Antennas

Radiated Transmission

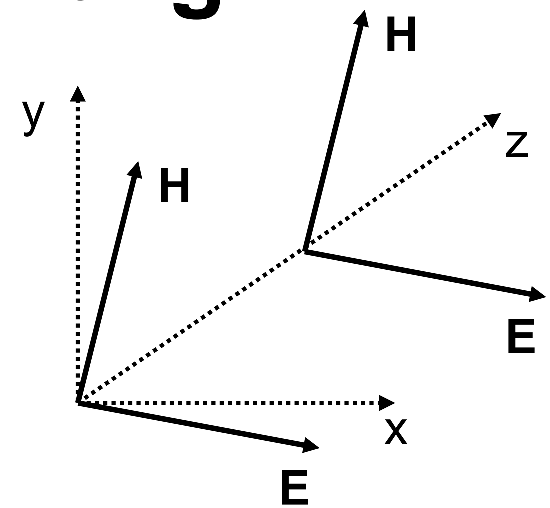
- The electromagnetic field can be “guided” by transmission lines and “radiated” in the space by suitable transducers called <u>antennas</u>
- The propagation in the free space is governed by Maxwell equations. At a sufficient distance from the source the radiated field is well represented by a <u>plane wave</u> (E and H vectors in a plane orthogonal to the propagation direction and orthogonal each other)

Polarization

- Linear: the direction of E ed H vectors don’t change during propagation
- Circular: E and H vectors rotate while propagating (they remain orthogonal each other)

Phasor representation of a plane wave propagating along *z*

- Linear polarization:



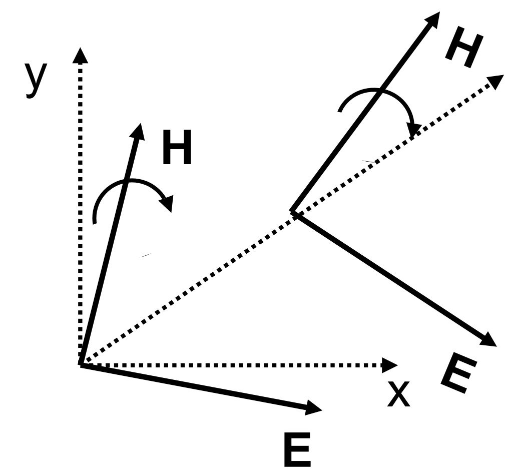
$$\mathbf{E} = \left(E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}} \right) e^{-j\beta z}$$

$$\mathbf{H} = \left(H_x \hat{\mathbf{x}} + H_y \hat{\mathbf{y}} \right) e^{-j\beta z}$$

- Circular polarization :

$$\mathbf{E} = \left(E_x \hat{\mathbf{x}} \pm jE_y \hat{\mathbf{y}} \right) e^{-j\beta z}$$

$$\mathbf{H} = \left(H_x \hat{\mathbf{x}} \pm jH_y \hat{\mathbf{y}} \right) e^{-j\beta z}$$



Relationship between E and H vectors of a plane wave

From Maxwell Equations:

$$E_x = Z_W H_x, \;\; E_y = -Z_W H_y$$

Z_W is called *intrinsic impedance* of the medium.
It has:

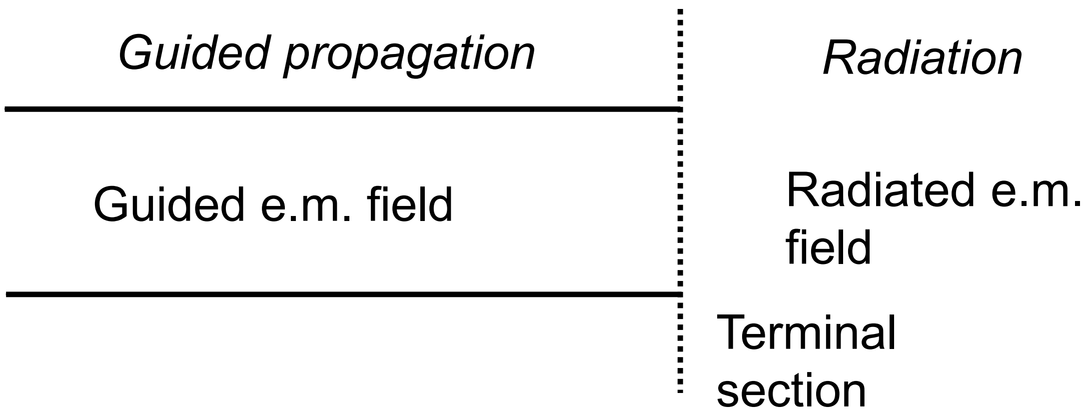
$$Z_W = \frac{377}{\sqrt{\varepsilon_r}} \text{ (Ohm)}$$

It also holds:

$$\left|E\right| = Z_W \left|H\right|$$

Generation of a plane wave in the space

- A transmission line terminating abruptly:



- The terminal section is a transition between guided propagation and radiation

Efficient generation of radiated em field

- The previously observed transition is a very bad way to produce radiation (most of the power is reflected back to the transmission line)
- An **Antenna** is a sort of matching structure that makes as efficient as possible the generation of radiated em power

Types of Antennas

- There are many different types of antennas, whose choice depends on a multiplicity of factors (frequency, size, cost, etc)
- In this course we will consider only the properties of the antennas impacting on the radio link.
  In other courses the physical configuration and the dimensioning techniques of the most important types of antennas are treated in details

E.M. Field generated by the antenna

- The generated field depends on two contributions:
  - the Near Field varying as $1/R^2$
  - Il Far Field varying as $1/R$
- At a point (R,θ,ϕ) sufficiently far from the antenna the near field is practically negligible and the far field is represented by a plane wave. The associated power density (magnitude of Poynting Vector) is given by:

$$S_R = \frac{dP_{rad}}{ds} = \frac{1}{2}\mathrm{Re}\left(\mathbf{E}\times\mathbf{H}^*\right) = \frac{1}{R^2}U\left(\theta,\phi\right)$$

- The function $U(\theta,\phi)$ is the radiation intensity and represents the power radiated per unit solid angle. If the power is isotropically radiated: $U=U_I=P_{rad}/4\pi$

Directional properties of the radiated power

- The power radiated by an antenna depends on the direction. The *directive gain* toward a given direction is expressed as:

$$D(\vartheta,\varphi)=\frac{\text{Radiation Intensity}}{\text{Isotropic Intensity}}=\frac{U(\vartheta,\varphi)}{P_{rad}/4\pi}$$

- The radiated power density at a point (R,θ,ϕ) can be then expressed as:

$$S_R(R,\vartheta,\varphi)=\frac{1}{R^2}U(\vartheta,\varphi)=\frac{P_{rad}}{4\pi R^2}D(\vartheta,\varphi)$$

- There is a direction $(\vartheta_{\max},\varphi_{\max})$ where D is maximum. The *directivity function* is the function D normalized to D_M:

$$f(\vartheta,\varphi)=\frac{D(\vartheta,\varphi)}{D_M} \quad \Rightarrow \quad S_R(R,\vartheta,\varphi)=\frac{P_{rad}}{4\pi R^2}D_M f(\vartheta,\varphi)$$

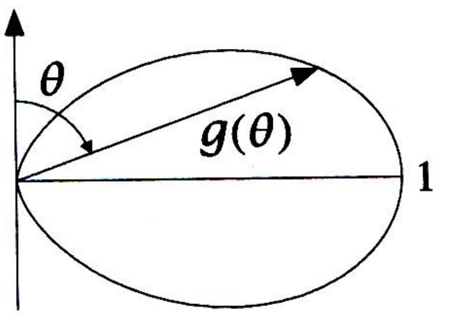
Meaning of D_M

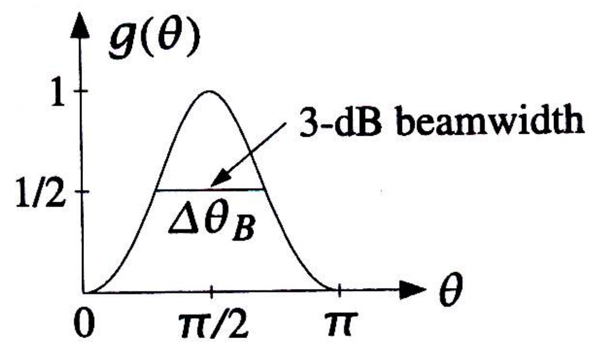
- D_M represents the ratio between the radiated power density in the direction where it is maximum divided by the power density obtained with an isotropic radiator

$$D_M = \frac{U\left(\vartheta_{\max}, \varphi_{\max}\right)}{P_{rad}/4\pi} = \frac{S_R\left(R, \vartheta_{\max}, \varphi_{\max}\right)}{\dfrac{P_{rad}}{4\pi R^2}}$$

Plots of directivity function

- The directivity function is usually displayed in a polar plot with one of the polar coordinates (θ,ϕ) assigned and the other assumed as variable:

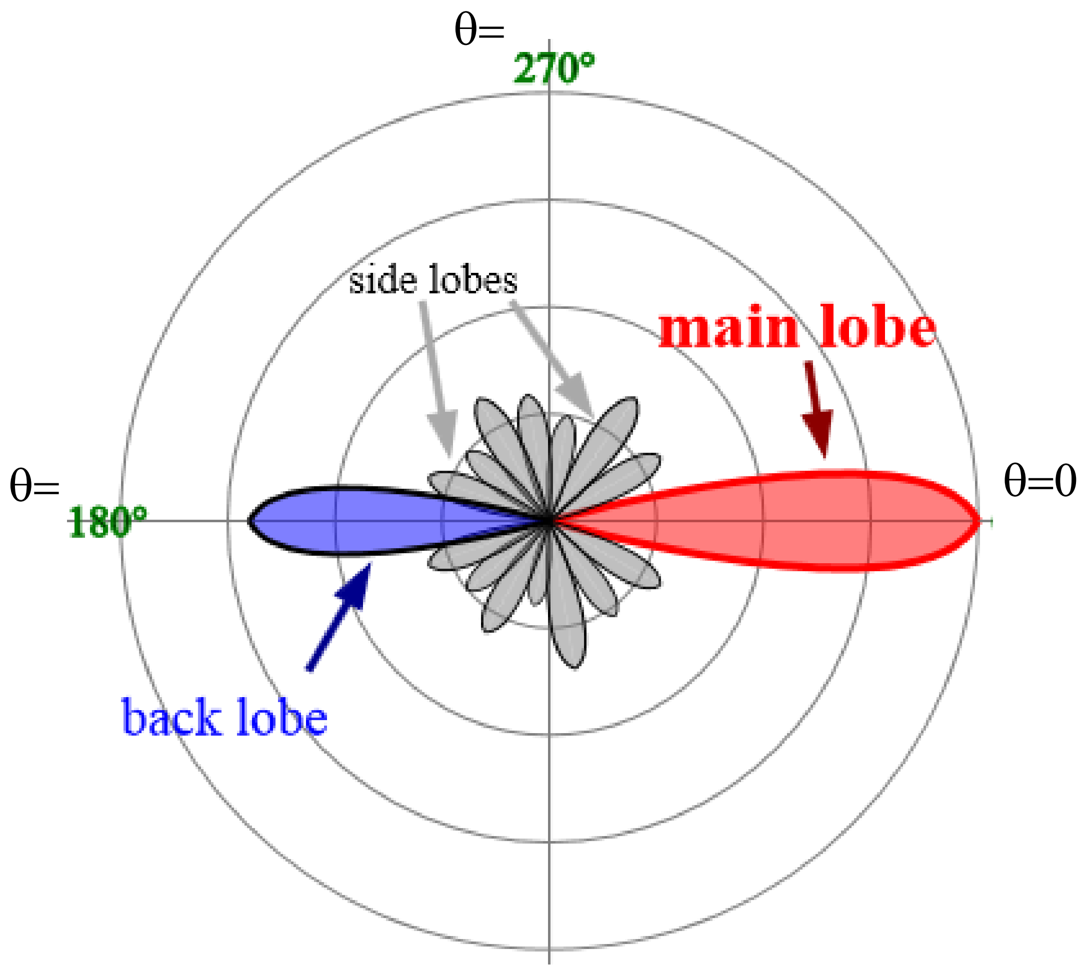




$$g(\vartheta) = \frac{\cos^2\left(0.5\pi\cos(\vartheta)\right)}{\sin^2(\vartheta)}$$

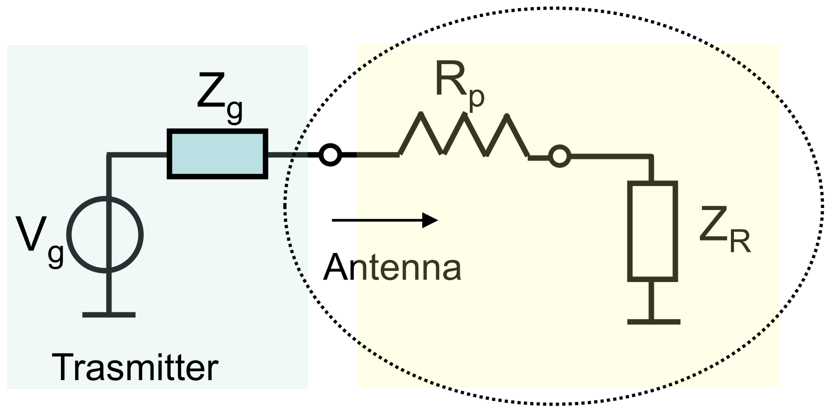
- The parameter $\Delta\vartheta_B = \vartheta_{3dB}$ is called *half-power beamwidth* and is defined by the two values of ϑ where *g* is halved with respect the direction of maximum

Directivity Diagram of a real antenna



Radiated Power and Electrical Power

- From the transmitter side the antenna can be represented as an electrical 1-port:



The impedance "seen" by the transmitter is determined by two contributions: Z_R (<u>radiation impedance</u>) and R_p (<u>loss resistance</u>). The power dissipated by Z_R constitutes the radiated power. Typically $R_p <<| Z_R |$

- In order to have the maximum power transferred to the antenna (P_T) we must have: $Z_g = (R_p + Z_R)^* \cong Z_R^*$. In this condition P_T is equal to the available power of the source.
  When the above condition is not verified, a matching network should be introduced between the antenna and the transmitter.
- The power actually radiated by the antenna can be expressed as:

$$P_{rad} = P_T \frac{\mathrm{Re}(Z_R)}{\mathrm{Re}(Z_R) + R_p} = \eta P_T$$

η is defined *efficiency factor* of the antenna

Antenna Gain

- La radiated power density can be the expressed as:

$$S_R(R,\vartheta,\varphi)=\frac{P_{rad}}{4\pi R^2}D_M f(\vartheta,\varphi)=\frac{P_T}{4\pi R^2}\eta D_M f(\vartheta,\varphi)$$

$$\boxed{S_R(R,\vartheta,\varphi)=\frac{P_T G}{4\pi R^2}f(\vartheta,\varphi)}$$

- $G=\eta D_M$ is defined *Gain* of the antenna
- $P_T G$ is referred as Effective Radiated Power (***ERP***) . It is often measured in logarithmic units (dBm or dBW)

Fields Intensity

- For a plane wave propagating in a medium with relative dielectric constant ε_r the E and H fields intensity are related to the power density (S_R) as follows (Z_0=intrinsic impedance of vacuum ≈ 377 Ω):

$$S_R = \frac{dP_{rad}}{ds} = \frac{1}{2}\mathrm{Re}\left(\mathbf{E}\times\mathbf{H}^*\right) = \frac{1}{2}\frac{\sqrt{\varepsilon_r}}{Z_0}\left|\mathbf{E}\right|^2 = \frac{1}{2}\frac{Z_0}{\sqrt{\varepsilon_r}}\left|\mathbf{H}\right|^2$$

The power density from a transmitting antenna (P_T, G) at distance *R* along the direction of maximum radiation is given by: $S_R = P_T \cdot G/(4\pi R^2)$.

Then:

$$\left|E\right| = \frac{1}{R}\sqrt{\frac{Z_0 \cdot P_{ERP}}{2\pi\sqrt{\varepsilon_r}}}\ \ \mathrm{V/m}, \qquad \left|H\right| = \frac{1}{R}\sqrt{\frac{\sqrt{\varepsilon_r}\cdot P_{ERP}}{2\pi \cdot Z_0}}\ \ \mathrm{A/m}$$

With $P_{ERP} = P_T \cdot G$ (*effective radiated power)*.

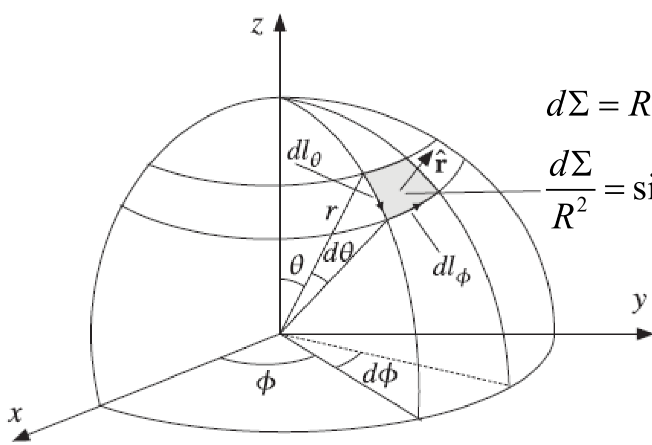
Example: P_T=100 W, G=10 dB, r=10m → E=24.5 V/m

Evaluation of G from the directivity function

- From the formula of the power density at distance R we can derive the following condition:

$$P_{rad} = \iint_{\Sigma} S_R\left(R,\vartheta,\varphi\right) d\Sigma = \frac{P_{rad} D_{MAX}}{4\pi R^2} \iint_{\Sigma} f\left(\vartheta,\varphi\right) d\Sigma$$

Σ=Surface of the sphere with radius R.



$$d\Sigma = R^2 \sin\left(\theta\right) d\theta d\varphi \quad \Rightarrow$$

$$\frac{d\Sigma}{R^2} = \sin\left(\theta\right) d\theta d\varphi$$

$$\frac{P_{rad} D_{MAX}}{4\pi R^2} \iint_{\Sigma} f\left(\vartheta, \varphi\right) d\Sigma = P_{rad} \Rightarrow D_{MAX} = \frac{4\pi}{\iint_{\Sigma} f\left(\vartheta, \varphi\right) \frac{d\Sigma}{R^2}}$$

Solid Angle ($\Delta\Omega$)

$$D_{MAX} = \frac{G}{\eta} = \frac{4\pi}{\int_0^{\pi} \int_0^{2\pi} f\left(\vartheta, \varphi\right) \sin\left(\theta\right) d\theta d\varphi}$$

$$G = 4\pi\eta \Big/ \int_0^{\pi} \int_0^{2\pi} f\left(\vartheta, \varphi\right) \sin\left(\theta\right) d\theta d\varphi$$

Beamwidth

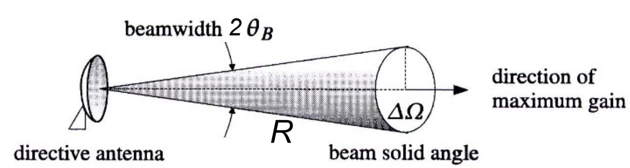
High gain antennas have most of the radiated power concentrated inside the main beam. If we assume that this beam is a cone centered in the antenna position, the half-aperture of the cone (θ_B) can be obtained (approximately) by the following expression:

$$\cos(\theta_B) = 1 - \frac{2}{D_{MAX}}$$

The **beamwidth** $\Delta\theta$ is two time θ_B: $\Delta\theta = 2\cos^{-1}\left(1 - \frac{2}{D_{MAX}}\right)$

Note that this expression of θ_B can be obtained by the last equation of the previous slide with $f(\theta,\varphi) = f(\theta)$ and:

$$f(\theta) = \begin{cases} 1 & \text{for} \quad 0 < \theta < \theta_B, \\ 0 & \text{for} \quad \theta_B < \theta < \pi, \end{cases}$$



Receiving Antenna

An electromagnetic plane wave incident on a receiving antenna produces an electric power P_r given by :

$$P_r = S_R \cdot A_e \cdot g\left(\vartheta, \varphi\right)$$

with S_R power density of the incident wave, $g(\theta,\phi)$ directivity function and A_e *effective area* of the receiving antenna.

From a physical point of view, the effective area is the ratio between the received power and the power density of the incident wave with the antenna pointed along the maximum of the directivity function. The internal electrical dissipation (efficiency factor) is included into A_e.

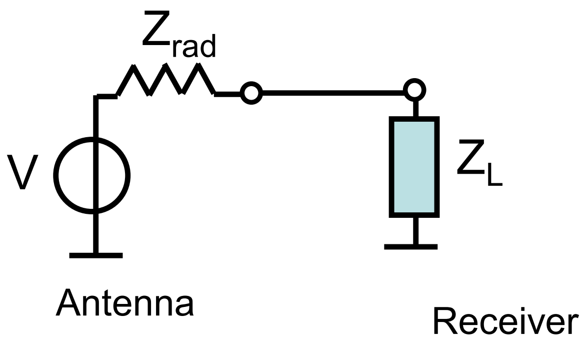
Relationship between A_e e G

- For all antennas it can be shown that A_e, G and the wavelength $\lambda = c/f$ are related as follows:

$$\frac{G}{A_e} = \frac{4\pi}{\lambda^2} = \frac{4\pi f^2}{c^2}$$

- The effective area is not equal to the physical area of the antenna (typically is a fraction of this latter)
- Antennas fall into two classes:
  - Fixed-area (A_e indipendent on frequency)
  - Fixed-gain (G indipendent on frequency)

Equivalent circuit of the receiving antenna



If the antenna is pointed in the direction of the incident wave and $Z_L = (Z_{rad})^*$:

$$P_L = k_{pol}^2 P_{r,\max} = k_{pol}^2 S_R \cdot A_e$$

Where k_{pol}^2 accounts for a different polarization of the incident wave with respect that of the antenna.

If the matching condition is not verified, i.e. $Z_L \neq (Z_{rad})^*$:

$$P_L = k_{pol}^2 P_{r,\max} \frac{4\,\mathrm{Re}\{Z_{rad}\}\,\mathrm{Re}\{Z_L\}}{\left|Z_{rad} + Z_L\right|^2} = k_{pol}^2 k_{mis}^2 P_{r,\max}$$

With:

$$k_{mis}^2 = \frac{4\,\mathrm{Re}\{Z_{rad}\}\,\mathrm{Re}\{Z_L\}}{\left|Z_{rad} + Z_L\right|^2}$$

Mismatch factor

Example: Dish antenna

Category: Fixed Area

Gain:

$$G = A_e \frac{4\pi}{\lambda^2} = A_e \frac{4\pi}{c^2} f^2$$

G <u>increseas quadratically with</u> *f*

The effective area A_e is proportional to the dish area:

$$A_e = e_a \frac{1}{4} \pi d^2$$

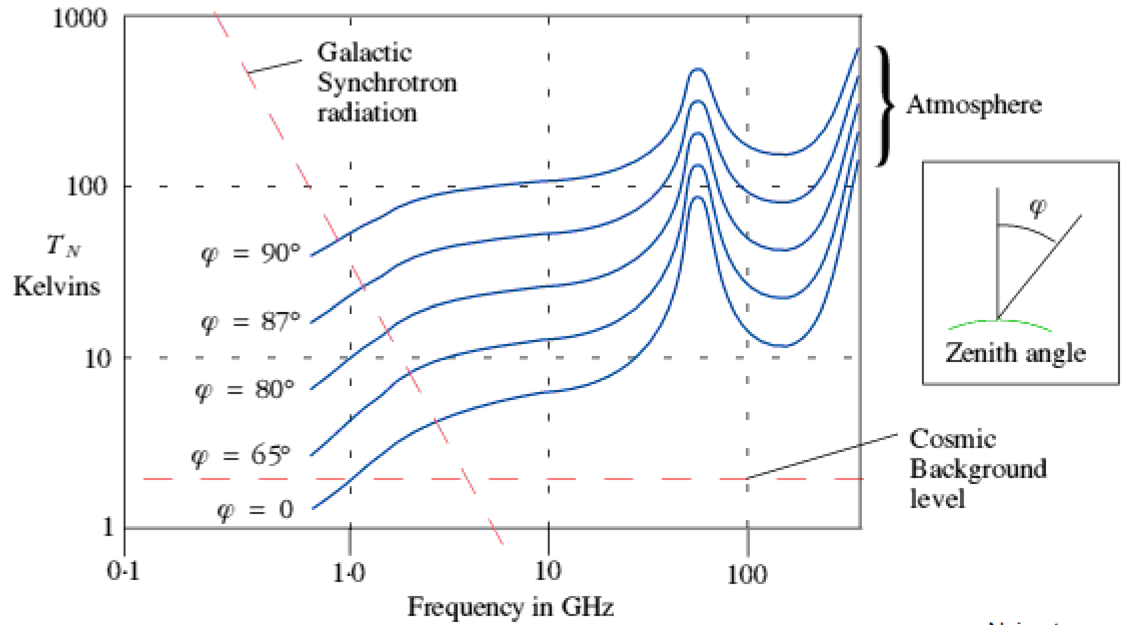
e_a= aperture efficiency (0.55-0.65)
d = Dish diameter
G <u>increseas quadratically with</u> *d*

The beamwidth $\Delta\theta$ is:

$$\Delta\theta = 2\cos^{-1}\left(1 - \frac{2}{D_{MAX}}\right) = 2\cos^{-1}\left(1 - \frac{2\eta}{\pi^2 \cdot e_a}\left(\frac{\lambda}{d}\right)^2\right)$$

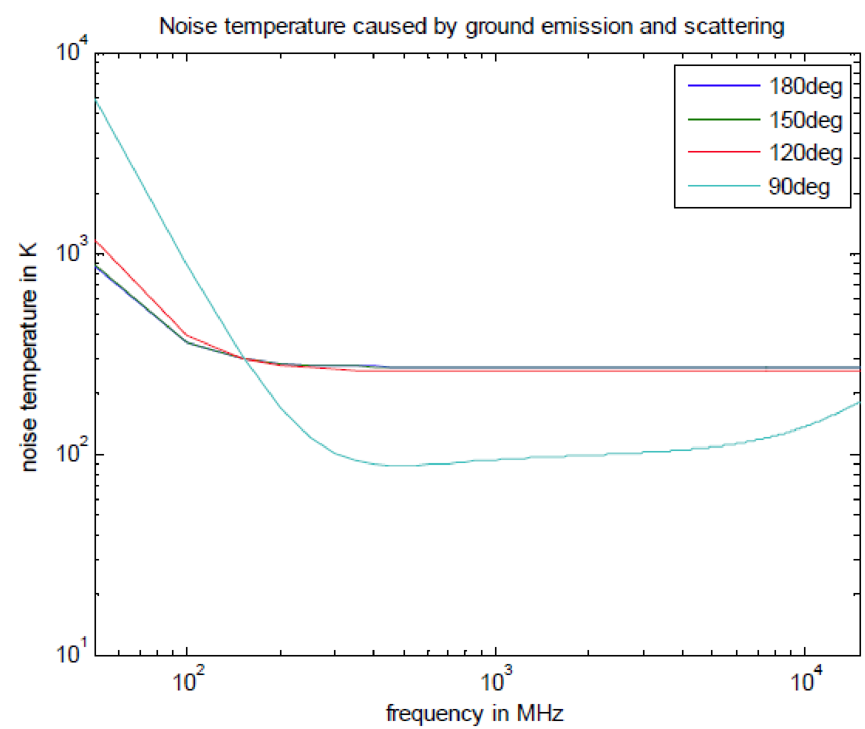
Noise at the antenna output

- In addition to the desired signal, the receiving antenna produces noise at its output which depends on several factors
- This kind of noise is typically generated by the intrinsic radiation associated to the materials
- The noise depends first of all on the pointing of the antenna; also the environment around the antenna may affect the noise generation
- Three main sources can be classified as follows:
  - Clear Sky noise: the antenna is directed toward the sky and there are not atmospheric perturbations
  - Ground noise: it is the noise radiated by the earth and partially captured by the main lobe of the antenna (also from the side lobes)
  - Perturbed atmosphere noise: the noise produced by rain, snow, fog, etc.



Sky Noise

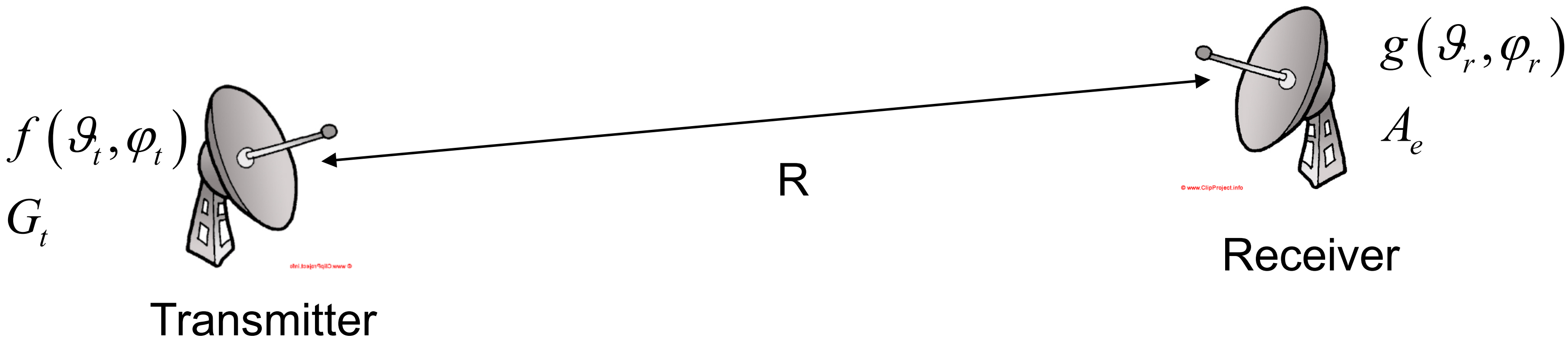
Ground Noise



Antenna parameters (summary)

- Gain G (transmission)
- Effective Area A_e (reception)
- Directivity function (same for T and R): $g(\theta,\phi)=f(\theta,\phi)$
- Antenna Impedance
- Beamwidth
- Polarization (E field direction)
- Side lobes level
- Equivalent noise temperature T_{ant}

The RF Link budget: Friis equation



Transmitter power density (at distance R)

$$S_R(R,\vartheta,\varphi)=\frac{P_t}{4\pi R^2}G_t\cdot f(\vartheta_t,\varphi_t)$$

Received power

$$P_r=S_R\cdot A_e\cdot g(\vartheta_r,\varphi_r)$$

Link Budget (Friis equation):

$$P_r=S_R\cdot A_e\cdot g(\vartheta_r,\varphi_r)=\frac{P_t}{4\pi R^2}G_t\cdot f(\vartheta_t,\varphi_t)\cdot A_e\cdot g(\vartheta_r,\varphi_r)$$

Logarithmic form of Friis Equation

Taking into account the relationship between A_e e G_r:

$$P_r = P_t \left(\frac{\lambda}{4\pi R} \right)^2 G_t \cdot G_r \cdot g\left(\vartheta_r, \varphi_r\right) \cdot f\left(\vartheta_t, \varphi_t\right)$$

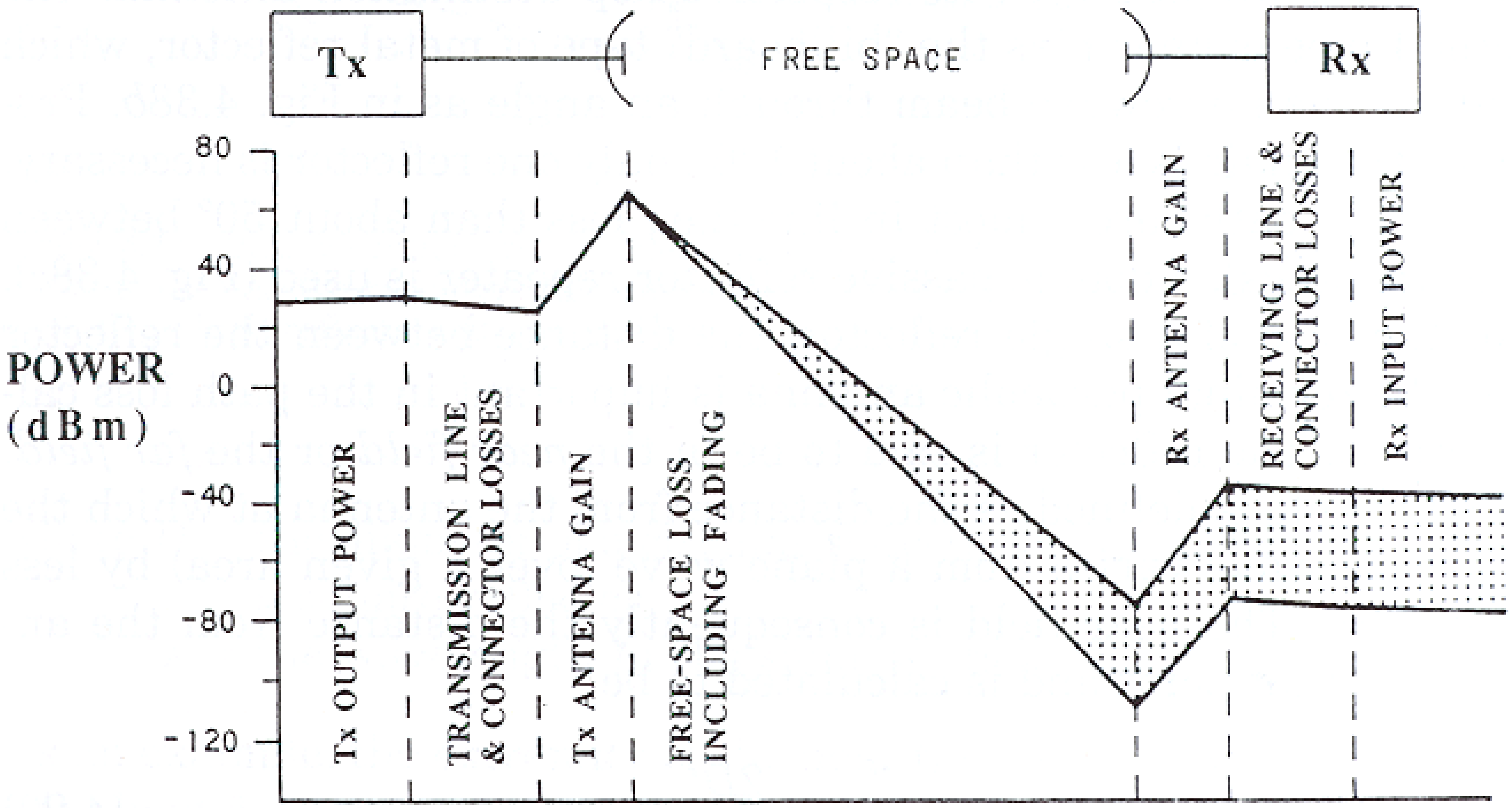
Introducing the logarithmic units (dB) it has:

$$P_{r,dBm} = P_{t,dBm} - 20 \cdot \log\left(\frac{4\pi R}{\lambda} \right) + G_{t,dB} + G_{r,dB} + f\left(\bar{\vartheta}_r, \bar{\varphi}_r\right)\Big|_{dB} + g\left(\bar{\vartheta}_t, \bar{\varphi}_t\right)\Big|_{dB}$$

Free space attenuation (additional contributions for atmospheric phenomena)

Note that $\left(\bar{\vartheta}_t, \bar{\varphi}_t\right)$ represent the coordinates beneath which the transmitting antenna is seen by the receiving antenna. Vice versa for $\left(\bar{\vartheta}_r, \bar{\varphi}_r\right)$.

Power Budget Diagram



$$Pr = Pt - Adt + Gt - FSL + Gr - Adr$$

System SNR

- Given the received signal (power Pr) with band B and assuming the noise limited to this band (with a filter), the SNR of the receiving system is given by:

$$SNR_{sys} = \frac{P_r}{K \cdot T_{sys} \cdot B}$$

- Introducing the expression of P_r obtained by the Friis equation:

$$SNR_{sys} = \frac{G_t \cdot P_t}{K \cdot B}\left(\frac{\lambda}{4\pi R}\right)^2 \frac{G_r}{T_{sys}} = P_{ERP}\frac{1}{L_f}\frac{1}{KB}\left(\frac{G_r}{T_{sys}}\right)$$

Where L_f is the free-space loss and the ratio G_r/T_{sys} is referred as the G/T ratio of the receiving antenna (it is measured in dB/°K)

Data Rate Limits

- We know that the system SNR limits the data rate between transmitting and receiving antenna. According to the Shannon's Theorem, the maximum data rate that can be achieved is (B signal bandwidth):

$$C = B\log_2\left(1 + SNR\right)$$

For $R<C$ is theoretically possible to realize an error-free transmission (with an ideal coding scheme). In practice there is a small but not zero probability of error represented by the BER (Bit-error-rate). BER is related to the ratio E_b/N_0 (according to the modulation scheme adopted); E_b is the energy per bit and N_0 is the system noise spectral density KT_{sys}.

T_b= time for receiving one bit → $R=1/T_b$
P_r= received power $=E_b/T_b=E_bR$

From the definition of system SNR:

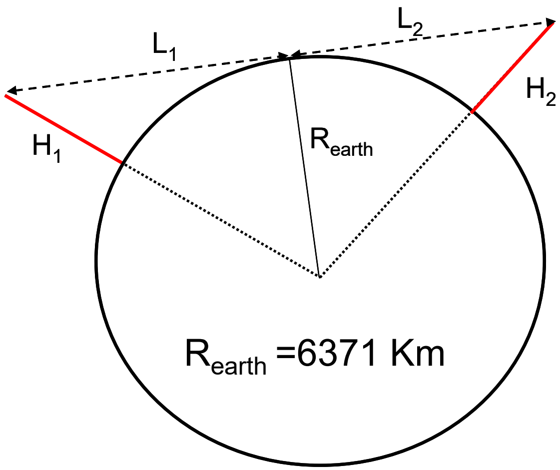
$$SNR_{sys} = \frac{P_r}{KT_{sys}B} = \left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right) = P_{ERP}\frac{1}{L_f}\frac{1}{KB}\left(\frac{G_r}{T_{sys}}\right)$$

For a pair of communicating antennas oriented for the maximum of the directivity diagrams:

$$R = \frac{P_{ERP}}{E_b/N_0}\frac{1}{K \cdot L_f}\left(\frac{G_R}{T_{sys}}\right)$$

R represents the maximum data rate which allows to obtain the imposed E_b/N_0 (and then the corresponding BER) over the link and with the assigned G/T and transmitted ERP

Terrestrial Radio Link: maximum length



The maximum length is limited by the Earth's surface curvature.

Ideal conditions:
- Earth is a sphere
- Wave scattering produced by the atmosphere is negligible

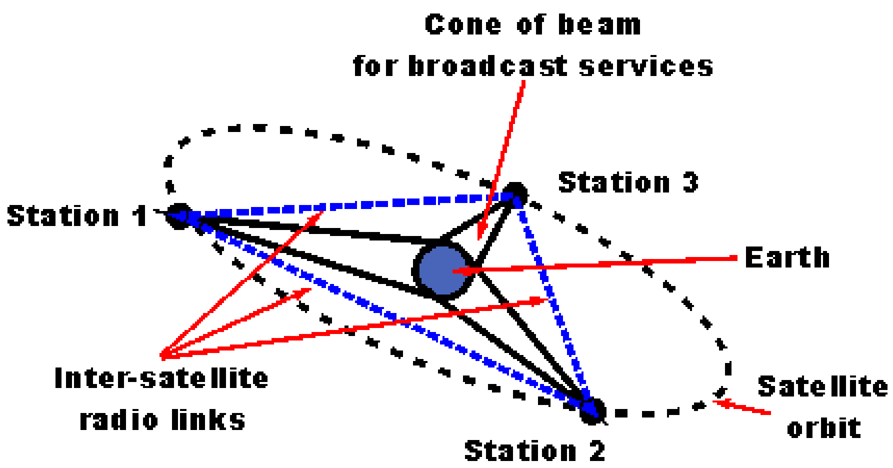
In ideal conditions the maximum distance is determined by the visibility condition of two points placed at heights H_1 and H_2:

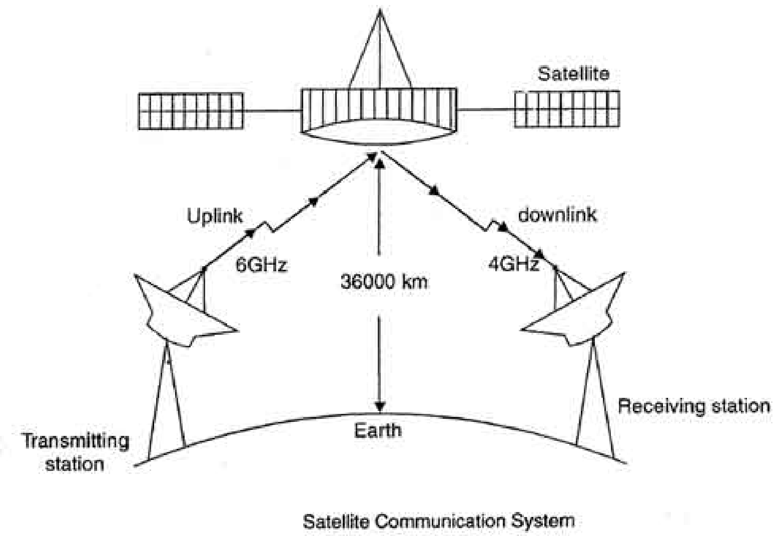
$$L_1 = \sqrt{\left(H_1 + R_{earth}\right)^2 - R_{earth}^2}, \qquad L_2 = \sqrt{\left(H_2 + R_{earth}\right)^2 - R_{earth}^2}$$

$$\left(L_{\max}\right)_{Km} = L_1 + L_2 \simeq 3.57\left(\sqrt{\left(H_1\right)_{meter}} + \sqrt{\left(H_2\right)_{meter}}\right)$$

Satellite Communications

- Using 3 geostationary satellites it is possible to cover the whole Earth's surface:



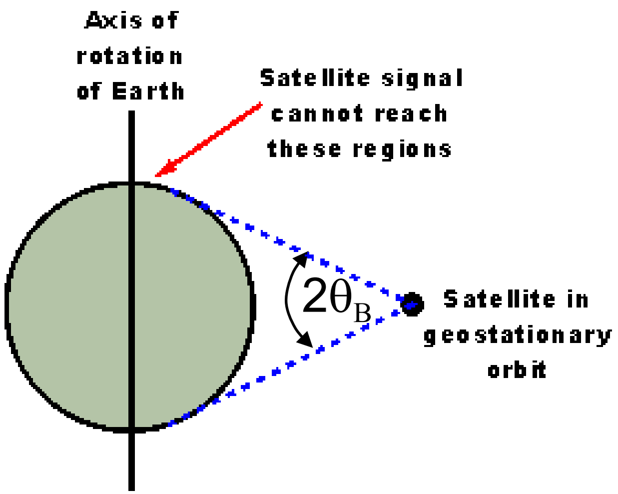


A geostationary satellite rotates in the same direction as the rotation of the Earth and has an approximate 24 hour period. This means that it remains in the same position relative to the Earth.

Antennas for Earth’s Stations

- They must exhibit a very large gain (the requested ERP from the earth’s station is extremely high for compensating the propagation loss)
- Dish antennas with a very large diameter are employed (10-30m) to this purpose
- The gain is up to 60 dB with the main beamwidth equal to a fraction of degree
- To maintain the correct pointing of the satellite an automatic system is necessary (a beacon signal transmitted from the satellite is used to this purpose)

Beamwidth of satellite antennas



- The value of θ_B is obtained by imposing the distance of geo satellite (~36000 Km) and the earth's radius (6371 Km). We get:

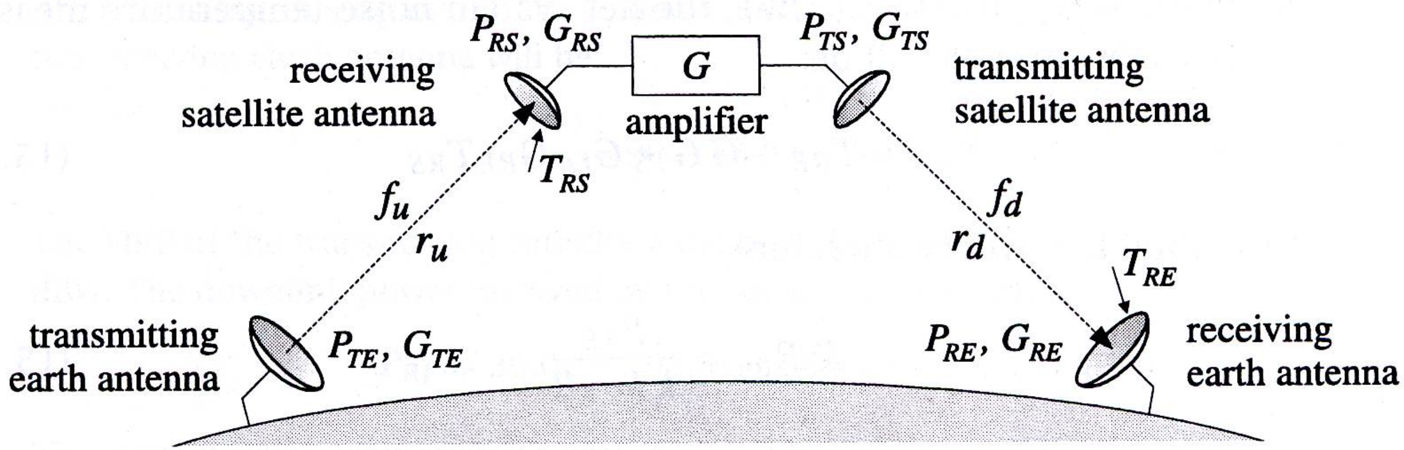
$$2\theta_B = 17.36°$$

The antenna (lossless dish) must exhibit the following parameters (η=1):

$$G = \left(\frac{2}{1-\cos(\theta_B)} \right) = 174.62 \ (\sim 22.4 \text{ dB}),$$

$$d = \frac{\lambda}{\pi}\sqrt{\frac{G}{e_a}} = 27.15 \text{ cm} \quad (\lambda\text{=5cm}, \ e_a = 0.6)$$

Satellite Link



f_u=Up-link frequency (6 GHz), *fd*=Down-link frequency (4 GHz)
r_u, r_d>36000 Km (depends on the angle beneath which the satellite is seen)
G_{TE}, G_{RS}, G_{TS}, G_{RE}= Gain of earth and satellite antennas
P_{TE}, P_{TS}=Transmitting powers (earth and satellite)
G = Gain of satellite transponder

Friis Equation (best pointing of the antennas):

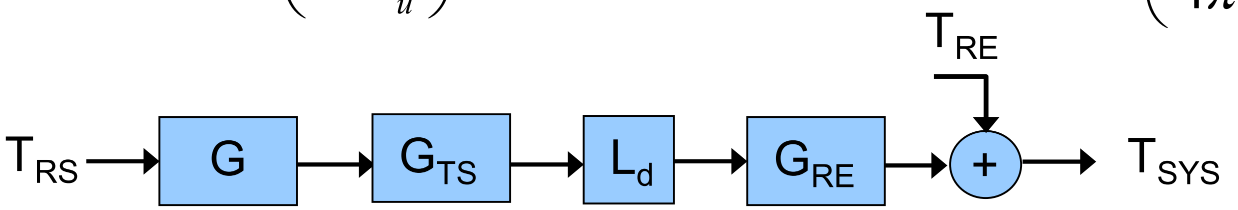
$$P_{RE} = P_{TE} \cdot G_{TE} \cdot G \cdot G_{RS} \cdot G_{TS} \cdot G_{RE} \cdot \left(\frac{\lambda_u}{4\pi r_u} \right)^2 \cdot \left(\frac{\lambda_d}{4\pi r_d} \right)^2$$

SNR of the satellite link

T_{RS}= Noise equivalent temperature of the satellite receiving system
T_{RE}= Noise equivalent temperature of the earth receiving station

$$SNR_u = \frac{P_{RS}}{KT_{RS}B} \quad \text{(Upper link)} \qquad SNR_d = \frac{P_{RE}}{KT_{RE}B} \quad \text{(Down link)}$$

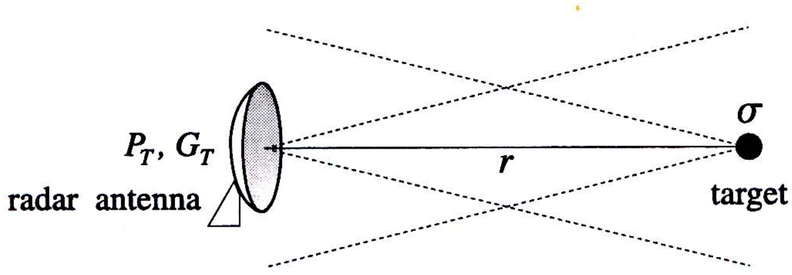
$$P_{RS} = P_{TE} \cdot G_{TE} G_{RS} \cdot \left(\frac{\lambda_u}{4\pi r_u} \right)^2 \qquad P_{RE} = P_{RS} \cdot G \cdot G_{TS} G_{RE} \cdot \left(\frac{\lambda_d}{4\pi r_d} \right)^2$$



$$T_{sys} = T_{RE} + T_{RS} \cdot G \cdot G_{TS} G_{RE} \cdot \left(\frac{\lambda_d}{4\pi r_d} \right)^2 = T_{RE} + T_{RS} \cdot \frac{P_{RE}}{P_{RS}}$$

$$SNR_{tot} = \frac{P_{RE}}{KT_{sys}B} = \frac{P_{RE}}{K\left(T_{RE} + T_{RS} \cdot \frac{P_{RE}}{P_{RS}} \right) B} = \frac{1}{\frac{KT_{RE}B}{P_{RE}} + \frac{KT_{RS}B}{P_{RS}}} = \boxed{\frac{1}{1/SNR_u + 1/SNR_d}}$$

Radar Equation



- The wave incident on the target is reflected back and a portion is intercepted by the radar antenna. The distance of the target is obtained from the delay of the reflected echo (the overall path length is 2r)
- The concept of *radar cross section* σ provides a measure of the effective area of the target:

$$\sigma = \frac{P_{\mathrm{target}}}{S_T}$$

P_{target}: Power reflected by the target
S_T=Power density incident on the target

Assuming the antenna pointed for the maximum of directivity, we have at the target position:

$$S_T = \frac{P_T G_T}{4\pi r^2} \qquad P_{\text{target}} = \sigma S_T = \sigma \frac{P_T G_T}{4\pi r^2}$$

The power density of the wave reflected back at the radar position:

$$S_{\text{target}} = \frac{P_{\text{target}}}{4\pi r^2} = \sigma \frac{P_T G_T}{\left(4\pi r^2\right)^2}$$

Finally the power at the radar receiver is given by:

$$P_R = A_R S_{\text{target}} = \sigma \frac{P_T G_T A_R}{\left(4\pi r^2\right)^2} = \sigma \frac{P_T G_T^2 \lambda^2}{\left(4\pi\right)^3 r^4} = P_T \cdot G_T^2 \cdot \frac{4\pi\sigma}{\lambda^2} \cdot \left(\frac{\lambda}{4\pi r}\right)^4$$

$$P_{R,dBm} = P_{T,dBm} + 2G_{T,dB} + 10\log_{10}\left(\frac{4\pi\sigma}{\lambda^2}\right) - 2L_f$$

The minimum detectable received power defines the maximum distance at which the target can be detected:

$$P_{R,\min} = \sigma \frac{P_T G_T A_R}{(4\pi)^2 r_{\max}^4} \quad \Rightarrow \quad r_{\max} = \left[\frac{\sigma P_T G_T A_R}{(4\pi)^2 P_{R,\min}} \right]^{1/4}$$