Exercise 1

It is known the directivity function of an antenna operating at 800 MHz:

$f(\theta) = \frac{3 \cdot \sqrt{3}}{2} \sin(\theta) \cos^2(\theta)$ for $0<\theta<90°$, $f(\theta)=0$ elsewhere.

1) Compute the value of θ where f is maximum (θ_{max}).
2) Compute the directivity gain D_M. Recall: $\int \cos^2(x) \cdot \sin^2(x) dx = \frac{x}{8} - \frac{\sin(4x)}{32}$
3) Evaluate the minimum efficiency of the antenna which determines the gain G equal to 5 dB
4) Assuming the antenna used in transmission with P_T=100W, evaluate the distance from the antenna (along the direction of maximum emission) where the radiated power density S_R is 10^{-3} W/m^2. What is the electric field intensity (V/m) at this distance?

Solution:

1) Derivative of f(θ) equal to zero:

$$\frac{df}{d\theta} = \cos(\theta) - 3\sin^2(\theta)\cos(\theta) = 0 \quad \Rightarrow \quad \theta_{\max} = \sin^{-1}\left(\sqrt{\frac{1}{3}}\right) = 0.6155 \text{ rad}$$

2) D_M computation:

$$D_M = \frac{4\pi}{\frac{3\sqrt{3}}{2} \int_0^{2\pi} \int_0^{\pi/2} \sin^2(\theta)\cos^2(\theta)\, d\varphi d\theta} = \frac{4}{3\sqrt{3}} \frac{1}{\left[\frac{\theta}{8} - \frac{\sin(4\theta)}{32}\right]_0^{\pi/2}} =$$

$$= \frac{4}{3\sqrt{3}} \frac{16}{\pi} = 3.9 \quad (5.93 \text{ dB})$$

3) $\eta = \frac{G}{D_M} = 0.806$

4)

$$S_R = \frac{P_T G}{4\pi R^2} \Rightarrow R = \sqrt{\frac{100 \cdot 10^{0.5}}{4\pi 10^{-3}}} = 158.63 \text{ m}$$

$$E = \frac{1}{R}\sqrt{\frac{Z_0 \cdot P_T \cdot G}{2\pi}} = \frac{1}{158.63}\sqrt{\frac{377 \cdot 100 \cdot 10^{0.5}}{2\pi}} = 0.87 \text{ V/m}$$

Exercise 2

A broadcasting transmitter operates in FM commercial band (88-108 MHz) with P_T=1KW. The antenna has the efficiency η=0.9 and the following directivity function:

$f(\theta,\varphi)=1$ for $0<\theta<90°$, $0<\varphi<360°$, $f(\theta,\varphi)=0$ elsewhere

In the surroundings of the FM station, it is planned to build civil buildings. The current legislation however imposes severe limits for the maximum field in such buildings: E_{max}<6 V/m, S_{max}<0.1 W/m^2 (S_{max} represent the maximum power density)

1) Evaluate the gain of the antenna and the emitted ERP power
2) Evaluate the minimum distance from the antenna for which both the above limits are respected

1) $$D_{MAX} = \frac{4\pi}{\int_0^{\pi/2} \sin\vartheta \, d\vartheta \cdot 2\pi} = \frac{2}{\left[-\cos\theta\right]_0^{\pi/2}} = 2$$

$$G = \eta \cdot D_{MAX} = 1.8 \quad (2.55\,dB)$$

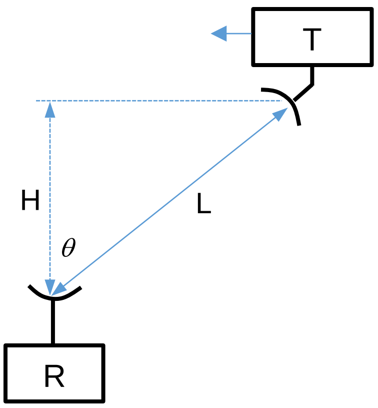
$$ERP = P_T \cdot G = 1.8 \text{ KW}$$

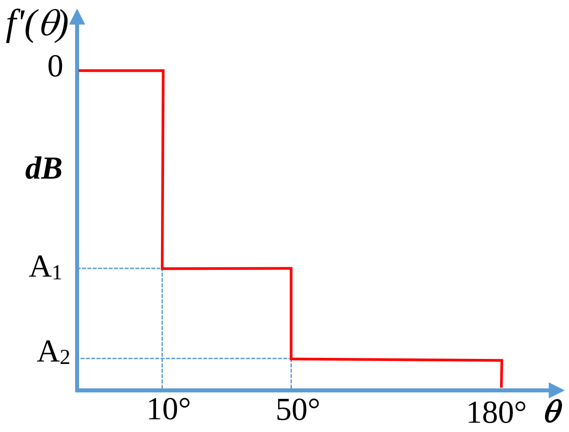
2) $$d_1 = \sqrt{\frac{ERP}{4\pi S_R}} = 37.85\,m$$

$$d_2 = \frac{1}{E_{MAX}} \sqrt{\frac{Z_0 \cdot ERP}{2\pi}} = 54.77$$

$$d_{MIN} = d_2 = 54.77\,m$$

Exercise 3





The transmitter T (P_T=20 W, f_0=4 GHz) is located on an aircraft flying parallel to ground at an altitude H=10 Km. The antenna on T is optimally directed toward the receiving station R on ground and has the gain G_T=10 dB.

The antenna at the ground station R is directed along the vertical direction and exhibits the following directivity diagram: $f(\theta,\varphi) = f'(\theta)\cdot\cos(\varphi/2)$ where $f'(\theta)$ is shown on the figure (A_1= -10 dB). Note that the range of φ is: $-\pi < \varphi < \pi$. The scheme on the above figure (left) refers to the plane φ=0.

1) Compute the parameter A_2 of the directivity diagram in order that the gain of the receiving antenna (G_R) is equal to 17 dB. Assume the radiation impedance Z_R=50 Ω e the loss resistance R_p=10Ω.
2) Compute the 3dB beamwidth $\Delta\varphi_{3dB}$ of the antenna in the plane θ=0.
3) Compute the power received by R when the aircraft crosses the positions where θ is equal 10° and 50° (with φ=0)
4) It is known the sensitivity of the receiver S=-90 dBm. Evaluate the maximum value of L where the signal is still detected (note that θ>50° for this distance)

Solution:

1) First the directivity gain D_{max} is computed:

$$D_{max} = \frac{4\pi}{\int_{-\pi}^{\pi}\int_{0}^{\pi} \sin(\varphi)\sin(\theta)\, f'(\theta)\, d\varphi\, d\theta} = \frac{4\pi}{4\left[(1-\cos(10°)) + A_1(\cos(10°)-\cos(50°)) + A_2(1+\cos(50°))\right]} =$$

$$= \frac{\pi}{0.0494 + A_2 1.6428} = \frac{G}{\eta} = 60.1425$$

Where $\eta = Z_{rad}/(Z_{rad}+R_p)$=0.8333 and G=10^1.7=50.1187
From the above equation it is found A_2=0.00173 (-27.6 dB)

2) The directivity in φ is:

$$f\left(\overline{\varphi}\right) = \cos\left(\frac{\overline{\varphi}}{2}\right) = 0.5 \Rightarrow \overline{\varphi} = 120°$$

The 3dB beamwidth is then $\varphi = 2\overline{\varphi} = 240°$

3) At i-th crossing the distance L is equal to:

L_i=H/cos(θ_i) with θ_i equal to 10° and 50°. The received power is obtained from the Friis eqution:

$$P_r = P_t + G_T + G_R - L_{f,i} + A_i$$

For θ_i =10°, L_1=H/cos(10°)=10.154 Km, $L_{f,i}$=20log(4πL_1/λ)=124.61 dB

For θ_i =50°, L_2=H/cos(50°)=15.557 Km, $L_{f,i}$=20log(4πL_2/λ)=128.32 dB

Replacing on the Friis equation (Pt=43dBm, G_T=10dB, G_R=17dB) it has:

Pr1=-64.616 dBm, Pr2=-85.94 dBm

4) From the Friis equation:

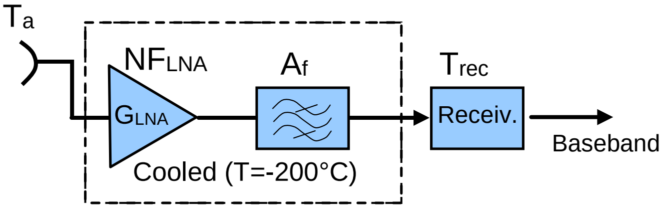
$$P_r = P_t + G_T + G_R - L_f + A_2 = -90 \Rightarrow L_f = 132.4 \text{ dB}$$

Then replacing the expression of L_f:

$$L = \frac{\lambda}{4\pi} 10^{L_f/20} = 24.88 \text{ Km}$$

Exercise 4

A space probe is sent in orbit around Saturn to take high-quality pictures of the planet surface. The probe has a dish antenna (4m diameter, aperture efficiency e_a=0.6) and a transmitter with P_T=30 W at 10 GHz. The receiving station on Earth employs an antenna with 70m diameter (e_a=0.65), connected to the receiver schematically depicted in the following figure.



T_a=10°K
NF_{LNA}=0.5 dB
G_{LNA}=10 dB
A_f=0.5 dB
T_{rec}= 100 °K

$K=1.38\cdot10^{-23}$

1) Evaluate the gain of receiving and transmitting antennas
2) The distance Saturn-Earth is $1.43\cdot10^9$ Km. Compute the received power P_r at the antenna output
3) Evaluate the equivalent system temperature (T_{sys}) of the receiving system
4) Assuming the requested E_b/N_0=10 dB, evaluate the maximum data rate R allowed by the system
5) If the signal bandwidth *B* is 0.1 MHz, what is the SNR_{sys}?

$\lambda = 3\cdot10^8/10^{10} = 0.03m$ $\quad P_T = 10\log_{10}(30) = 14.77$ dBW

1) $G_T = e_{aT}\left(\frac{\pi d_T}{\lambda}\right)^2 = 1.053\cdot10^5$ (50.22 dB)

$G_R = e_{aR}\left(\frac{\pi d_R}{\lambda}\right)^2 = 3.49\cdot10^7$ (75.43 dB)

2) $P_R = P_T - 20\log\left(\frac{4\pi\cdot1.43\cdot10^{12}}{0.03}\right) + G_T + G_R = -155.12$ dBW

3) $T_{sys} = T_a + T_{LNA} + \frac{T_0(a_f-1)}{g_{lna}} + T_{Rec}\cdot a_f/g_{LNA}$

$T_0 = 273 - 200 = 73°K$ $\quad T_{LNA} = T_0(10^{NF/10}-1) = 8.907$ °K

$a_f = 10^{A_f/10} = 1,122$ $\quad T_f = T_0(a_f-1) = 8,907$°K $\quad g_{lna} = 10$

$T_{sys} = 31.018$ °K

4) $R = \frac{P_R}{K\cdot T_{sys}\cdot E_b/N_0} = 71.82$ Kbit/sec

5) $SNR = P_R/(K T_{sys}\cdot B) = 8.56$ dB

6) $S = 10\cdot10^7$ bit $\quad T = \frac{S}{R} = 1390$ sec.