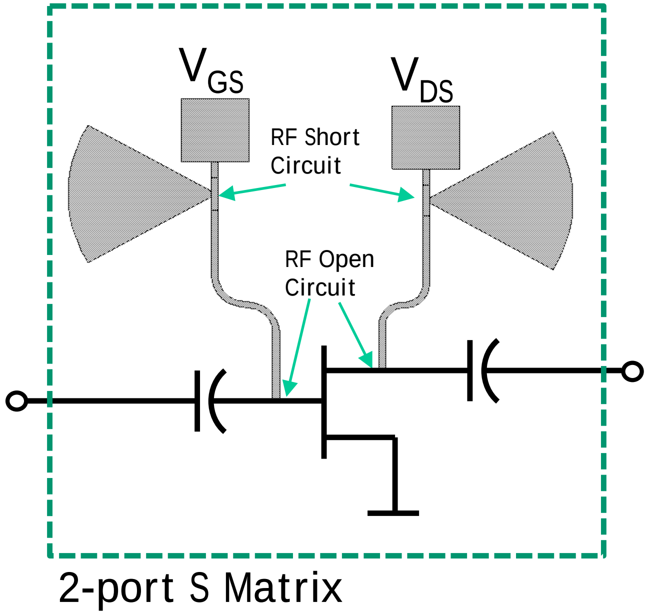
Microwave devices are usually fabricated for a given terminal reference. The most used is the **common source** configuration

Biasing Networks

- Conceptually (lumped elements):

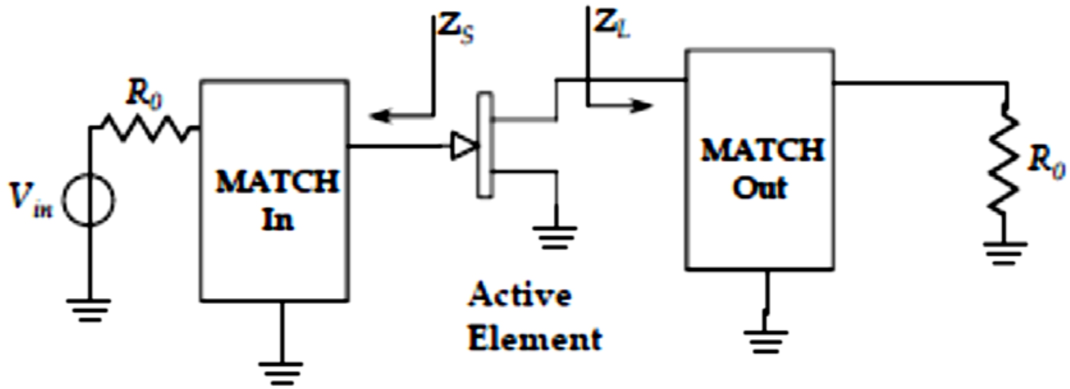


- Microwave implementation:



For small signal amplifiers the S parameters are provided by the manufacturers (for various biasing conditions)

General Topology of a single stage microwave amplifier



Task of MATCH networks (*in* and *out*):

To present at the active device suitable impedances Z_L and Z_S

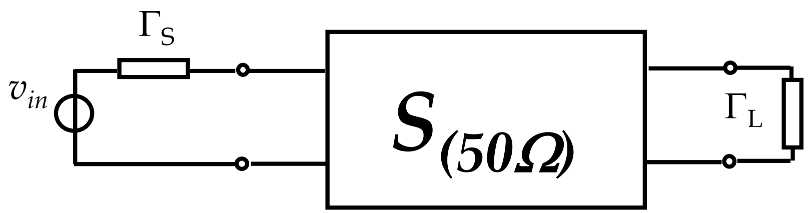
Design Steps

The design of a small signal microwave amplifier at a single frequency is composed of the following steps:

- Choice of active device
- Evaluation of optimum load for the device (Z_L and Z_S)
- Synthesis of MATCH networks

Active device representation

The active device is represented by means of its measured scattering parameter <u>in the specified bias condition</u>. The reference impedance is typically 50 Ohm

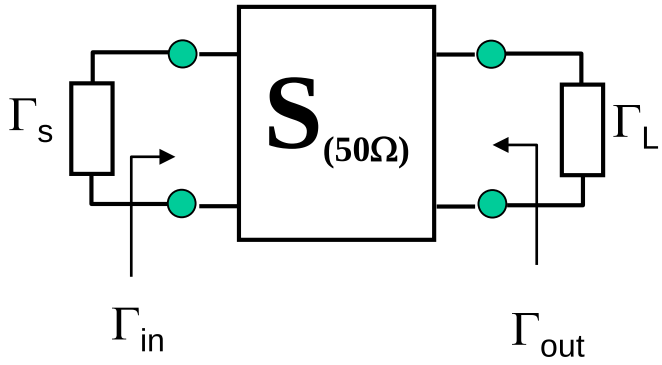


$$\Gamma_S = \frac{Z_S - 50}{Z_S + 50}$$

$$\Gamma_L = \frac{Z_L - 50}{Z_L + 50}$$

Using suitable formulas, it is possible to evaluate the Transducer Gain and the reflection coefficients at input and output.

Evaluation of Γ_{in}, Γ_{out} and G_T



All reflection coefficients are defined with respect 50 Ω:

$$\Gamma_S = \frac{Z_S - 50}{Z_S + 50}, \Gamma_L = \frac{Z_L - 50}{Z_L + 50}$$

$$\Gamma_{in} = \frac{Z_{in} - 50}{Z_{in} + 50} = s_{11} + \frac{\Gamma_L \cdot s_{12} \cdot s_{21}}{\left(1 - \Gamma_L \cdot s_{22}\right)}$$

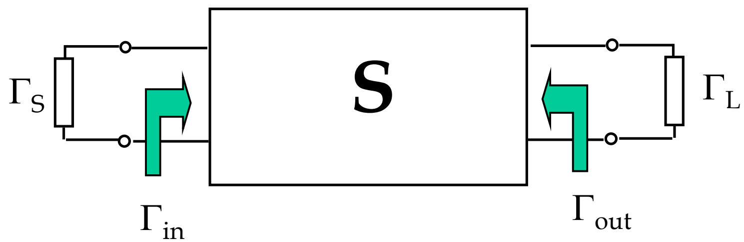
$$\Gamma_{out} = \frac{Z_{out} - 50}{Z_{out} + 50} = s_{22} + \frac{\Gamma_S \cdot s_{12} \cdot s_{21}}{\left(1 - \Gamma_S \cdot s_{11}\right)}$$

$$G_T = \left|s_{21}\right|^2 \frac{(1 - \left|\Gamma_S\right|^2) \cdot (1 - \left|\Gamma_L\right|^2)}{\left|(1 - \Gamma_S \cdot s_{11}) \cdot (1 - \Gamma_L \cdot s_{22}) - \Gamma_S \Gamma_L s_{12} s_{21}\right|^2}$$

Stability Issue

A 2-port network operating as an amplifier must be **<u>stable</u>**, i.e. the output signal must remain of finite amplitude for an input exciting signal with finite amplitude

A 2-port is said **unconditionally stable** if, for whatever value of Γ_L e Γ_S, it has:



$$\left|\Gamma_{in}\right| = \left|S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}\right| < 1$$

$$\left|\Gamma_{out}\right| = \left|S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S}\right| < 1$$

If both the conditions are not satisfied the 2-port is said **potentially instable**

Stability Conditions

Given the **S** parameters, the conditions Γ_{in}<1, Γ_{out}<1 are verified for whatever value of Γ_S, Γ_L if:

$$K = \frac{1-|s_{11}|^2 - |s_{22}|^2 + |s_{11}\cdot s_{22} - s_{12}\cdot s_{21}|^2}{2|s_{12}\cdot s_{21}|} > 1, \qquad Det[\mathbf{S}] < 1$$

When these equations are verified (and the 2port is **unconditionally stable**) a pair $\Gamma_{S,opt}$, $\Gamma_{L,opt}$ does exist which determines <u>the conjugate matching both at input and output</u>. In this condition G_T is maximum and can be expressed as:

$$G_{T,MAX} = \left|\frac{s_{21}}{s_{12}}\right|\left(K - \sqrt{K^2 - 1}\right)$$

Expressions for $\Gamma_{S,opt}$, $\Gamma_{L,opt}$

$$\Gamma_{S,opt} = \frac{C_g^*\left[B_g - \left(B_g^2 - 4|C_g|^2\right)^{\frac{1}{2}}\right]}{2|C_g|^2}$$

$$\Gamma_{L,opt} = \frac{C_L^*\left[B_L - \left(B_L^2 - 4|C_L|^2\right)^{\frac{1}{2}}\right]}{2|C_L|^2}$$

$$B_g = 1 + |s_{11}|^2 - |s_{22}|^2 - |s_{11}s_{22} - s_{21}s_{12}|^2 \qquad B_L = 1 - |s_{11}|^2 + |s_{22}|^2 - |s_{11}s_{22} - s_{21}s_{12}|^2$$

$$C_g = s_{11} - (s_{11}s_{22} - s_{21}s_{12}) \cdot s_{22}^* \qquad C_L = s_{22} - (s_{11}s_{22} - s_{21}s_{12}) \cdot s_{11}^*$$

NOTE: The above equations hold for $s_{12} \neq 0$. They are the solution of the system:

$$\Gamma_{s,opt} = (\Gamma_{in})^* = \left(s_{11} + \frac{s_{21}s_{12}\Gamma_{L,opt}}{1 - s_{22}\Gamma_{L,opt}}\right)^*, \qquad \Gamma_{L,opt} = (\Gamma_{out})^* = \left(s_{22} + \frac{s_{21}s_{12}\Gamma_{S,opt}}{1 - s_{11}\Gamma_{S,opt}}\right)^*$$

Potentially Instable Device

- When *k<1* a pair (Γ_S, Γ_L) does not exist for which G_T is maximum. In case of instability G_T becomes in fact infinity.
- The admissible values of Γ_S, Γ_L must however satisfy the conditions

$$\left|\Gamma_{in}\right| = \left|s_{11} + \frac{s_{21}s_{12}\Gamma_L}{1 - s_{22}\Gamma_L}\right| < 1, \qquad \left|\Gamma_{out}\right| = \left|s_{22} + \frac{s_{21}s_{12}\Gamma_S}{1 - s_{11}\Gamma_S}\right| < 1$$

These constraints, reported graphically on the Smith Chart, allow to identify the admissible values of di Γ_S e Γ_L.

- As a guideline for the choice of G_T the *maximum stable gain* can be considered:

$$G_{T,\max} = \left|\frac{s_{21}}{s_{12}}\right|$$

The selected value of G_T should be smaller of this value.

Admissible region for Γ_S

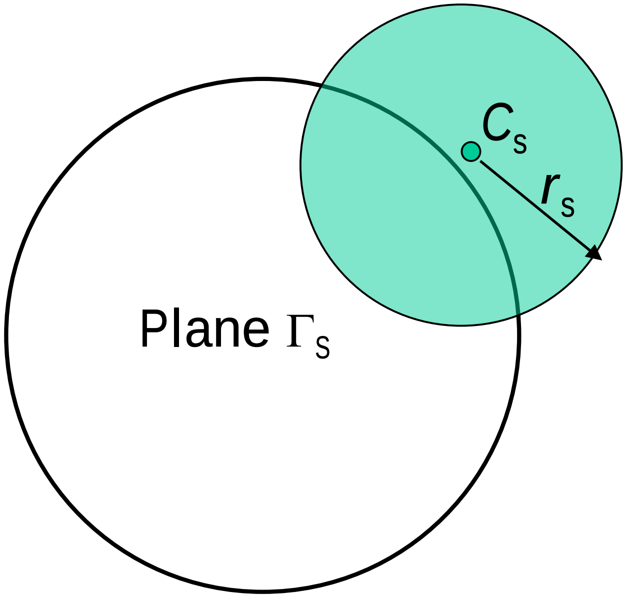
The equation defining the boundary of this region is:

$$\left|\Gamma_{out}\right| = \left| s_{22} + \frac{s_{21}s_{12}\Gamma_S}{1 - s_{11}\Gamma_S} \right| = 1$$

The equation define a circle with centre and radius given by:

$$C_S = \frac{s_{22}\Delta^* - s_{11}^*}{\left|\Delta\right|^2 - \left|s_{11}\right|^2}$$

$$r_s = \frac{\left|s_{12}s_{21}\right|}{\left|\Delta\right|^2 - \left|s_{11}\right|^2}$$



Admissible region per Γ_L

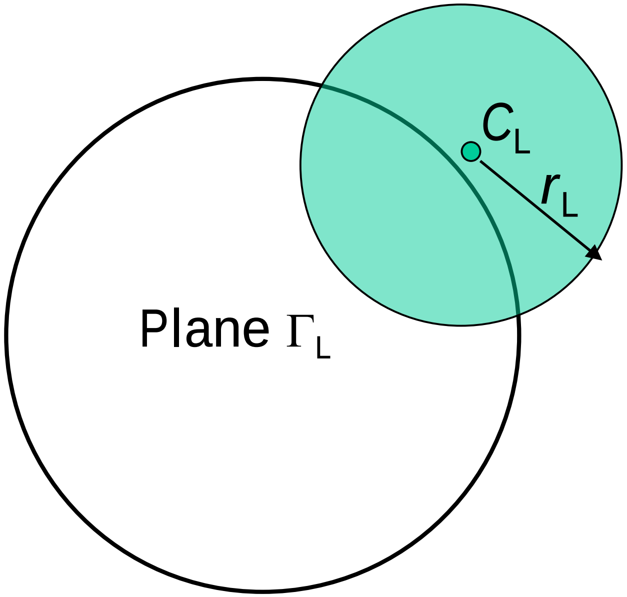
The equation defining the boundary of this region is:

$$\left|\Gamma_{in}\right| = \left| s_{11} + \frac{s_{21}s_{12}\Gamma_L}{1 - s_{22}\Gamma_L} \right| = 1$$

The equation define a circle with centre and radius given by:

$$C_L = \frac{s_{11}\Delta^* - s_{22}^*}{\left|\Delta\right|^2 - \left|s_{22}\right|^2}$$

$$r_L = \frac{\left|s_{12}s_{21}\right|}{\left|\Delta\right|^2 - \left|s_{22}\right|^2}$$



Identification of admissible region

The circles seen in the previous slide define the boundary between the admissible and not admissible regions.

To identify which of the two is the stable region the values of Γ_{in} (Γ_{out}) for Γ_L (Γ_S)=0 must be observed. Taking into account that Γ_{in} (Γ_{out}) coincides in this case with s_{11} (s_{22}), it has:

The stable region for Γ_L (Γ_S) is **outside** the instability circle if:

- $|s_{11}|$ ($|s_{22}|$) <1 and the circle <u>does not enclose</u> the centre of the chart
- $|s_{11}|$ ($|s_{22}|$) >1 and the circle <u>encloses</u> the centre of the chart

The stable region for Γ_L (Γ_S) is **inside** the instability circle if :

- $|s_{11}|$ ($|s_{22}|$) >1 and the circle <u>does not enclose</u> the centre of the chart
- $|s_{11}|$ ($|s_{22}|$) <1 and the circle <u>encloses</u> the centre of the chart

Amplifier design with potentially instable devices

In this case there is not a unique solution. It is however possible:

- To choice Γ_L in the stable region and compute Γ_S for the maximum G_T (also the resulting Γ_S must be inside the stable region)
- To choice Γ_S in the stable region and compute Γ_L for the maximum G_T (also the resulting Γ_L must be inside the stable region)

To maximize G_T the conjugate matching condition must be imposed, i.e:

$$\Gamma_s = (\Gamma_{in})^* = \left(s_{11} + \frac{s_{21} s_{12} \Gamma_L}{1 - s_{22} \Gamma_L} \right)^* \quad \Leftarrow \quad \text{Case 1}$$

$$\Gamma_L = (\Gamma_{out})^* = \left(s_{22} + \frac{s_{21} s_{12} \Gamma_S}{1 - s_{11} \Gamma_S} \right)^* \quad \Leftarrow \quad \text{Case 2}$$

Circles with constant G_p

From the definition of Power Gain G_p the following expression is derived:

$$G_p = |s_{21}|^2 \frac{(1-|\Gamma_L|^2)}{1-|s_{11}|^2+|\Gamma_L|^2\cdot\left(|s_{22}|^2-|\Delta|^2\right)-2\,\mathrm{Re}\left[\Gamma_L\left(s_{22}-\Delta\cdot s_{11}^*\right)\right]}$$

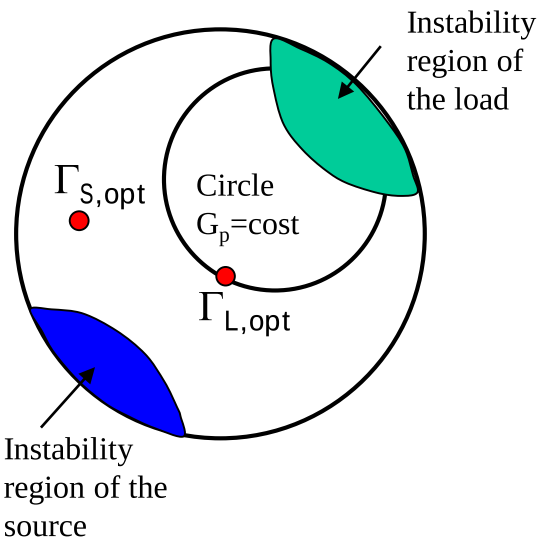
Note that G_p does not depend on Γ_S. Moreover it always larger than or equal to G_T; it is equal when the input is matched: $\Gamma_{in} = (\Gamma_s)^*$. Drawing the previous equation in the plane Γ_L, we found a curve on this plane where G_p is constant; this curve is a circle with the following center and radius ($g_p=G_p/|s_{21}|^2$):

$$C_p = \frac{g_p\left(s_{22}^*-\Delta^*\cdot s_{11}\right)}{1+g_p\cdot\left(|s_{22}|^2-|\Delta|^2\right)},\qquad r_p = \frac{\left(1-2k\,|s_{12}s_{21}|\,g_p+|s_{12}s_{21}|^2\,g_p^2\right)^{1/2}}{1+g_p\cdot\left(|s_{22}|^2-|\Delta|^2\right)}$$

Design with assigned G_p

- Select an active device with the requested G_T (maximum stable gain)
- Draw the circle $G_p = G_T$ on the Smith Chart representing Γ_L
- Select a point on this circle in the stable region; this is the selected $\Gamma_{L,opt}$
- Compute $\Gamma_{S,opt}$ by imposing the matching at the input. Verify that also $\Gamma_{S,opt}$ is in the stable region of Γ_S.

$$\Gamma_{s,opt} = \left(s_{11} + \frac{s_{21} s_{12} \Gamma_{L,opt}}{1 - s_{22} \Gamma_{L,opt}} \right)^*$$



Note: Once $\Gamma_{S,opt}$ is assigned to the source, the input is matched and, as a consequence, the transducer power gain coincides numerically with the imposed power gain

Circles with constant G_a

From the definition of Available Power Gain G_a the following expression is derived :

$$G_a = |s_{21}|^2 \frac{(1-|\Gamma_S|^2)}{1-|s_{22}|^2 + |\Gamma_S|^2 \cdot \left(|s_{11}|^2 - |\Delta|^2\right) - 2\,\mathrm{Re}\left[\Gamma_S \left(s_{11} - \Delta \cdot s_{22}^*\right)\right]}$$

Note that G_a does not depend on Γ_L. Moreover it always larger than or equal to G_T; it is equal when the output is matched: $\Gamma_{out} = (\Gamma_L)^*$. Drawing the previous equation in the plane Γ_S, we found a curve on this plane where G_a is constant; this curve is a circle with the following center and radius ($g_a = G_a / |s_{21}|^2$):

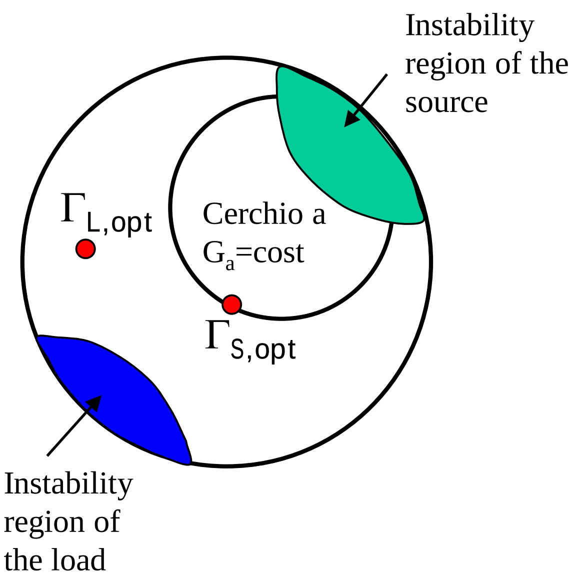
$$C_a = \frac{g_a \left(s_{11}^* - \Delta^* \cdot s_{22}\right)}{1 + g_a \cdot \left(|s_{11}|^2 - |\Delta|^2\right)}, \qquad r_a = \frac{\left(1 - 2k\,|s_{12}s_{21}|\,g_a + |s_{21}s_{12}|^2\, g_a^2\right)^{1/2}}{1 + g_a \cdot \left(|s_{11}|^2 - |\Delta|^2\right)}$$

Design with assigned Gp

- Select an active device with the requested G_T (maximum stable gain)
- Draw the circle $G_a = G_T$ on the Smith Chart representing Γ_S
- Select a point on this circle in the stable region; this is the selected $\Gamma_{S,opt}$
- Compute $\Gamma_{L,opt}$ by imposing the matching at output. Verify that also $\Gamma_{L,opt}$ is in the stable region of Γ_L.

$$\Gamma_{L,opt} = \left(s_{22} + \frac{s_{21} s_{12} \Gamma_{S,opt}}{1 - s_{11} \Gamma_{S,opt}} \right)^*$$

Note: Once $\Gamma_{L,opt}$ is assigned to the load, the output is matched and, as a consequence, the transducer power gain coincides numerically with the imposed available power gain



Design results

Once the values of $\Gamma_{L,opt}$ and $\Gamma_{S,opt}$ have been obtained with once of the method presented, we have:

Case 1 (Assigned Gp)

- Trasduced Gain imposed
- Input Matched (NOT the output)

Case 2 (Assigned Ga)

- Trasduced Gain imposed
- Output Matched (NOT the input)

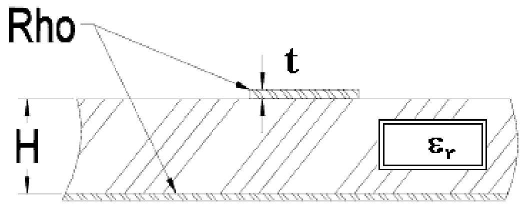
NOTE: If lossless network are used for realized $\Gamma_{L,opt}$ and $\Gamma_{S,opt}$, also the input or the output of the amplifier is matched

Example: Design of a 12 GHz Amplifier

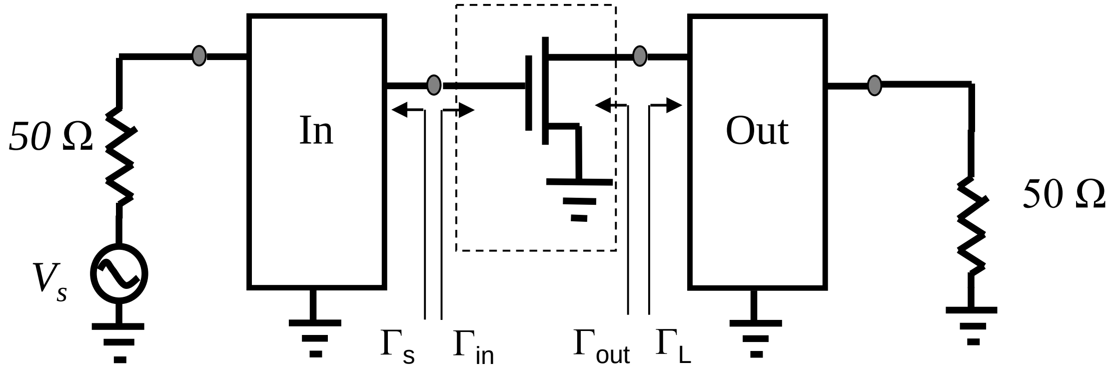
<u>Active device</u>
NEC70000 (GaAs Mesfet)
Max Gain at 12 GHz: 13.24 dB

<u>Amplifier requirements</u>
Frequency band: 11.9-12.1 GHz
Transducer gain in band: 12.5 ± 0.5 dB
Output Matching: < -15 dB

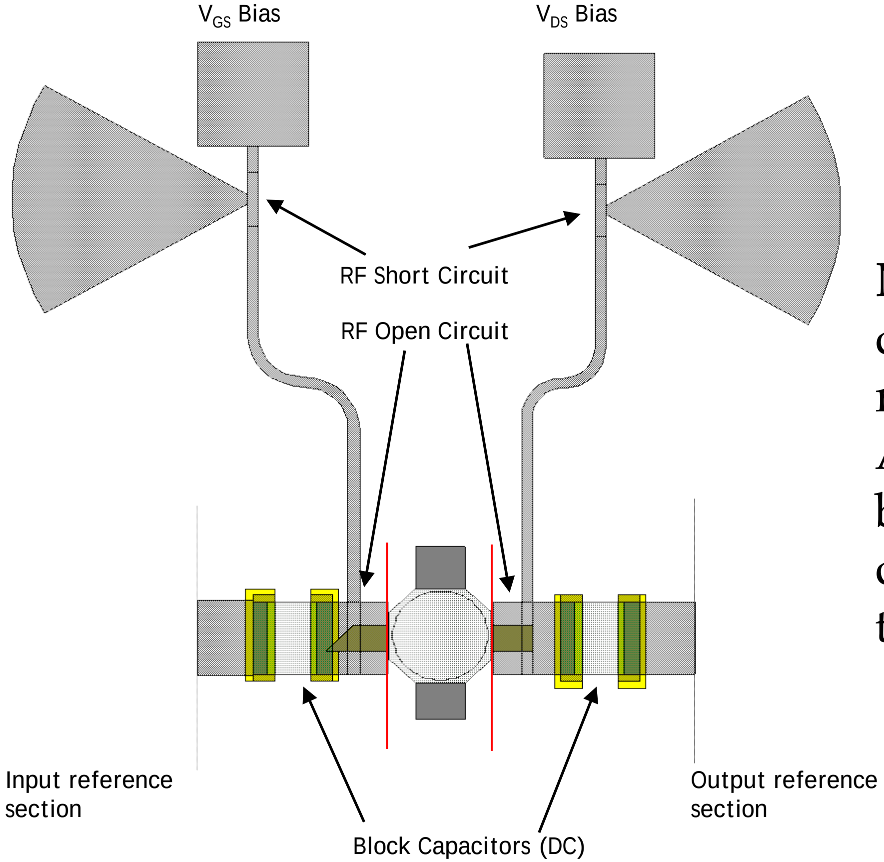
<u>Substrate</u>
$\varepsilon_r = 9.8$
H = 0.6 mm
t = 50 μm
Rho (resistivity normalized to Gold) = 1



<u>Topology:</u>



Biasing network of the active device



NOTE: The S parameters delivered by the manufacturer refers to the red sections.
After the biasing network has been assigned, the S parameters changes to the ones referred to the black sections

Scattering parameters of the biased transistor

S parameters at 12 GHz

S11 (Mag, Phase deg) 0.704 , 88.14
S12 (Mag, Phase deg) 0.099 , -115.7
S21 (Mag, Phase deg) 2.067 , -70.71
S22 (Mag, Phase deg) 0.467 , 146. 19

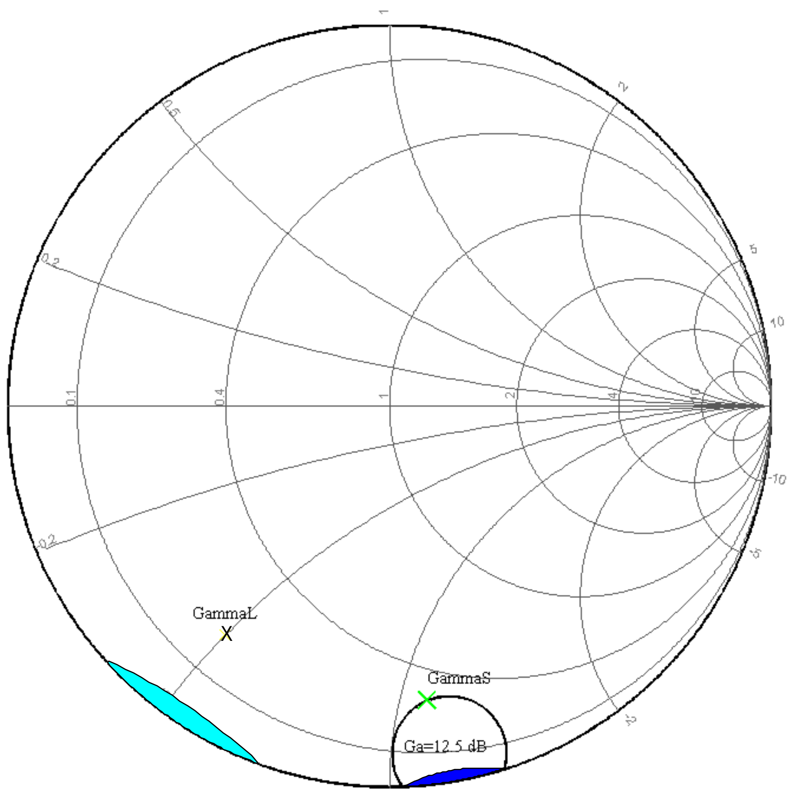
Stability Coefficient: $K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |S_{11}S_{22} - S_{12}S_{21}|^2}{2|S_{12}S_{21}|} = 0.914 < 1$

$\rightarrow$ Potentially Instable

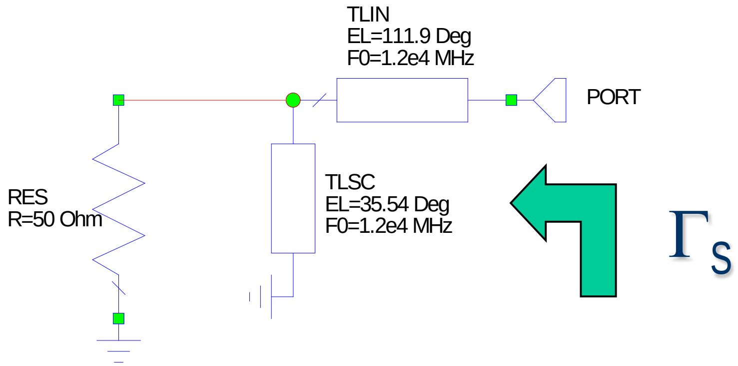
Maximum Stable Gain (dB): $10 \log \left| \frac{S_{21}}{S_{12}} \right| = 13.24$

Evaluation of Γ_L and Γ_S

- Assigned Ga : 12.5 dB
- Choice of Γ_S =0.778∠-82.7455
- Evaluation of $\Gamma_L = (\Gamma_{out})^*$=0.74 ∠-125.68
- Matching networks (single stub topology)



Input Network



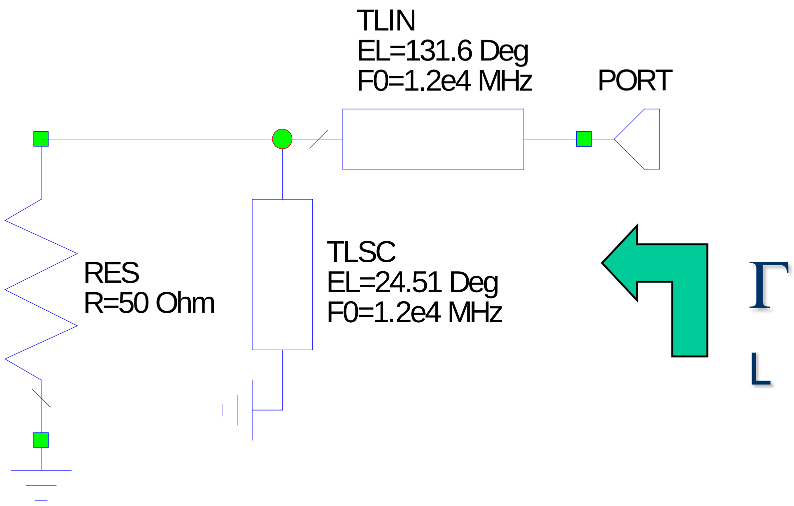
Line : $\Phi = \beta L = 111.91°$

Stub admittance: $j\,B = -j2.475 = -j\,\cot(\beta L_S)$

Stub length (s.c.) :

$$\Phi_S = \tan^{-1}\left(\frac{1}{2.475}\right) = 22^\circ$$

Output Network



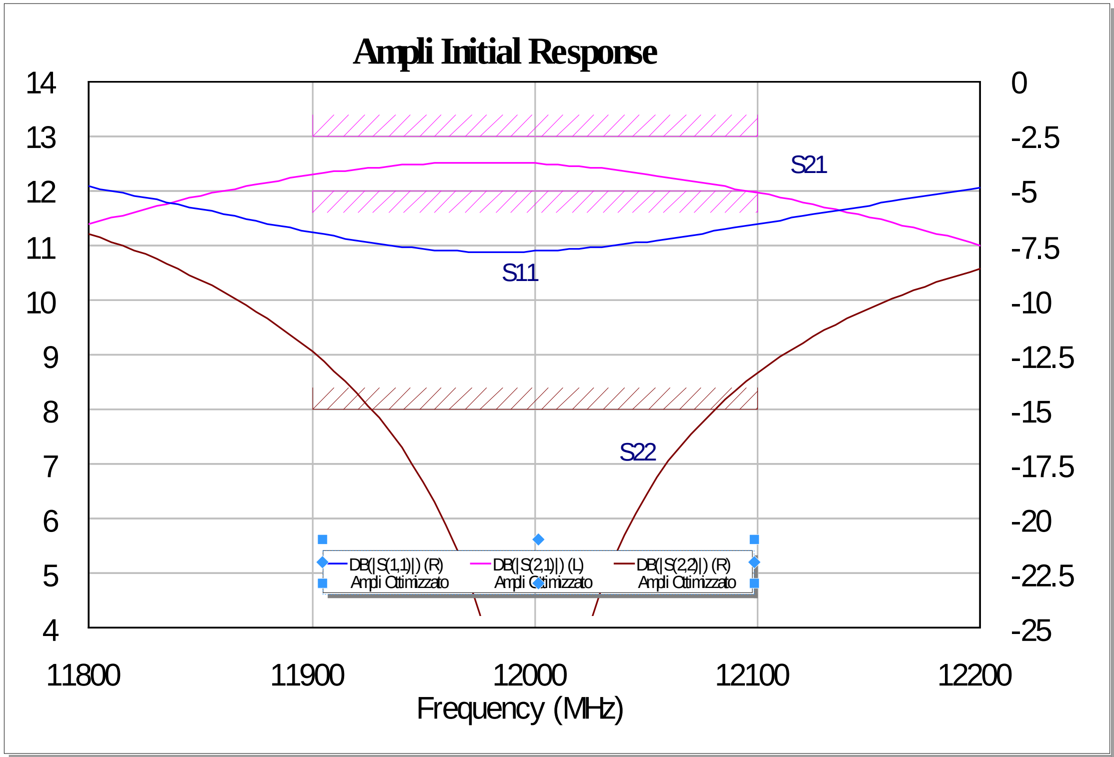
Line : $\Phi = \beta L = 131.64°$

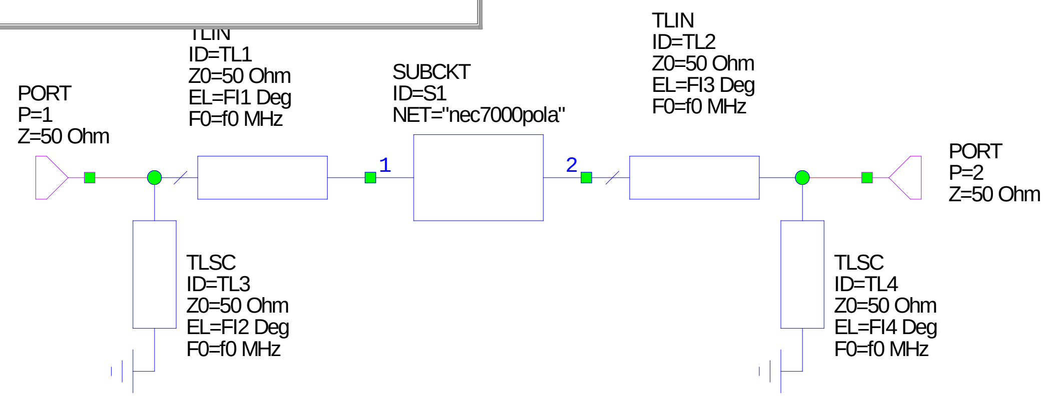
Stub admittance: $j\,B = -j2.193 = -j\,\mathrm{cotan}(\beta L_S)$

Stub length (s.c.) :

$$\Phi_S = \tan^{-1}\left(\frac{1}{2.193}\right) = 24.51^{\circ}$$

Simulated response





Optimized Response