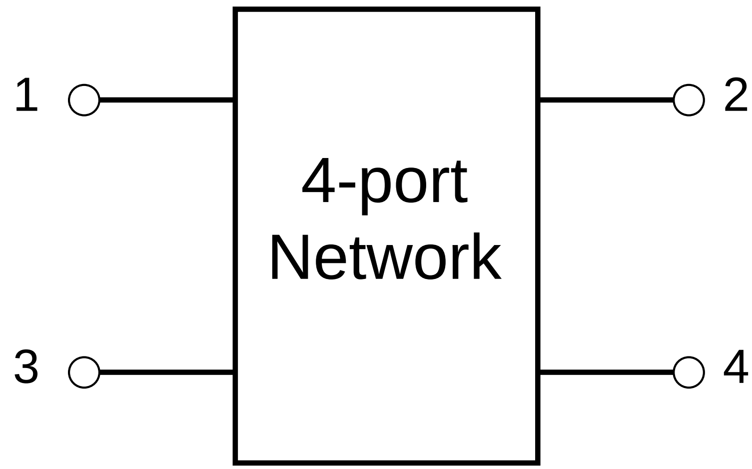
Directional Coupler



A directional coupler is a 4-port network exhibiting:

- All ports matched on the reference load (i.e. $S_{11}=S_{22}=S_{33}=S_{44}=0$)
- Two pair of ports uncoupled (i.e. the corresponding $S_{i,j}$ parameters are zero). Typically the uncoupled ports are (1,3) and (2,4) or (1,4) and (2,3)

Characteristic Parameters

It is here assumed that the uncoupled ports are (1-4) and (2-3). The S parameters the ideal coupler must exhibit are:

$$S_{11}=S_{22}=S_{33}=S_{44}=0,\ S_{14}=S_{41}=S_{23}=S_{32}=0.$$

In this case the ports (1-2), (1-3), (3-4), (2-4) are coupled.
Let define **C** (**Coupling**) as:

$$C=|S_{13}|^2, \quad C_{dB}=-20\log(|S_{13}|)$$

If the network is assumed lossless and reciprocal, the unitary condition of the S matrix determines the following relationships:

$$|S_{11}|^2+|S_{12}|^2+|S_{13}|^2+|S_{14}|^2=1 \ \Rightarrow\ |S_{12}|^2+C=1 \ \Rightarrow\ |S_{12}|=\sqrt{1-C}$$

$$|S_{21}|^2+|S_{22}|^2+|S_{23}|^2+|S_{24}|^2=1 \ \Rightarrow\ |S_{12}|^2+|S_{24}|^2=1 \ \Rightarrow\ |S_{24}|^2=|S_{13}|^2=C$$

$$|S_{31}|^2+|S_{32}|^2+|S_{33}|^2+|S_{34}|^2=1 \ \Rightarrow\ C+|S_{34}|^2=1 \ \Rightarrow\ |S_{34}|=|S_{12}|=\sqrt{1-C}$$

Then, exciting the network at port 1 (2), the output power is divided between ports 3 and 2 (4 and 1) according the factors C and 1-C. No power comes out of port 4 (3)

Further implications of lossless condition (symmetric structure)

$$S_{12}S_{24}^{*} + S_{13}S_{34}^{*} = 0 \quad \Rightarrow \quad |S_{12}S_{24}|e^{j(\Phi_{12}-\Phi_{24})} = |S_{13}S_{34}|e^{j(\pi+\Phi_{13}-\Phi_{34})}$$

$$\Phi_{12} - \Phi_{24} = \pi + \Phi_{13} - \Phi_{34}$$

We assume that the network is symmetric: $S_{12}=S_{34}$, $S_{13}=S_{24}$

Then $\Phi_{12}=\Phi_{34}$ and $\Phi_{13}=\Phi_{24}$. From the previous condition:

$$\boldsymbol{\Phi_{12}=\Phi_{24}\pm\pi/2}$$

This means that the outputs are in quadrature each other.

It can be demonstrated that it is sufficient to impose the matching condition to the four ports of a lossless and reciprocal network to get a directional coupler

Parameters of a real directional coupler

In a real coupler the matching at the ports is not zeros at all the frequencies. It is then specified the minimum **Return Loss** in the operating bandwidth.
The coupling parameter **C**, it is in general referred to the port with the smallest coupling.
Moreover, in a real device a not zero power arrives to the uncoupled port. To characterize this unwanted effect the **isolation parameter (I)** is introduced:

I=Power to the coupled port/Power to the uncoupled port

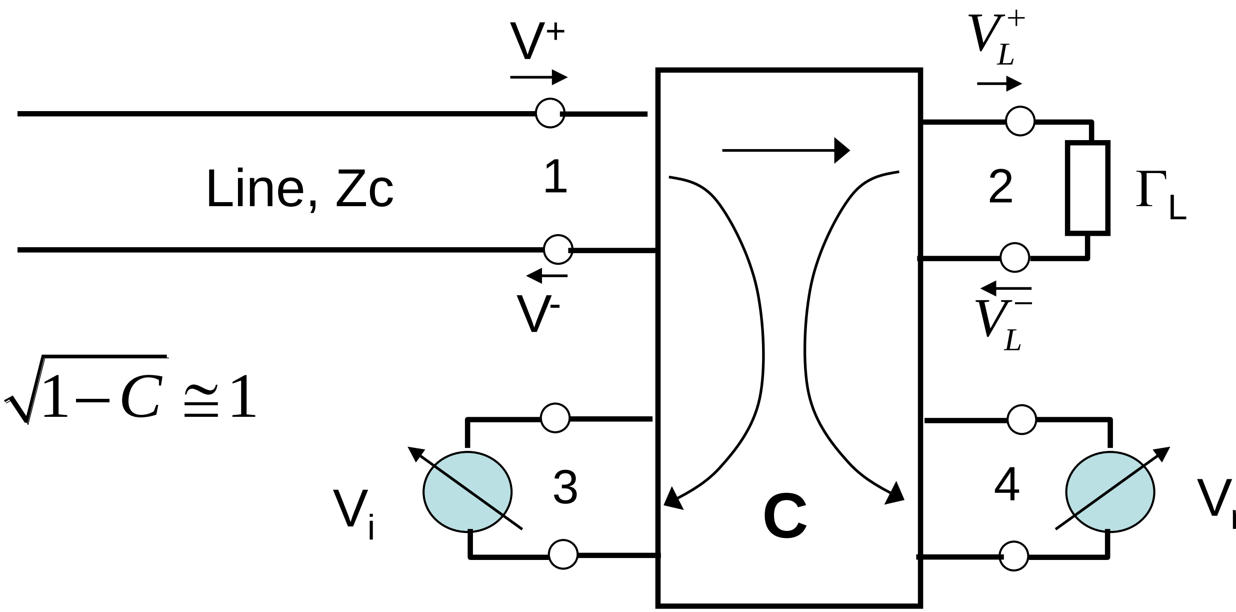
For the coupler considered in the previous slides (assuming port 3 that with the lowest coupling):

$$I = \left|S_{13}\right|^2 \Big/ \left|S_{14}\right|^2$$

Note that for the ideal coupler I is infinite

Use of the directional coupler (1)

Measure of the reflection coefficient



$$C << 1, \qquad S_{12} = \sqrt{1 - C} \cong 1$$

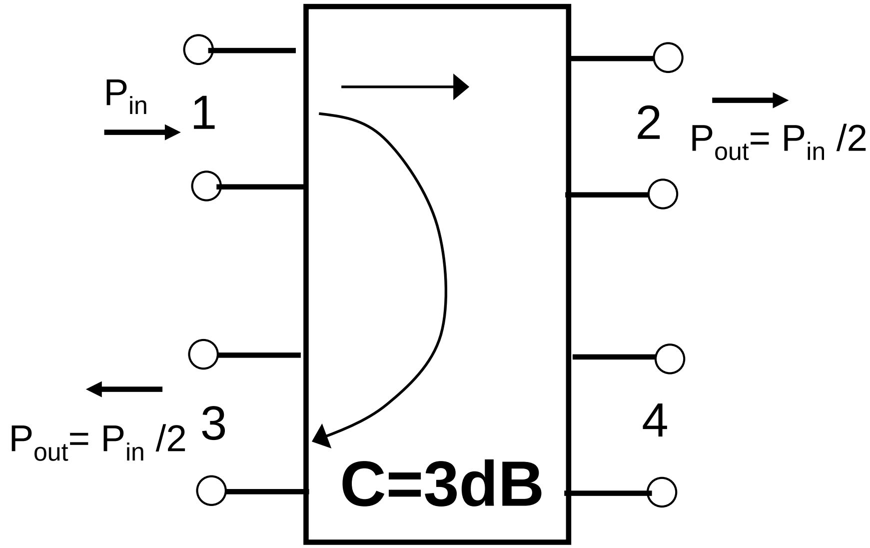
$$V_L^+ \cong V^+$$

$$V_i \doteq V^+ S_{13} = j\sqrt{C} \cdot V_L^+, \qquad V_r \doteq V_L^- S_{24} = j\sqrt{C} \cdot V_L^-$$

$$\frac{V_r}{V_i} = \frac{j\sqrt{C} \cdot V_L^-}{j\sqrt{C} \cdot V_L^+} = \frac{V_L^-}{V_L^+} = \Gamma_L$$

Use of the directional coupler (2)

Power divider (C=3 dB)



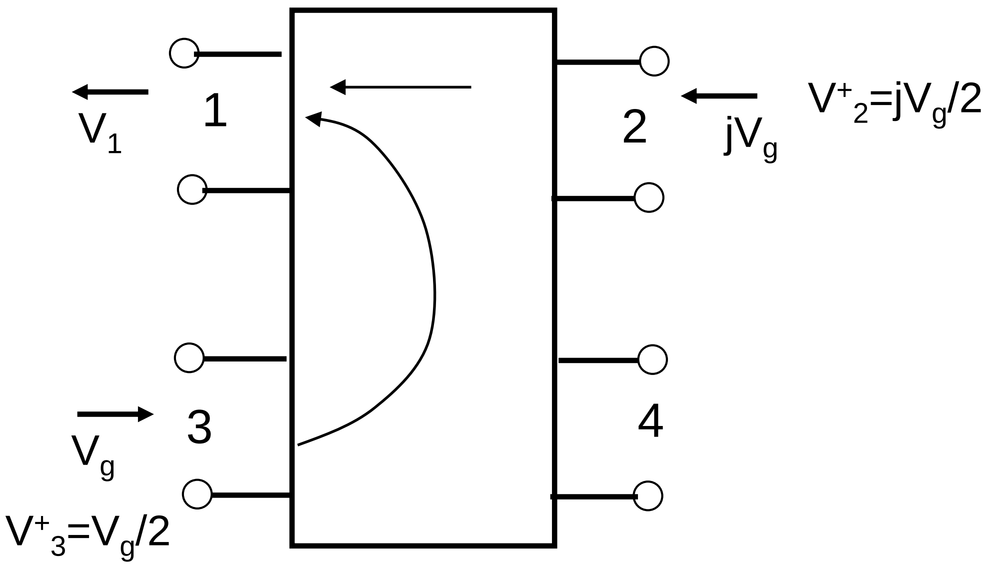
Ports closed on the reference load

$$V_2 = V_2^- = V_1^+ S_{12} = \sqrt{\frac{1}{2}} \cdot V_1 \Rightarrow P_2 = \frac{1}{2} P_1,$$

$$V_3 = V_3^- = V_1^+ S_{13} = \sqrt{\frac{1}{2}} \cdot V_1 \Rightarrow P_3 = \frac{1}{2} P_1$$

Use of the directional coupler (3)

Power combiner (C=3 dB)



$$P_2 = P_3 = \frac{1}{8}\frac{\left|V_g\right|^2}{R_0}$$

$$S_{12} = S_{34} = \frac{1}{\sqrt{2}}$$

$$S_{24} = S_{13} = \frac{j}{\sqrt{2}}$$

$$V_1 = V_1^- = V_2^+ S_{12} + V_3^+ S_{13} = \cdot\left(j\frac{V_g}{2\sqrt{2}} + j\frac{V_g}{2\sqrt{2}} \right) = j\frac{V_g}{\sqrt{2}}, \qquad V_4 = 0$$

$$P_1 = \frac{1}{2}\frac{\left|V_1\right|^2}{R_0} = \frac{1}{4}\frac{\left|V_g\right|^2}{R_0} = P_2 + P_3$$

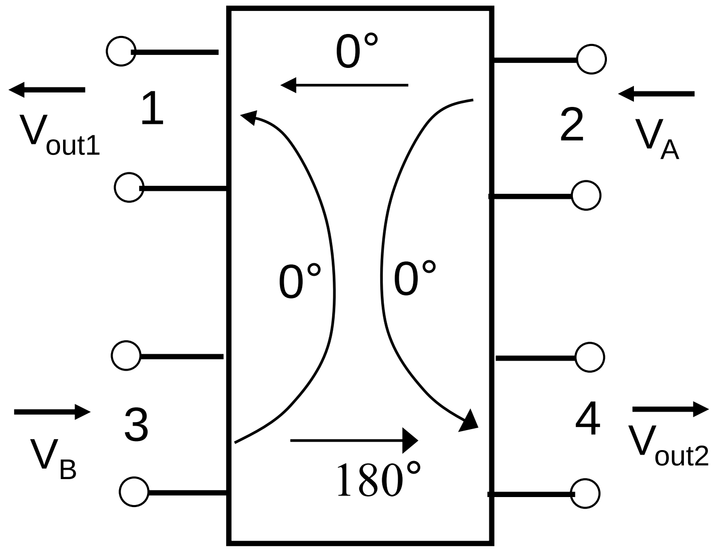
Only if P_2 e P_3 are equal

Use of the directional coupler (4)

Sum and difference of voltages (C=3 dB)

$\Phi_{12}=\Phi_{13}=\Phi_{24}=0$

$\rightarrow \Phi_{34}=\pi$



Ports closed on the reference load

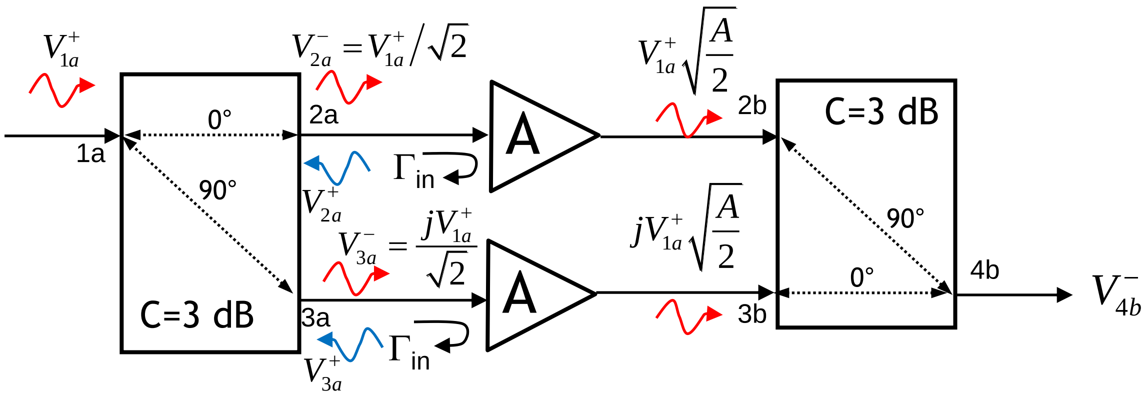
C=3dB

$$V_{out1} = V_1 = V_A S_{12} + V_B S_{13} = \sqrt{\frac{1}{2}} \cdot \left(V_A + V_B\right),$$

$$V_{out2} = V_4 = V_A S_{24} + V_B S_{34} = \sqrt{\frac{1}{2}} \cdot \left(V_A - V_B\right)$$

Use of the directional coupler (5)

Balanced Amplifier



Gain:

$$V_{2b}^{+}=\sqrt{A}\cdot V_{1a}^{+}/\sqrt{2},\quad V_{3b}^{+}=j\sqrt{A}\cdot V_{1a}^{+}/\sqrt{2}$$

$$V_{4b}^{-}=j\frac{V_{2b}}{\sqrt{2}}+\frac{V_{3b}}{\sqrt{2}}=j\sqrt{A}\cdot V_{1a}^{+}\quad\Rightarrow\quad\frac{P_{out}}{P_{in}}=A$$

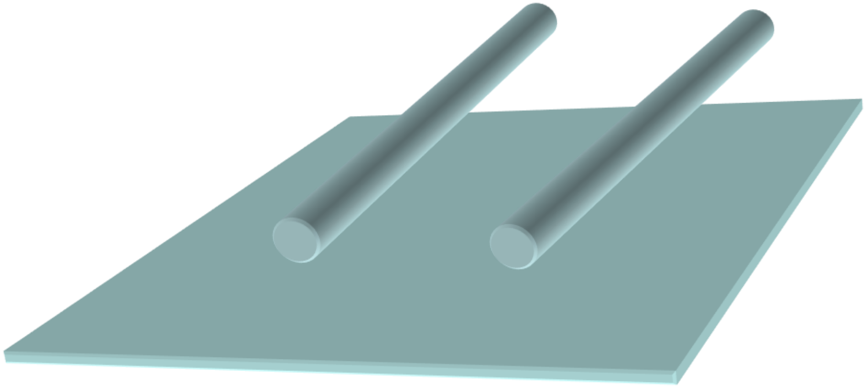
Reflection:

$$V_{2a}^{-}=V_{1a}^{+}/\sqrt{2},\quad V_{2a}^{+}=\Gamma_{in}V_{1a}^{+}/\sqrt{2},\quad V_{3a}^{-}=jV_{1a}^{+}/\sqrt{2},$$

$$V_{3a}^{+}=j\Gamma_{in}V_{1a}^{+}/\sqrt{2},\quad V_{1a}^{-}=\frac{1}{\sqrt{2}}\left(jV_{3a}^{+}+V_{2a}^{+}\right)=-\Gamma_{in}V_{1a}^{+}/2+\Gamma_{in}V_{1a}^{+}/2$$

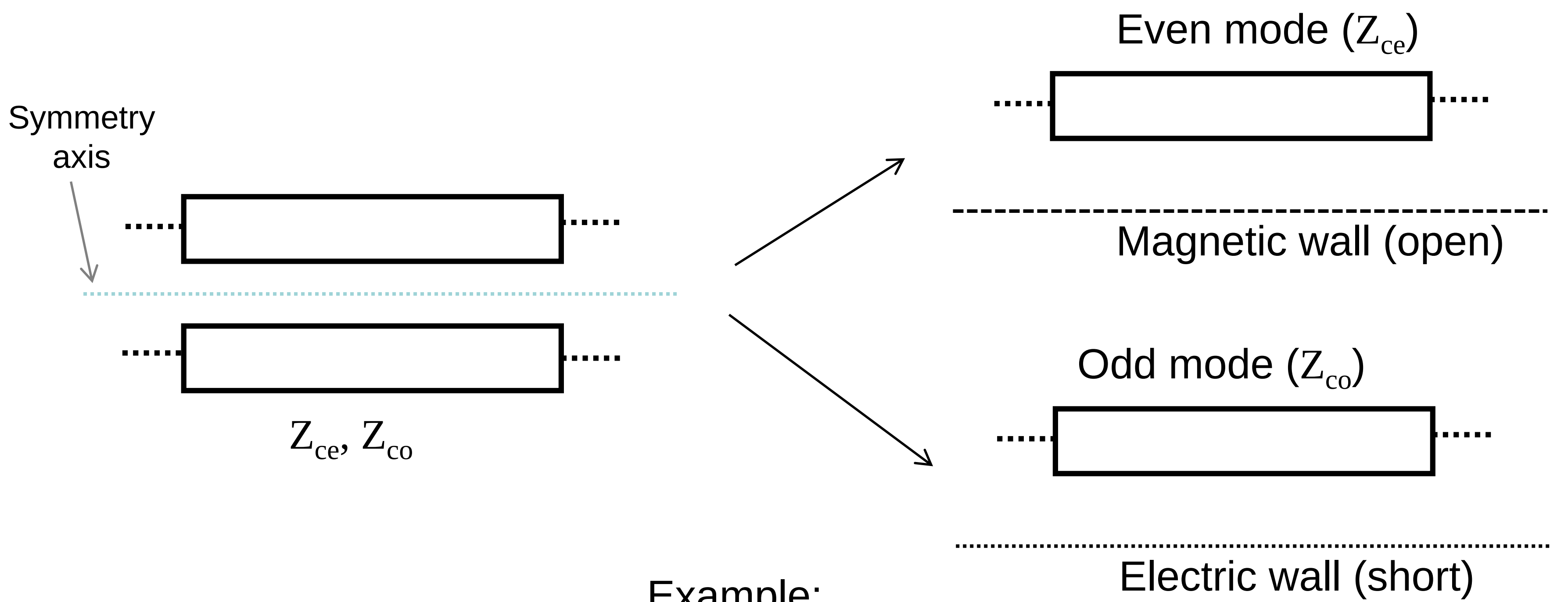
$$\Gamma_{in}=\frac{V_{1a}^{-}}{V_{1a}^{+}}=0$$

Coupled TEM lines

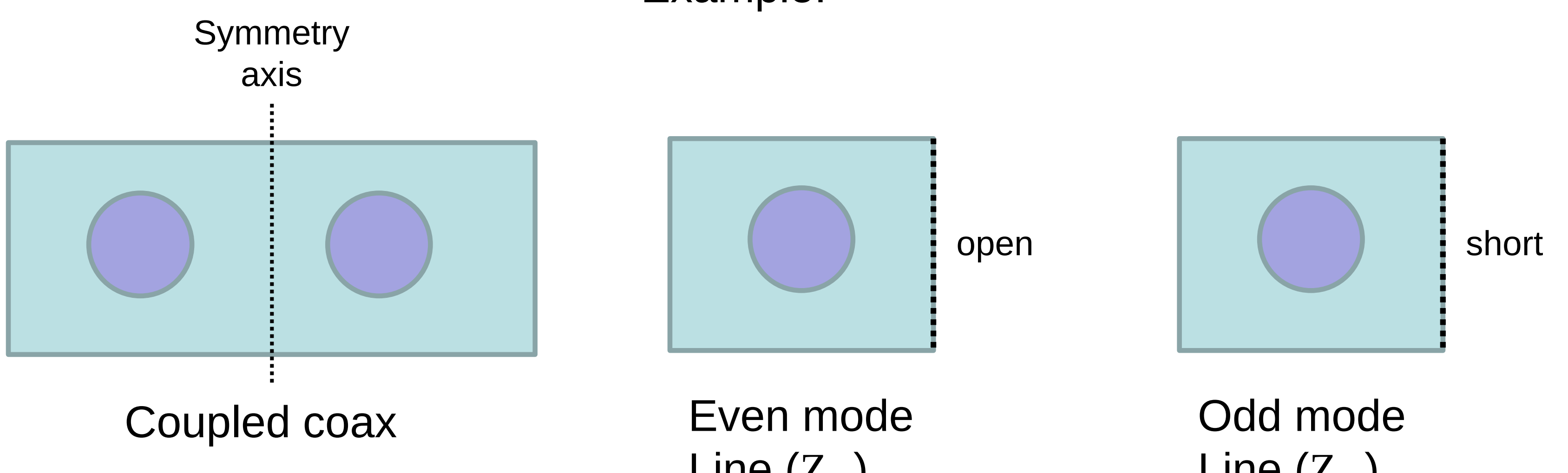


When two transmission lines are placed close together, the propagation in each line is influenced on the other. We talk in this case of coupled-line propagation.
There are two possible modes of propagation which are called even and odd (symmetrical structure). Each mode is characterized by its characteristic impedance whose meaning is illustrated in the next slide

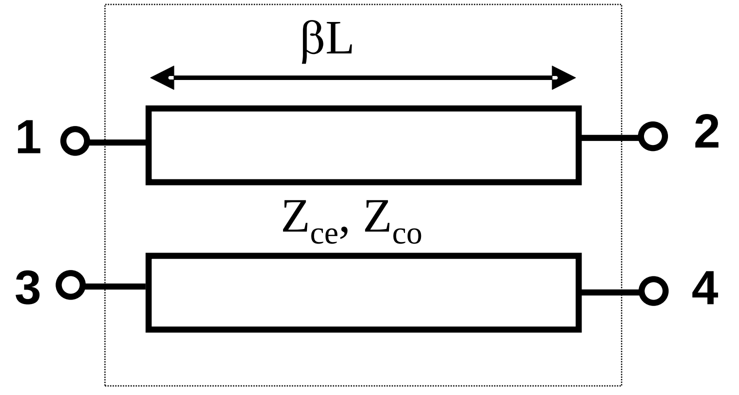
Modes in TEM coupled lines



Example:



Circuit model of two coupled lines with finite length



Goal: compute the 4-port Z matrix (or Y, or S).

Hypothesis: equal lines (two symmetry axis)

Evaluation method: matrix eigenvalues

Eigenvalues and Eigenvectors of a matrix

The eigenvalues S_λ of a square matrix **S** are the solutions of the equation:

$$\det\left[\mathbf{S} - S_\lambda \mathbf{U}\right] = 0$$

The eigenvectors x_λ associated to **S** represent the solution of the homogenous system of equations:

$$\mathbf{S} \cdot x_\lambda = S_\lambda x_\lambda$$

A matrix of order *n* has *n* eigenvalues and n eigenvectors (each eigenvectors has n elements). The eigenvectors are defined up to a constant.

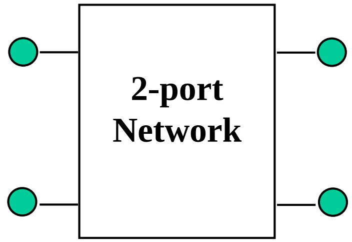
<u>Properties</u>

If a N-port network is excited with a vectors of currents representing an eigenvector of Z, you see the same impedance at all ports, and its value is just the eigenvalue. The same holds for all the other matrices (Y, S, …)

If there are symmetry axis in the network, the eigenvalues can be derived by on suitably defined circuits (*eigencircuits*). The eigenvectors are obtained by induction (the excitations must determine either an open or a short along the symmetry axis).

Once the eigenvalues and eigenvectors are known, simple equations define the elements of the corresponding matrices.

Example: 2-port network

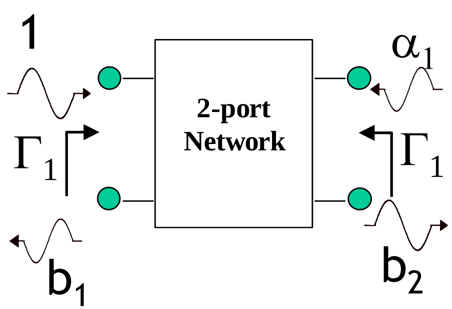


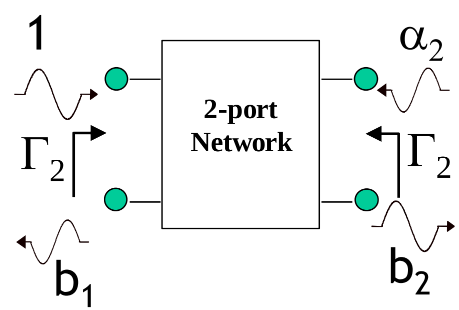
$$\begin{vmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{vmatrix}$$

Let assume that the two eigenvectors are known (we have assigned arbitrarily the largest element of each x_i equal to 1

Eigenvector 1: $x_1 = \begin{matrix} +1 \\ \alpha_1 \end{matrix}$

Eigenvector 2: $x_2 = \begin{matrix} +1 \\ \alpha_2 \end{matrix}$





$$\frac{b_1}{1} = \frac{b_2}{\alpha_1} = \Gamma_1 \quad \text{(Eigenvalue 1)}$$

$$\frac{b_1}{1} = \frac{b_2}{\alpha_2} = \Gamma_2 \quad \text{(Eigenvalue 2)}$$

2-port network (cont.)

Eigenvector 1:

$$b_1 = s_{11}\cdot 1 + s_{12}\cdot \alpha_1$$

$$b_2 = s_{11}\cdot 1 + s_{22}\cdot \alpha_1$$

$$\frac{b_1}{1} = \frac{b_2}{\alpha_1} = \Gamma_1$$

Eigenvector 2:

$$b_1 = s_{11}\cdot 1 + s_{12}\cdot \alpha_2$$

$$b_2 = s_{11}\cdot 1 + s_{22}\cdot \alpha_2$$

$$\frac{b_1}{1} = \frac{b_2}{\alpha_2} = \Gamma_2$$

$$s_{11} = \frac{\alpha_2\Gamma_1 - \alpha_1\Gamma_2}{\alpha_2 - \alpha_1}, \qquad s_{22} = \frac{\alpha_1\Gamma_1 - \alpha_2\Gamma_2}{\alpha_1 - \alpha_2}, \qquad s_{12} = s_{21} = \frac{\Gamma_1 - \Gamma_2}{1/\alpha_1 - 1/\alpha_2}$$

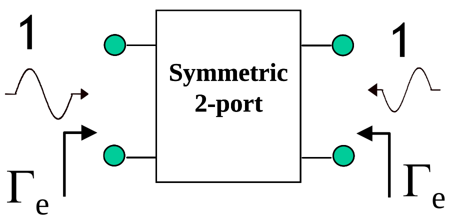
Relationship between eigenvalues of S, Y, Z

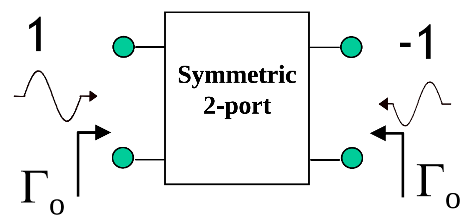
$$S_\lambda = \frac{Z_\lambda - Z_0}{Z_\lambda + Z_0} = \frac{Y_0 - Y_\lambda}{Y_0 + Y_\lambda} \qquad Z_\lambda = Z_0\,\frac{1 + S_\lambda}{1 - S_\lambda} = \frac{1}{Y_\lambda}$$

Evaluation of the Eigenvectors

In general there is no way to derive the eigenvectors of a generic N-port network. Only is the network is symmetric we can deduct the eigenvectors by exploiting the effect produced by the eigenvectors excitation.

Let's consider for instance the case of a symmetric 2-port (S11=S22). Being the network symmetric we can obtain the same reflection coefficient at port 1 and port 2 only when the incident waves at the two ports have the same magnitude. As a consequence the only possible values for α_1 and α_2 are 1 and -1:





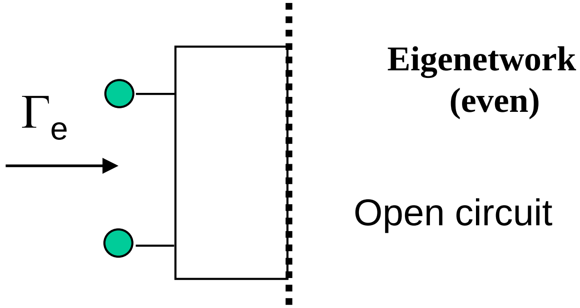
Eigenvector 1 (Even): $x_e = \begin{matrix} +1 \\ +1 \end{matrix}$

Eigenvector 2 (Odd): $x_e = \begin{matrix} +1 \\ -1 \end{matrix}$

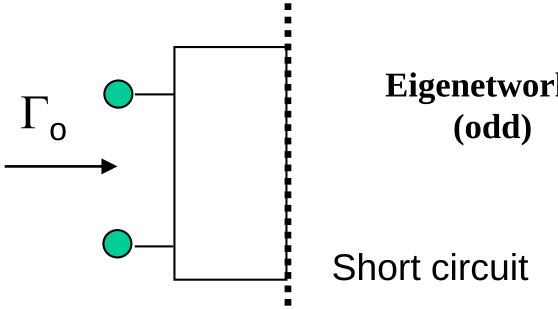
$$s_{11} = s_{22} = \frac{\Gamma_e + \Gamma_o}{2}, \qquad s_{12} = s_{21} = \frac{\Gamma_e - \Gamma_o}{2}, \qquad \Gamma_e = s_{11} + s_{12}, \qquad \Gamma_o = s_{11} - s_{12}$$

2-port symmetric network (cont.)

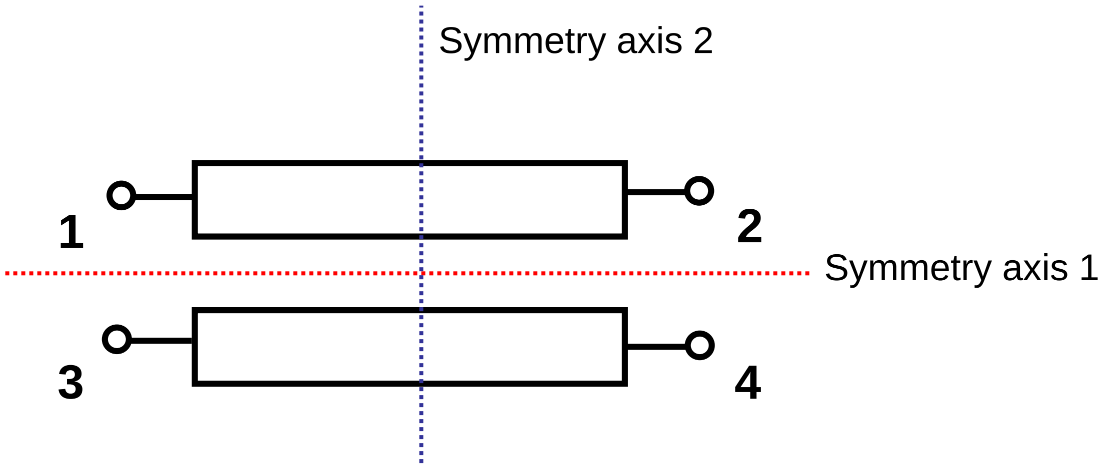
If the first eigenvector is considered , two equal excitations (amplitude and phase) at the two ports determines, for the network symmetry, an open circuit along the symmetry axis. The first eigenvalue is the imput reflection coefficient (or impedance or admittance) of the *even eigenetwork* (one port):



The second eigenvector consists in two equal but opposite excitations: a short circuit is then determined along the symmetry axis. The second eigenvalue can be computed from the *odd eigenetwork* (one port):



Eigenvalues evaluation of 2-coupled lines



Exciting the network with an eigenvector, an electric or a magnetic wall is obtained along the two symmetry axis. With reference to Z matrix, the exciting currents for each eigenvector result:

$$I_{\lambda 1} = \{+1, +1, +1, +1\} \Rightarrow$$ Axis 1: Magnetic, Axis 2: Magnetic

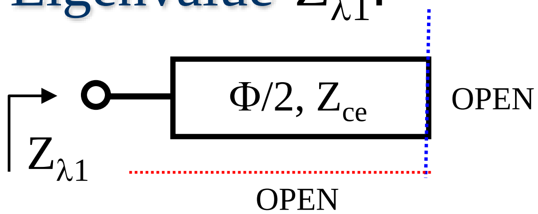
$$I_{\lambda 2} = \{+1, -1, +1, -1\} \Rightarrow$$ Axis 1: Magnetic, Axis 2: Electric

$$I_{\lambda 3} = \{+1, +1, -1, -1\} \Rightarrow$$ Axis 1:Electric, Axis 2: Magnetic

$$I_{\lambda 4} = \{+1, -1, -1, +1\} \Rightarrow$$ Axis 1: Electric, Axis 2: Electric

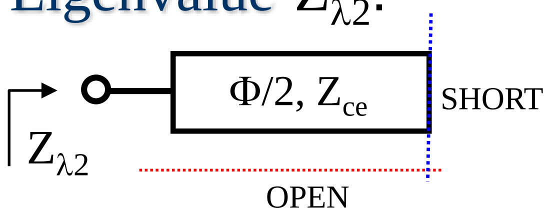
Evaluation of the eigenvalues using the eigenetwork

Eigenvalue $Z_{\lambda 1}$:



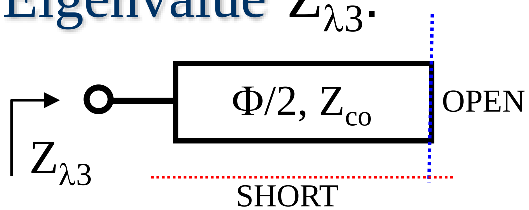
$$Z_{\lambda 1} = -jZ_{ce}\cot\left(\Phi/2\right)$$

Eigenvalue $Z_{\lambda 2}$:



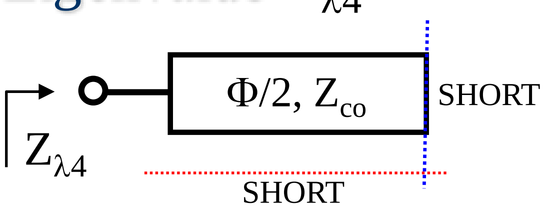
$$Z_{\lambda 2} = jZ_{ce}\tan\left(\Phi/2\right)$$

Eigenvalue $Z_{\lambda 3}$:



$$Z_{\lambda 3} = -jZ_{co}\cot\left(\Phi/2\right)$$

Eigenvalue $Z_{\lambda 4}$:



$$Z_{\lambda 4} = jZ_{co}\tan\left(\Phi/2\right)$$

Evaluation of Z Matrix

From the definition of Z, imposing each eigenvector as excitation, the four independent elements of Z are obtained:

$$Z_{\lambda 1} = \frac{V_1}{I_0} = Z_{11} + Z_{12} + Z_{13} + Z_{14}$$

$$Z_{\lambda 2} = \frac{V_1}{I_0} = Z_{11} - Z_{12} + Z_{13} - Z_{14}$$

$$Z_{\lambda 3} = \frac{V_1}{I_0} = Z_{11} + Z_{12} - Z_{13} - Z_{14}$$

$$Z_{\lambda 4} = \frac{V_1}{I_0} = Z_{11} - Z_{12} - Z_{13} + Z_{14}$$

$$\Rightarrow$$

$$Z_{11} = \frac{1}{4}\left(Z_{\lambda 1} + Z_{\lambda 2} + Z_{\lambda 3} + Z_{\lambda 4}\right)$$

$$Z_{12} = \frac{1}{4}\left(Z_{\lambda 1} - Z_{\lambda 2} + Z_{\lambda 3} - Z_{\lambda 4}\right)$$

$$Z_{13} = \frac{1}{4}\left(Z_{\lambda 1} + Z_{\lambda 2} - Z_{\lambda 3} - Z_{\lambda 4}\right)$$

$$Z_{14} = \frac{1}{4}\left(Z_{\lambda 1} - Z_{\lambda 2} - Z_{\lambda 3} + Z_{\lambda 4}\right)$$

Using $Z_{\lambda i}$, the eigenvalues of the other matrices (Y, S) can be obtained. The above formulas can be then used for computing the elements of also these matrices

Expression of Z matrix elements

$$Z_{11} = \frac{j}{4}\left(-Z_{ce} \cot\left(\frac{\Phi}{2}\right) + Z_{ce} \tan\left(\frac{\Phi}{2}\right) - Z_{co} \cot\left(\frac{\Phi}{2}\right) + Z_{co} \tan\left(\frac{\Phi}{2}\right) \right)$$

$$Z_{12} = \frac{j}{4}\left(-Z_{ce} \cot\left(\frac{\Phi}{2}\right) - Z_{ce} \tan\left(\frac{\Phi}{2}\right) - Z_{co} \cot\left(\frac{\Phi}{2}\right) - Z_{co} \tan\left(\frac{\Phi}{2}\right) \right)$$

$$Z_{13} = \frac{j}{4}\left(-Z_{ce} \cot\left(\frac{\Phi}{2}\right) + Z_{ce} \tan\left(\frac{\Phi}{2}\right) + Z_{co} \cot\left(\frac{\Phi}{2}\right) - Z_{co} \tan\left(\frac{\Phi}{2}\right) \right)$$

$$Z_{14} = \frac{j}{4}\left(-Z_{ce} \cot\left(\frac{\Phi}{2}\right) - Z_{ce} \tan\left(\frac{\Phi}{2}\right) + Z_{co} \cot\left(\frac{\Phi}{2}\right) + Z_{co} \tan\left(\frac{\Phi}{2}\right) \right)$$

Compact expressions of Z and Y elements

$$Z_{11} = -j\frac{\left(Z_{ce} + Z_{co}\right)}{2}\cot\left(\Phi\right)$$

$$Z_{12} = -j\frac{\left(Z_{ce} + Z_{co}\right)}{2}\frac{1}{\sin\left(\Phi\right)}$$

$$Z_{13} = -j\frac{\left(Z_{ce} - Z_{co}\right)}{2}\cot\left(\Phi\right)$$

$$Z_{14} = -j\frac{\left(Z_{ce} - Z_{co}\right)}{2}\frac{1}{\sin\left(\Phi\right)}$$

$$Y_{11} = -j\frac{\left(Y_{ce} + Y_{co}\right)}{2}\cot\left(\Phi\right)$$

$$Y_{12} = j\frac{\left(Y_{ce} + Y_{co}\right)}{2}\frac{1}{\sin\left(\Phi\right)}$$

$$Y_{13} = -j\frac{\left(Y_{ce} - Y_{co}\right)}{2}\cot\left(\Phi\right)$$

$$Y_{14} = j\frac{\left(Y_{ce} - Y_{co}\right)}{2}\frac{1}{\sin\left(\Phi\right)}$$

Special cases

$$Z_\lambda = j\left\{-Z_{ce}\cot\left(\Phi/2\right),\ \ Z_{ce}\tan\left(\Phi/2\right),\ \ -Z_{co}\cot\left(\Phi/2\right),\ \ Z_{co}\tan\left(\Phi/2\right)\right\}$$

$\Phi=\beta L=180°$

The eigenvalues of Z are [0, ∞, 0, ∞], so those of S result:
$S_{\lambda i}$= [-1, 1, -1, 1]. The scattering matrix elements are then:

$$S_{11}=0,\ \ S_{12}=-1,\ \ S_{13}=0,\ \ S_{14}=0$$

Note that line 2 is completely uncoupled from line 1!

Perfect matching at the four ports

There is a value of the load Z_0 for which the ports are all matched (S_{11}=S_{22}=S_{33}=S_{44}=0) independently on βL:

$$Z_0=\sqrt{Z_{ce}\cdot Z_{co}}$$

Note that the matching at the ports does not imply the absence of reflected waves along the two lines. It means that the reflected waves are canceled only at the ports

Derivation of the matching condition

Eigenvalues of **S**

$$Z_\lambda = j\left\{-Z_{ce}\cot(\Phi/2),\ \ Z_{ce}\tan(\Phi/2),\ \ -Z_{co}\cot(\Phi/2),\ \ Z_{co}\tan(\Phi/2)\right\} = jX_{\lambda i}$$

$$S_{\lambda i} = \frac{jX_{\lambda i} - Z_0}{jX_{\lambda i} + Z_0}$$

Parameter S_{11}:

$$S_{11} = \frac{1}{4}\left(S_{\lambda 1} + S_{\lambda 2} + S_{\lambda 3} + S_{\lambda 4}\right) = 0$$

There are only two cases where the above condition can be satisfied independently on $\Phi=\beta L$, i.e.:

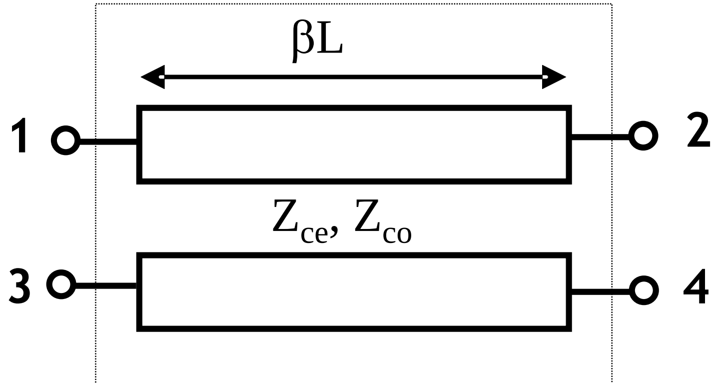
$$\begin{matrix} (S_{\lambda 1} + S_{\lambda 2}) = 0 \\ (S_{\lambda 3} + S_{\lambda 4}) = 0 \end{matrix} \quad \Rightarrow \quad \begin{matrix} X_{\lambda 1}\cdot X_{\lambda 2} = -Z_0^2 \\ X_{\lambda 3}\cdot X_{\lambda 4} = -Z_0^2 \end{matrix} \quad \Rightarrow \quad \begin{matrix} Z_{ce}^2 = Z_0^2 \\ Z_{co}^2 = Z_0^2 \end{matrix}$$

NOT Admissible

$$\begin{matrix} (S_{\lambda 1} + S_{\lambda 4}) = 0 \\ (S_{\lambda 2} + S_{\lambda 3}) = 0 \end{matrix} \quad \Rightarrow \quad \begin{matrix} X_{\lambda 1}\cdot X_{\lambda 4} = -Z_0^2 \\ X_{\lambda 3}\cdot X_{\lambda 2} = -Z_0^2 \end{matrix} \quad \Rightarrow \quad \begin{matrix} \sqrt{Z_{ce}\cdot Z_{co}} = Z_0 \\ \sqrt{Z_{ce}\cdot Z_{co}} = Z_0 \end{matrix}$$

Admissible

Coupled TEM lines as directional coupler



Using the admissible condition we obtain the match at all port imposing:

$$\left(S_{\lambda 1}+S_{\lambda 4}\right)=0$$
$$\left(S_{\lambda 2}+S_{\lambda 3}\right)=0$$

$$\Rightarrow \quad Z_0=\sqrt{Z_{ce}\cdot Z_{co}}$$

This condition also implies that port 4 is uncoupled : $S_{14}=\frac{1}{4}\left(S_{\lambda 1}-S_{\lambda 2}-S_{\lambda 3}+S_{\lambda 4}\right)=0$

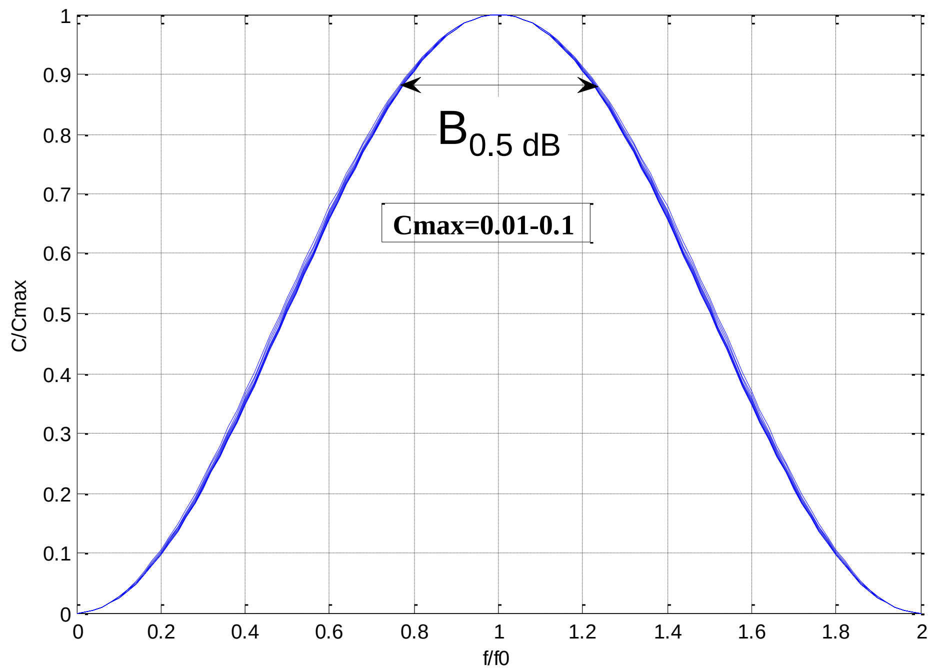
The maximum value $|S_{13}|^2$ defines the coupling: $C=(|S_{13}|^2)_{max}$
It is obtained for $\beta L=\pi/2$:

$$\left|S_{13}\right|^2_{\max}=\left|\frac{1}{4}\left(S_{\lambda 1}+S_{\lambda 2}-S_{\lambda 3}-S_{\lambda 4}\right)\right|^2_{\max}=\left|\frac{1}{2}\left(S_{\lambda 1}+S_{\lambda 2}\right)\right|^2_{\max}=\left|\frac{Z_{ce}-Z_{co}}{Z_{ce}+Z_{co}}\right|^2$$

Variation of frequency

Matching and Isolation are frequency independent and equal to zero and infinity respectively (ideal lossless TEM line).
The coupling varies with the frequency according to the following expression:

$$C(\Phi) = \frac{C_{\max}}{1+(1-C_{\max})\cdot\cot^2(\Phi)}, \qquad C_{\max} = \frac{Z_{ce}-Z_{co}}{Z_{ce}+Z_{co}}$$



For C_{max}<0.1 the variation of C is practically independent on C_{max}.
Note that the band for C/C_{max}< 0.5 dB is about 0.44 f_0

f_0 is the frequency for which $\beta L=\pi/2$

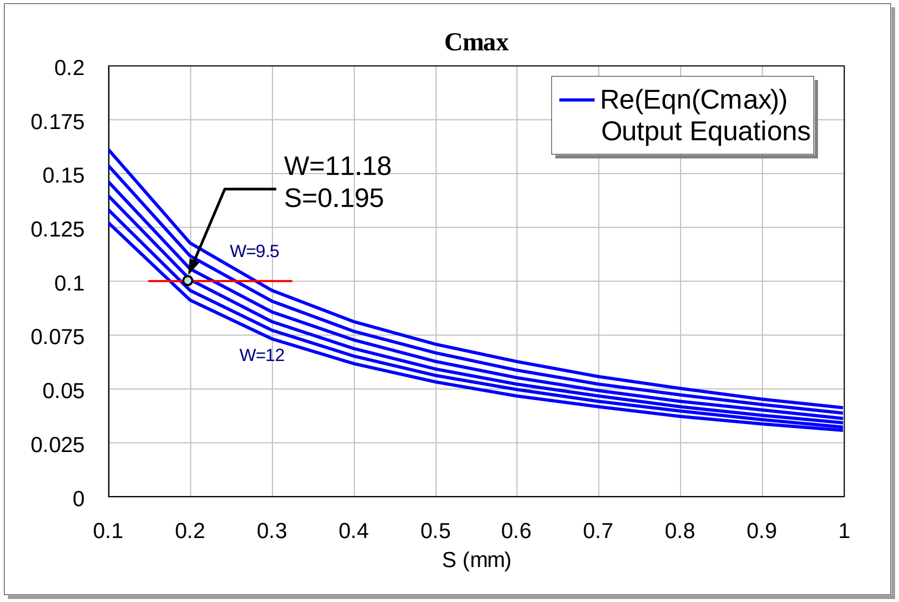
Practical Restrictions

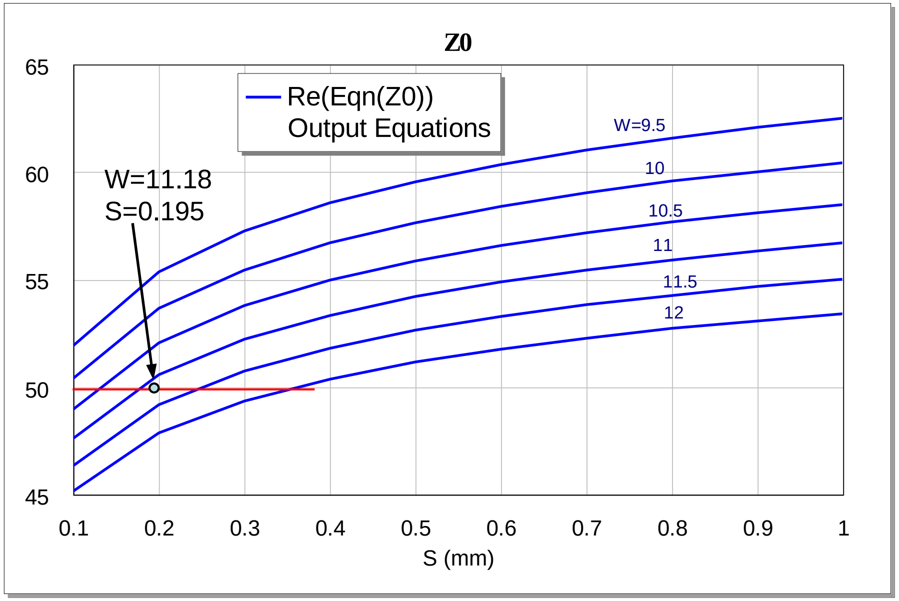
They concern mainly the maximum value of C_{max}. In fact, increasing C_{max}, the lines become closer and closer until the practical implementation is no more possible with sufficient accuracy. Typically the maximum value of C_{max} must be lower than 0.1 (C_{dB}=10)

<u>Example</u>: Design a stripline coupler with C=0.1 with Z0=50 Ω using the following figures reporting the values of

$$C_{\max} = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}, \quad Z_0 = \sqrt{Z_{ce} \cdot Z_{co}}$$

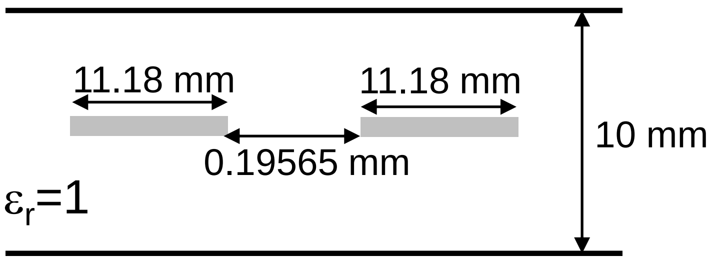
as a function of S and W (lines separation and width). Frequency: 1 GHz





Solution:

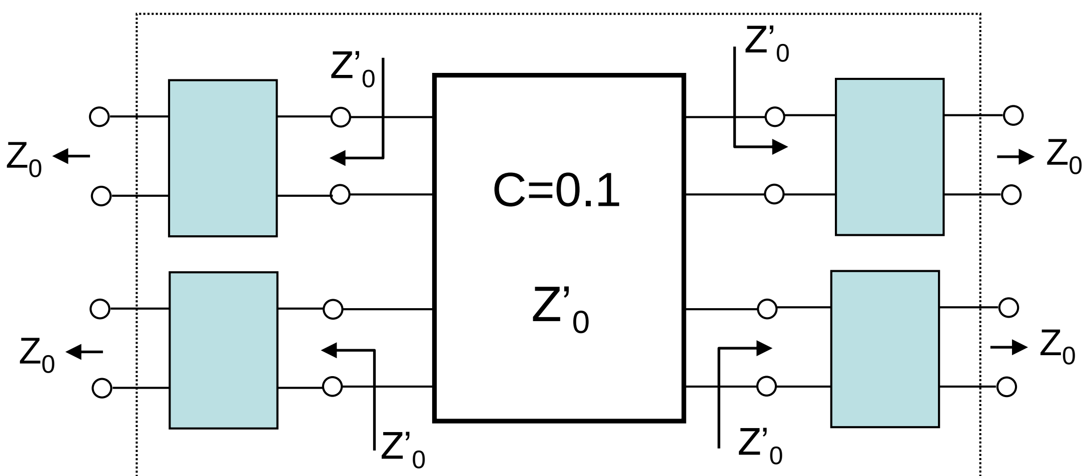
We draw the line C=0.1 on the first graph and the line Z_0=50 on the second graph. A point on each of these lines has to be then found, which is characterized by the same pair of values (S, W) .



$$\beta L = \frac{\omega_0 L}{c} = \frac{\pi}{2}$$

$$L = 75\ \text{mm}$$

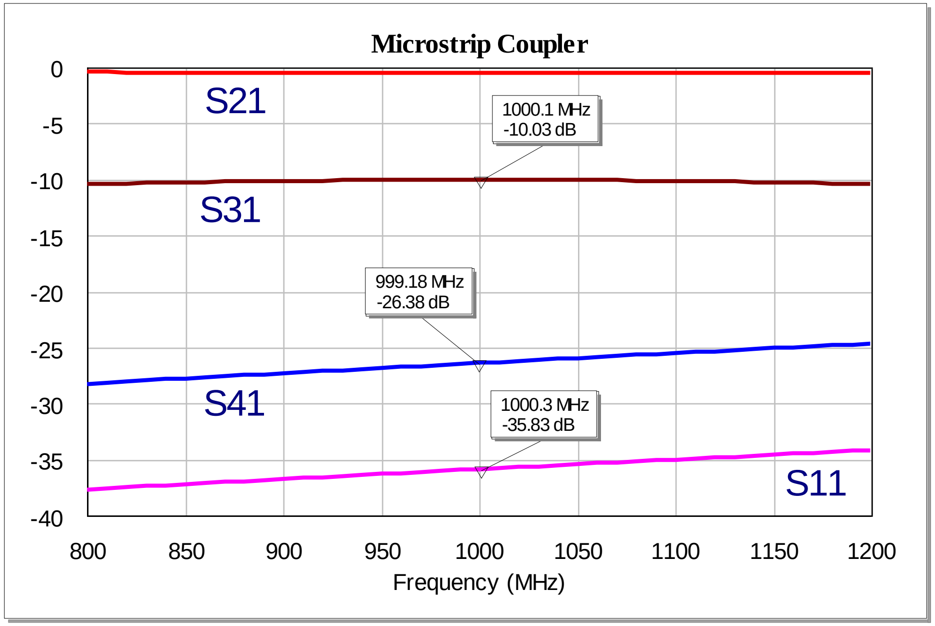
<u>Note</u>: If a coupler dimensioned for the requested C but with a different value of Z_0 is available, impedance transforming networks can be used in place of redesigning a new coupler.

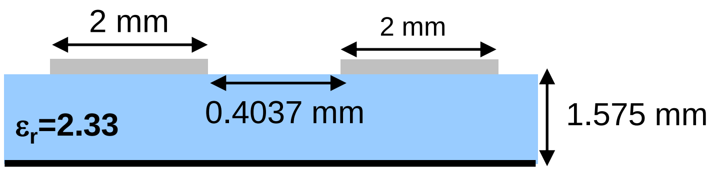


Couplers with quasi-TEM Lines

If quasi-TEM coupled lines are considered (Microstrip), the phase velocity of the even and odd modes are not exactly coincident. Strictly speaking, that would not allow to apply the model here assumed for the characterization of the directional coupler.

In the practice, until the difference between the two velocities is not too large, the same phase velocity can be assumed for both modes (equal to the average of the actual values), assuming the lines as TEM. There are however some differences comparing the performances with the ideal TEM coupler (a perfect matching is no more possible)





$$m = 0.1 \;\Rightarrow\; S = 0.4037 \text{ mm}$$

$$Z_0 = \sqrt{Z_{cp} \cdot Z_{cd}} = 74.12$$

$$\varepsilon_{eff,p} = 1.97, \quad \varepsilon_{eff,d} = 1.71$$

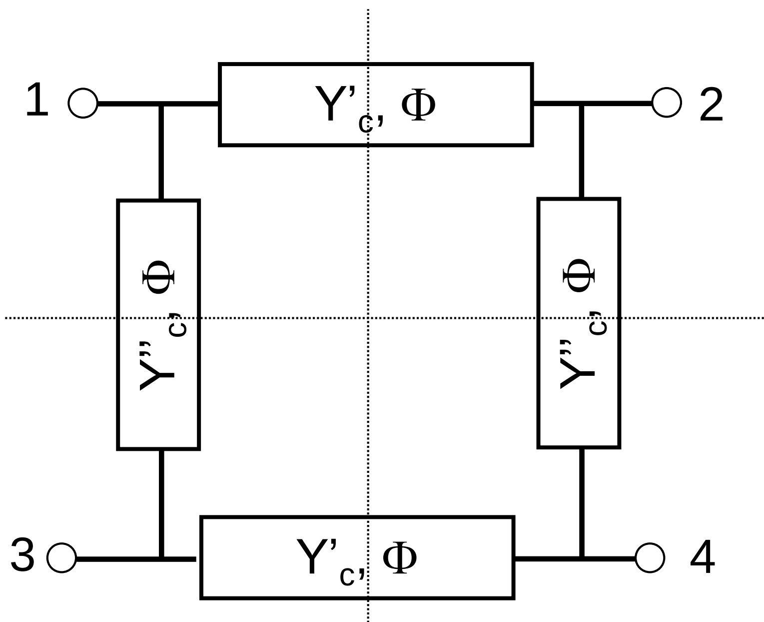
$$\varepsilon_{eff,medio} = 1.84, \quad \beta L = \frac{\omega_0 L}{c}\sqrt{\varepsilon_{eff,medio}} = \frac{\pi}{2}$$

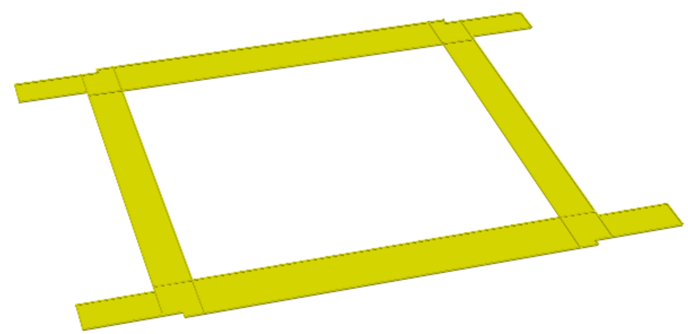
$$L = 55.24 \text{ mm}$$

Coupler with lumped couplings

To realize couplers with a large coupling (C<10 dB) structure with lumped couplings are used. In planar technology two kinds of such couplers are employed, the **branch-line** and the **rat-race**.

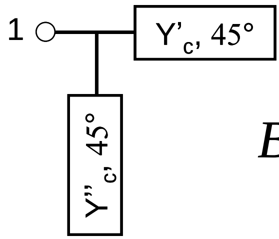
Branch-line





$$\Phi = \frac{\pi}{2} \quad \left(L = \frac{\lambda_0}{4} \right)$$

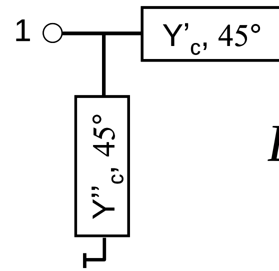
Evaluation of the eigenvalues



$$B_s = Y_c' + Y_c''$$

$$Y_{\lambda 1} = j\tan\left(45°\right)\left(Y_c' + Y_c''\right) = j\left(Y_c' + Y_c''\right)$$

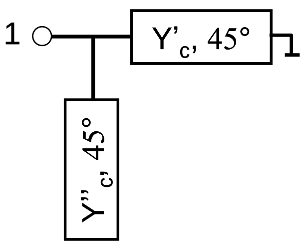
$$S_{\lambda 1} = \left(Y_0 - jB_s\right)/\left(Y_0 + jB_s\right)$$



$$B_d = Y_c' - Y_c''$$

$$Y_{\lambda 3} = jY_c'\tan\left(45°\right) - jY_c''\cot\left(45°\right) = j\left(Y_c' - Y_c''\right)$$

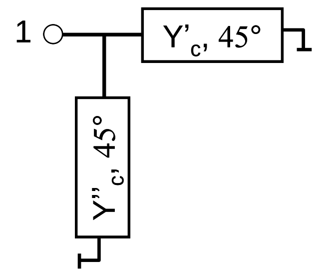
$$S_{\lambda 3} = \left(Y_0 - jB_d\right)/\left(Y_0 + jB_d\right)$$



$$Y_{\lambda 2} = -jY_c'\tan\left(45°\right) + jY_c''\tan\left(45°\right)$$

$$= -j\left(Y_c' - Y_c''\right)$$

$$S_{\lambda 2} = \left(Y_0 + jB_d\right)/\left(Y_0 - jB_d\right)$$



$$Y_{\lambda 4} = -jY_c'\tan\left(45°\right) - jY_c''\cot\left(45°\right)$$

$$= -j\left(Y_c' + Y_c''\right)$$

$$S_{\lambda 4} = \left(Y_0 + jB_s\right)/\left(Y_0 - jB_s\right)$$

Coupling conditions

All ports matched, ports 1-3/2-4 uncoupled: $S_{11}=0$ and $S_{13}=S_{24}=0$

$$S_{11}=\frac{1}{4}\left(S_{\lambda 1}+S_{\lambda 2}+S_{\lambda 3}+S_{\lambda 4}\right)=0$$

$$S_{13}=\frac{1}{4}\left(S_{\lambda 1}+S_{\lambda 2}-S_{\lambda 3}-S_{\lambda 4}\right)=0$$

$$\Downarrow$$

$$S_{\lambda 1}+S_{\lambda 2}=0,\quad S_{\lambda 3}+S_{\lambda 4}=0$$

This condition is actually feasible, giving in fact:

$$\frac{B_s B_d}{Y_0^2}=1 \quad \Rightarrow \quad \boxed{Y_c'^2-Y_c''^2=Y_0^2}$$

For S_{12} e S_{14} we have:

$$S_{12} = \frac{1}{4}\left(S_{\lambda 1} - S_{\lambda 2} + S_{\lambda 3} - S_{\lambda 4}\right) = \frac{1}{2}\left(S_{\lambda 1} - S_{\lambda 4}\right) = \frac{-2jb_s}{1+b_s^2}$$

$$S_{14} = \frac{1}{4}\left(S_{\lambda 1} - S_{\lambda 2} - S_{\lambda 3} + S_{\lambda 4}\right) = \frac{1}{2}\left(S_{\lambda 1} + S_{\lambda 4}\right) = \frac{1-b_s^2}{1+b_s^2}, \qquad b_s = \frac{B_s}{Y_0}$$

Moreover, the unitary condition of **S** implies the following relation between the phases of S_{12} and S_{14}:

$$\phi_{12} - \phi_{14} = \pm\frac{\pi}{2}$$

Then, being $\phi_{12}=-\pi/2 \rightarrow \phi_{14}=\pi$. The parameter b_s must be then >1 for having S_{14} negative. By imposing now the requested coupling ($|S_{14}|^2=C$), we can obtain the requested value of b_s:

$$-S_{14} = \frac{b_s^2 - 1}{1+b_s^2} = \sqrt{C}, \quad \Rightarrow \quad b_s = \sqrt{\frac{1+\sqrt{C}}{1-\sqrt{C}}} = \frac{Y_c' + Y_c''}{Y_0}$$

Taking into account the first found condition ($Y_c'^2 - Y_c''^2 = Y_0^2$) we finally obtain the expressions of Y'_c e Y''_c:

$$\boxed{Y_c' = Y_0 \frac{1}{\sqrt{1-C}}, \quad Y_c'' = Y_0 \sqrt{\frac{C}{1-C}}}$$

Branch-line coupler with C=0.5 (3 dB)

The couplers with C=3 dB are identified with the word **Hybrid**.
To realize an hybrid of branch-line type the characteristic impedances of the lines must be:

$$Z_c' = Z_0\sqrt{1-0.5} = 35.35\ \Omega,\quad Z_c'' = Z_0\sqrt{\frac{0.5}{0.5}} = 50\ \Omega \qquad (Z_0 = 50\ \Omega)$$

Practical restrictions

One can easily verify that, with C tending to 0: $Z'_c \rightarrow Z_0$ and $Z''_c \rightarrow \infty$. In the practice, even with con C=0.1 (10 dB) the corresponding value of Z''_c is very difficult to realize ($Z''_c = 3Z_0$). Usually, C must be between 3 and 6 dB.

Frequency dependence

For this device, both matching and isolation vary with frequency (the nominal value is obtained at the frequency where the length of the lines is $\lambda_0/4$). Also the coupling depends on f (the max is again at f_0).
The bandwidth for a given value of maximum coupling increases with C_{max};
Usually, the frequency variation of matching and isolation is more pronounced than that of the coupling.

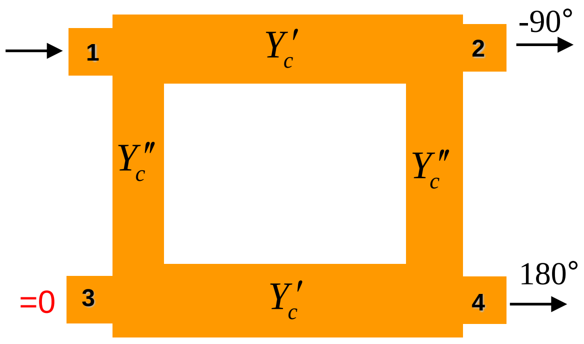
Branch Line: a summary

Conditions to be imposed:

$$S_{11} = S_{33} = S_{22} = S_{44} = 0, \qquad S_{13} = S_{24} = 0$$

$$|S_{14}|^2 = |S_{23}|^2 = C$$

$$|S_{12}|^2 = |S_{34}|^2 = 1 - C$$



Design equations:

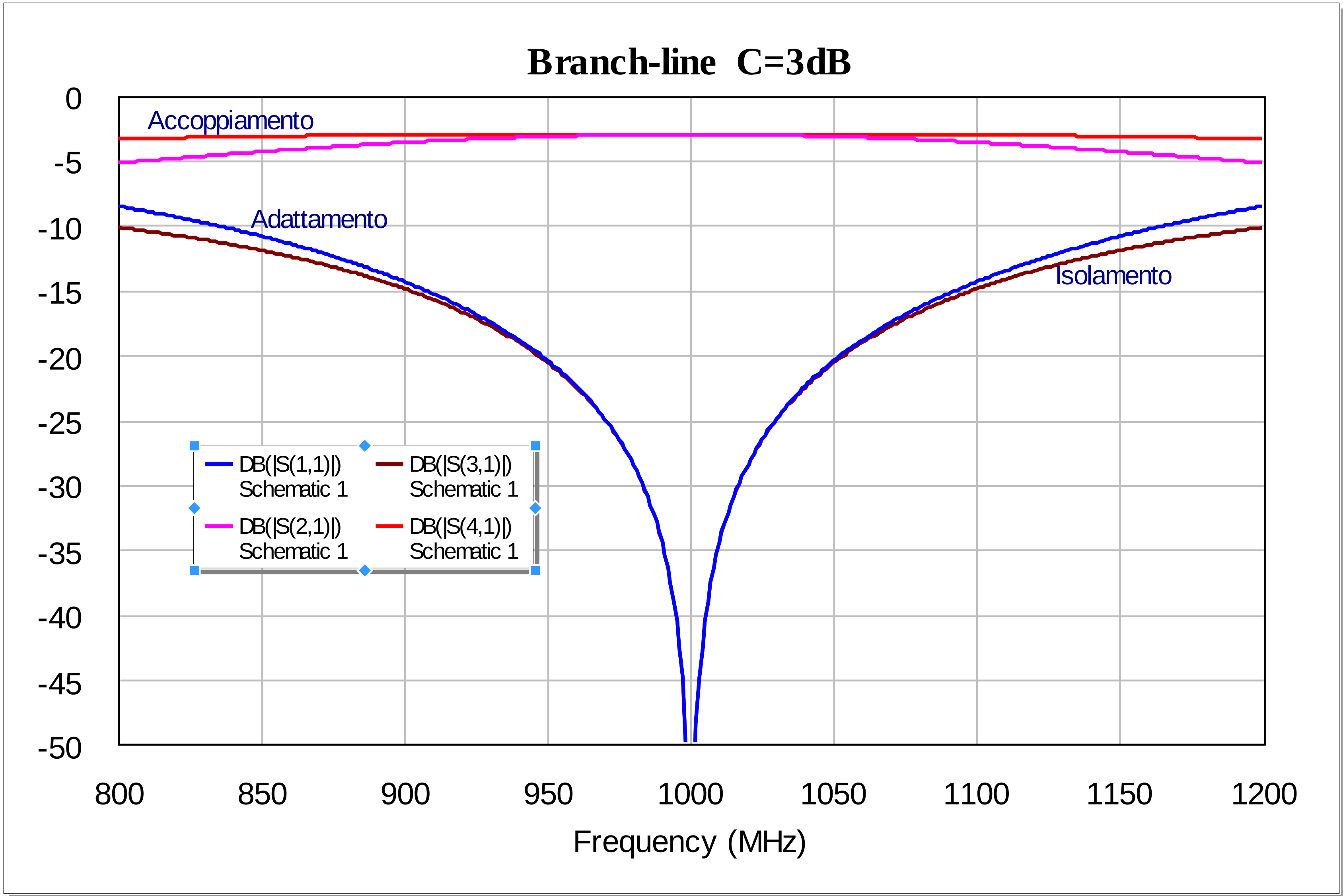
$$Y_c' = Y_0 \frac{1}{\sqrt{1-C}}, \quad Y_c'' = Y_0 \sqrt{\frac{C}{1-C}}$$

S parameters obtained:

$$S_{14} = S_{23} = -\sqrt{C}, \quad S_{12} = S_{34} = -j\sqrt{1-C}$$

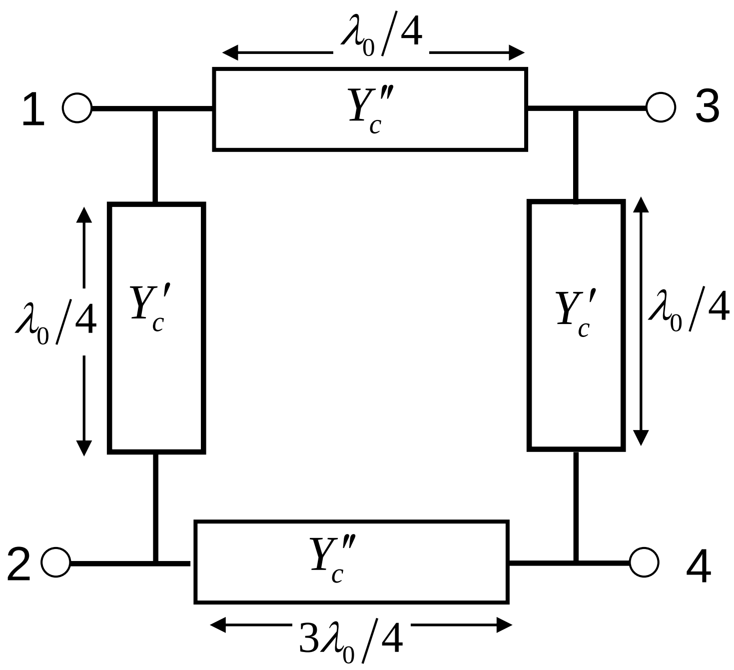
For C=0.5 (3dB), it has (assuming $Z_0 = 1/Y_0 = 50\Omega$):

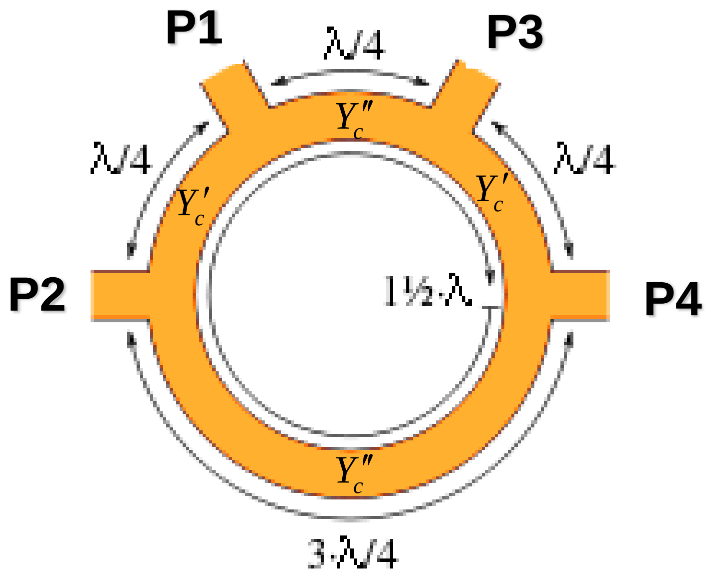
$$Z_c' = Z_0\sqrt{1-0.5} = 35.35\ \Omega, \quad Z_c'' = Z_0\sqrt{\frac{0.5}{0.5}} = 50\ \Omega \qquad (Z_0 = 50\ \Omega)$$



Branch-line C=3dB
Accoppiamento
Adattamento
Isolamento
DB(|S(1,1)|)
Schematic 1
DB(|S(3,1)|)
Schematic 1
DB(|S(2,1)|)
Schematic 1
DB(|S(4,1)|)
Schematic 1
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
800
850
900
950
1000
1050
1100
1150
1200
Frequency (MHz)

Rat-race coupler





There is only one symmetry axis (vertical)

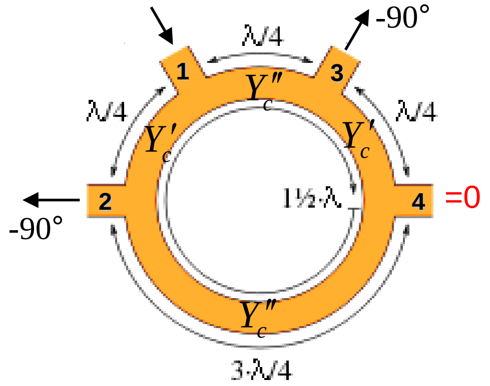
It is anyhow possible to still use the eigenvalues method for finding the dimensioning equations

Conditions to be imposed:

$$S_{11}=S_{33}=S_{22}=S_{44}=0, \qquad S_{23}=S_{14}=0$$

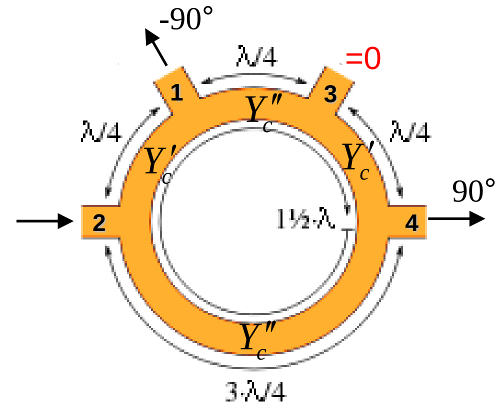
$$|S_{13}|^2=|S_{24}|^2=C$$

$$|S_{12}|^2=|S_{21}|^2=1-C$$



Design equations:

$$Y_c'=Y_0\cdot\sqrt{1-C}, \quad Y_c''=Y_0\cdot\sqrt{C}$$



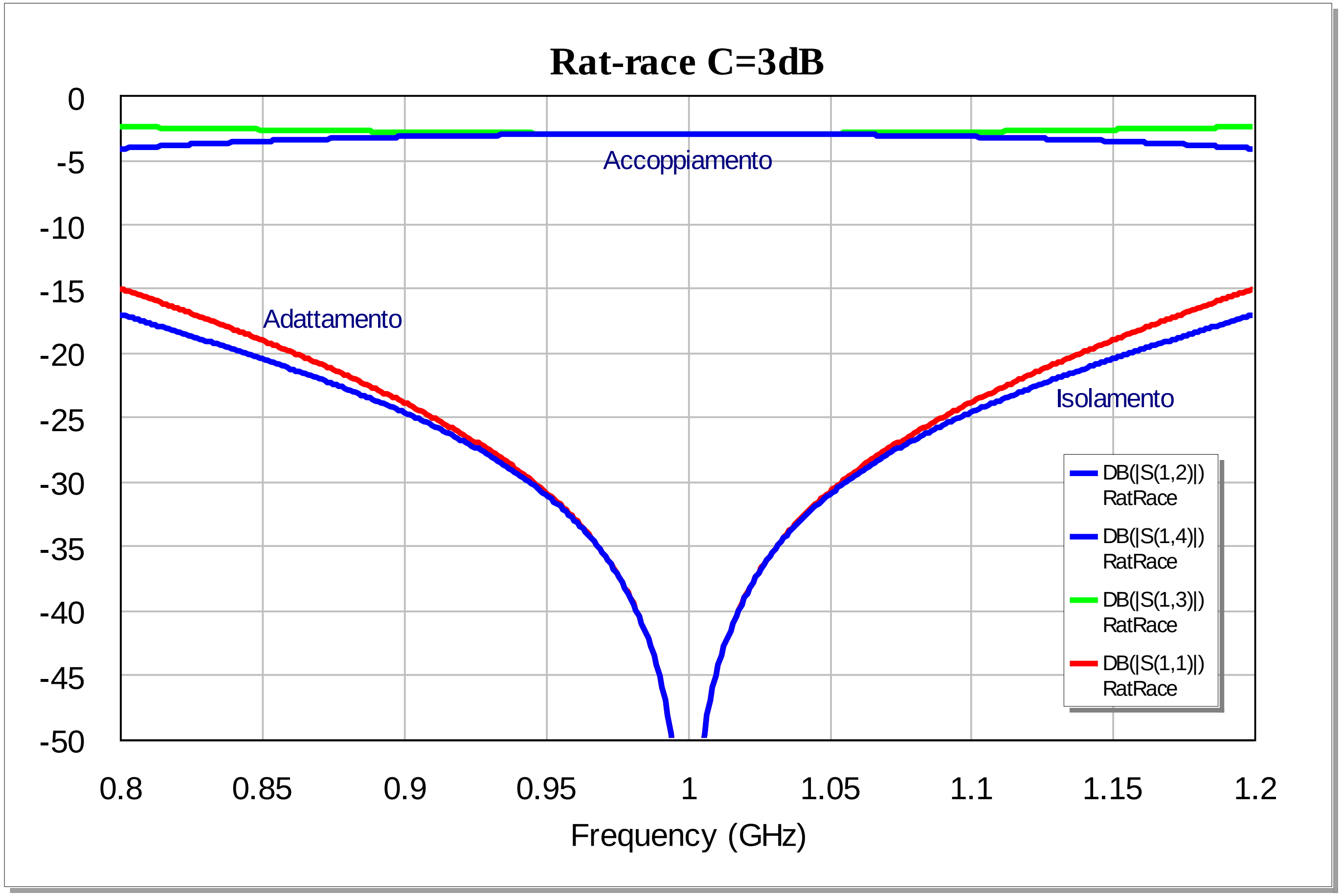
S parameters obtained:

$$S_{13}=-j\sqrt{C}, \; S_{24}=j\sqrt{C}, \; S_{12}=S_{34}=-j\sqrt{1-C}$$

For C=0.5 (3dB), we obtain (assuming $Z_0=1/Y_0=50\Omega$):

$$Z_c'=\frac{Z_0}{\sqrt{1-C}}=\frac{Z_0}{\sqrt{0.5}}=70.707\Omega, \qquad Z_c''=\frac{Z_0}{\sqrt{C}}=\frac{Z_0}{\sqrt{0.5}}=70.707\Omega$$

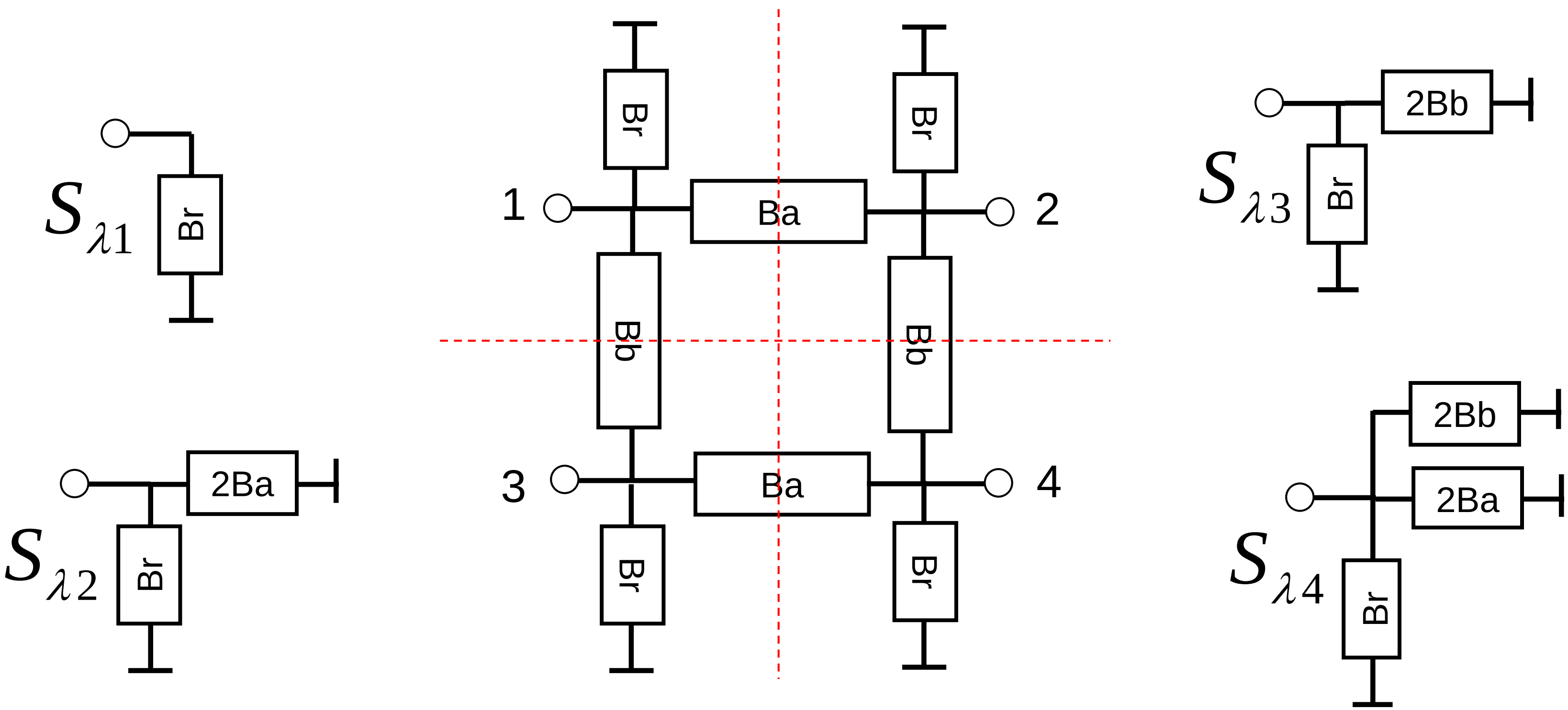
Parameters dependence on *f*



- The bandwidth is larger than that of the branch-line
- With the decreasing of the coupling the bandwidth increases
- The practical feasibility limits the coupling between about 3 and 8 dB

Couplers with lumped elements

In some cases, the practical implementation of planar couplers with C in the range 10-15 dB may be difficult using distributed elements. A possible alternative is represented by a structure similar to the branch line but employing lumped components (for the most critical elements).
Le consider the following (symmetrical) 4-port configuration, constituted by lumped susceptances (also the eigen-networks are shown in the figure):



Eigenvalues expressions ($b_i=B_i/Y_o$):

$$S_{\lambda 1}=\frac{1-jb_r}{1+jb_r},\ S_{\lambda 2}=\frac{1-j\left(b_r+2b_a\right)}{1+j\left(b_r+2b_a\right)},\ S_{\lambda 3}=\frac{1-j\left(b_r+2b_b\right)}{1+j\left(b_r+2b_b\right)},\ S_{\lambda 4}=\frac{1-j\left(b_r+2b_a+2b_b\right)}{1+j\left(b_r+2b_a+2b_b\right)},$$

Now impose $S_{11}=0$ and $S_{13}=0$ (uncoupled ports 1-3):

$$S_{11}=\frac{1}{4}\left(S_{\lambda 1}+S_{\lambda 2}+S_{\lambda 3}+S_{\lambda 4}\right)=0$$

$$S_{13}=\frac{1}{4}\left(S_{\lambda 1}+S_{\lambda 2}-S_{\lambda 3}-S_{\lambda 4}\right)=0$$

$$S_{\lambda 1}+S_{\lambda 2}=0,\quad S_{\lambda 3}+S_{\lambda 4}=0$$

$$b_r=-\left(b_a+b_b\right)$$

Imposing the coupling $C=|S_{14}|^2$:

$$S_{12}=\frac{1}{4}\left(S_{\lambda 1}-S_{\lambda 2}-S_{\lambda 3}+S_{\lambda 4}\right)=\frac{1}{2}\left(S_{\lambda 1}-S_{\lambda 3}\right)=\frac{j4b_b}{\left(1-b_a^2+b_b^2\right)-j2b_a}$$

If we assume S_{12} real, $b_a^2-b_b^2=1$, and: $\left|S_{13}\right|=\sqrt{C}=\dfrac{b_b}{b_a}$

Finally:

$$\boxed{b_a=\sqrt{\frac{1}{1-C}},\qquad b_b=\sqrt{\frac{C}{1-C}},\qquad b_r=-\left(b_a+b_b\right)}$$

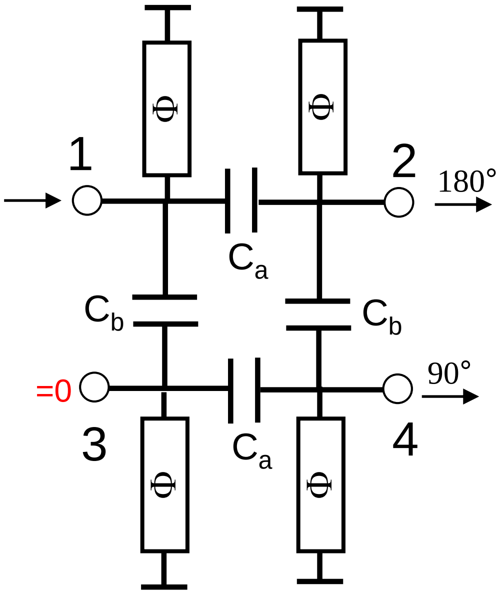
Example of implementation

We observe that Ba and Bb are always positive, so can be implemented with capacitances. Br is instead negative and can be realized with a short circuited stub with characteristic impedance Z_0.

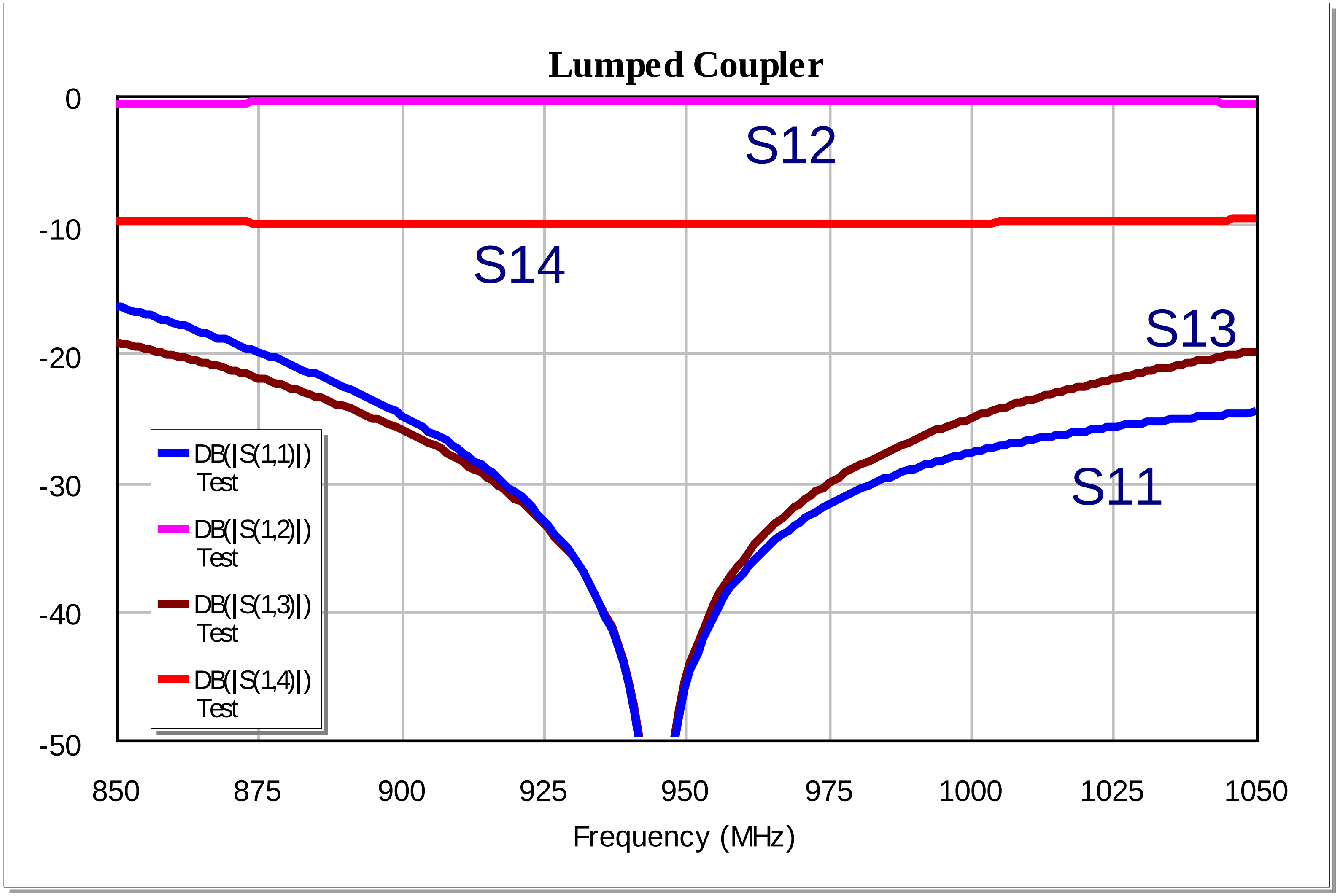
Assume CdB=10 dB (C=0.1), f0=945 MHz and Y_0=1/50. From the previous formulas we get: b_a=1.054, b_b=0.3333, b_r=-1.387.
The capacitances implementing Ba and Bb are given by: Ca=3.55 pF, Cb=1.12 pF. The susceptance B_r is realized with a short circuited stub with Z_c=50 W and βL=atan(-1/b_r)=35.78°. The final circuit is shown below together with the expressions of S12 and S14

$$S_{12} = \frac{1+(\omega C_a)^2 - (\omega C_b)^2}{1-(\omega C_a)^2 + (\omega C_b)^2 - j2\omega C_a}, \quad \text{for } \omega=\omega_0 \;\rightarrow S_{12} = j\frac{1}{\omega_0 C_a}$$

$$S_{14} = \frac{j2\omega C_b}{1-(\omega C_a)^2 + (\omega C_b)^2 - j2\omega C_a}, \quad \text{for } \omega=\omega_0 \;\rightarrow S_{14} = -\frac{C_b}{C_a}$$



Frequency response of the lumped coupler



3-port lossless networks

A 3-port reciprocal lossless network cannot be matched at the 3 ports (i.e. $S_{11}=S_{22}=S_{33}=0$ not possible).
In fact, imposing the unitary of S:

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad \Rightarrow \quad |S_{12}|^2 = |S_{13}|^2 = |S_{23}|^2 = 0.5$$

$$|S_{13}|^2 + |S_{23}|^2 = 1$$

These conditions are incompatible so <u>it is impossible to have $S_{11}=S_{22}=S_{33}=0$</u>

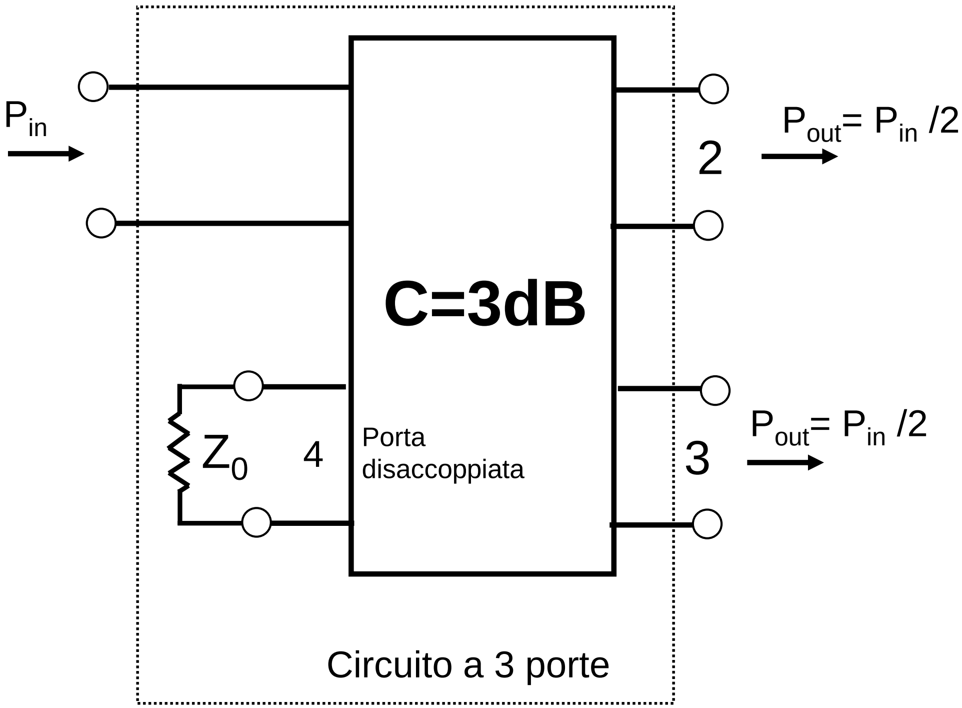
$$S_{13}S_{23}^* = 0$$

$$S_{12}S_{23}^* = 0 \quad \Rightarrow \quad |S_{12}|=0 \text{ or } |S_{23}| = 0 \text{ or } |S_{23}| = 0$$

$$S_{12}S_{13}^* = 0$$

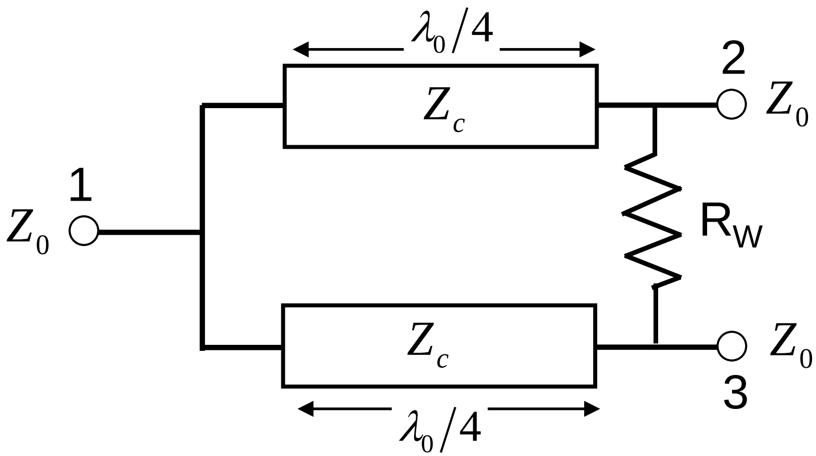
Is it still possible to realize a power splitter with a 3-port?

<u>Possible solution</u>: a 3 dB hybrid with the uncoupled port closed on a matched load



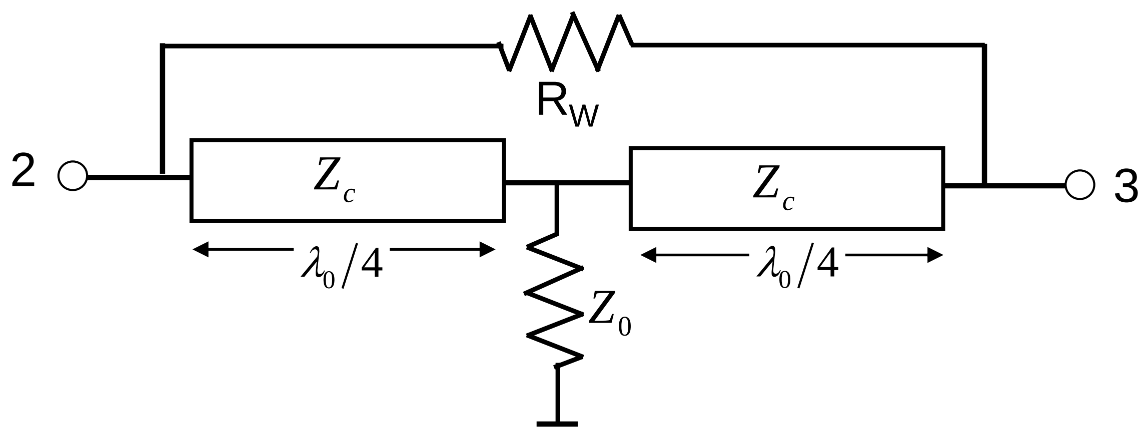
The 3-port network is lossy due to the presence of Z_0. This resistor however does not dissipate power because the port 4 is uncoupled. Pin is then split between ports 2 and 3 without losses

A 3-port divider: the Wilkinson network

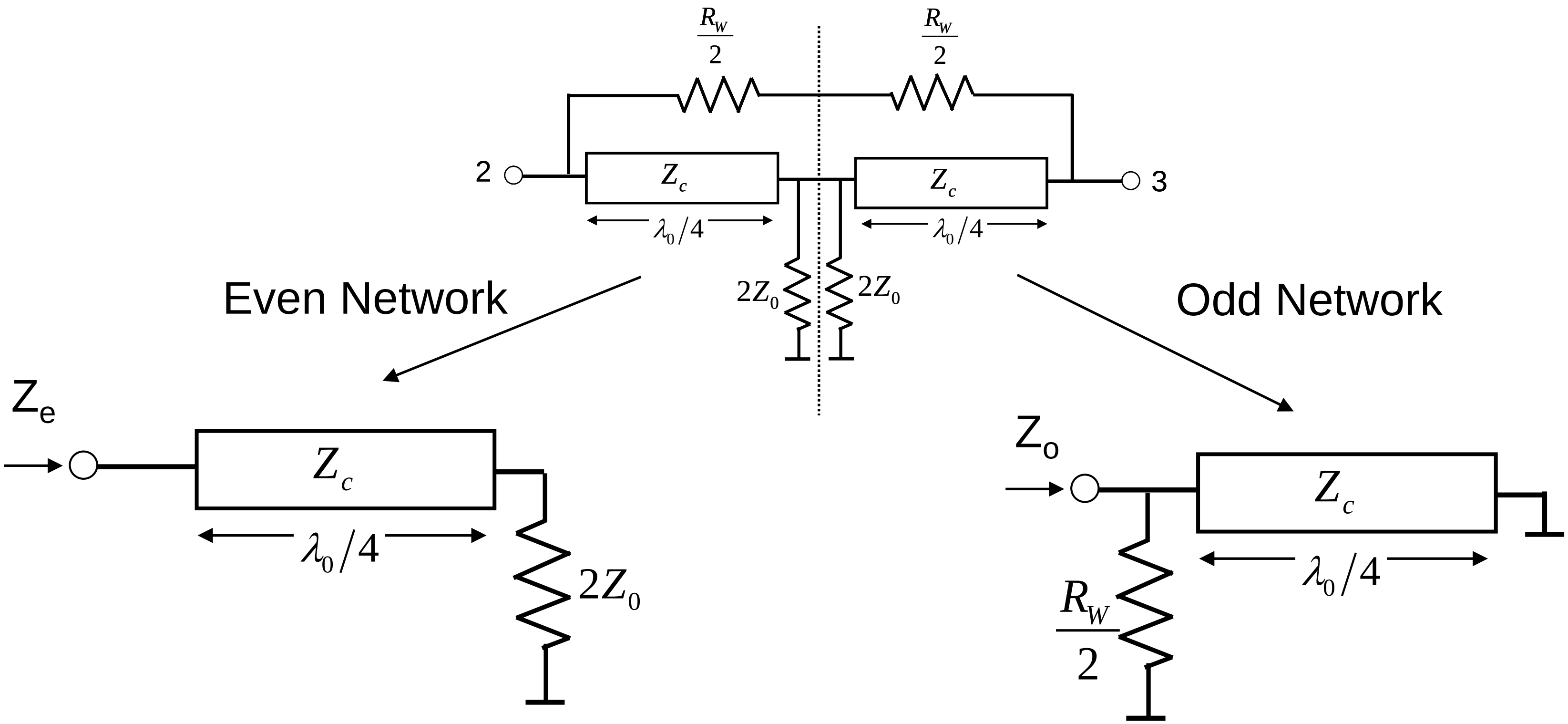


Due to the presence of R_W the 3-port network is lossy, so the condition $S_{11}=S_{22}=S_{33}=0$ can be imposed

2-port network obtained by closing port 1 with Z_0:



$S_{22}=S_{33}$ and S_{23} can be computed thorugh the eigenvalues of this network



$$Z_e = \frac{Z_c^2}{2Z_0}, \quad \Gamma_e = \frac{Z_c^2 - 2Z_0^2}{Z_c^2 + 2Z_0^2}$$

$$Z_o = \frac{R_W}{2}, \quad \Gamma_o = \frac{R_W - 2Z_0}{R_W - 2Z_0}$$

$$S_{22} = \frac{\Gamma_e + \Gamma_o}{2} = 0, \quad S_{23} = \frac{\Gamma_e - \Gamma_o}{2} = 0 \quad \Rightarrow \quad \Gamma_e = 0, \quad \Gamma_o = 0$$

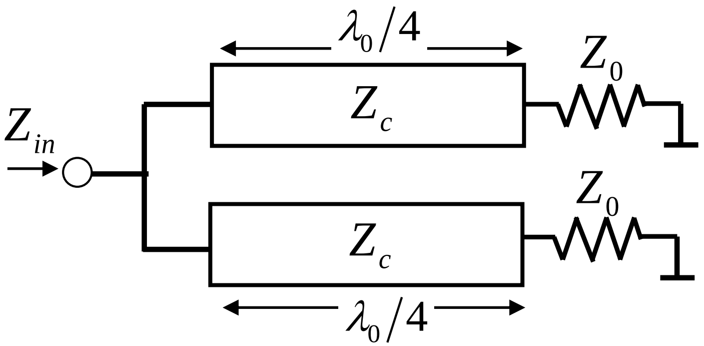
$$\Downarrow$$

$$\boxed{Z_c = \sqrt{2} \cdot Z_0, \qquad R_W = 2Z_0}$$

For Z_0=50Ω → Z_c=70.7Ω, R_W=100Ω

Evaluation of S_{11}

Exciting port 1 with ports 2 and 3 closed with Z_0, there is no current through R_W, so it can be discarded.



$$Z_{in} = \frac{1}{2}\frac{Z_c^2}{Z_0}, \quad \Gamma_{in} = S_{11} = \frac{Z_c^2 - 2Z_0^2}{Z_c^2 + 2Z_0^2}$$

Being $Z_c = \sqrt{2} \cdot Z_0$ also S_{11}=0.

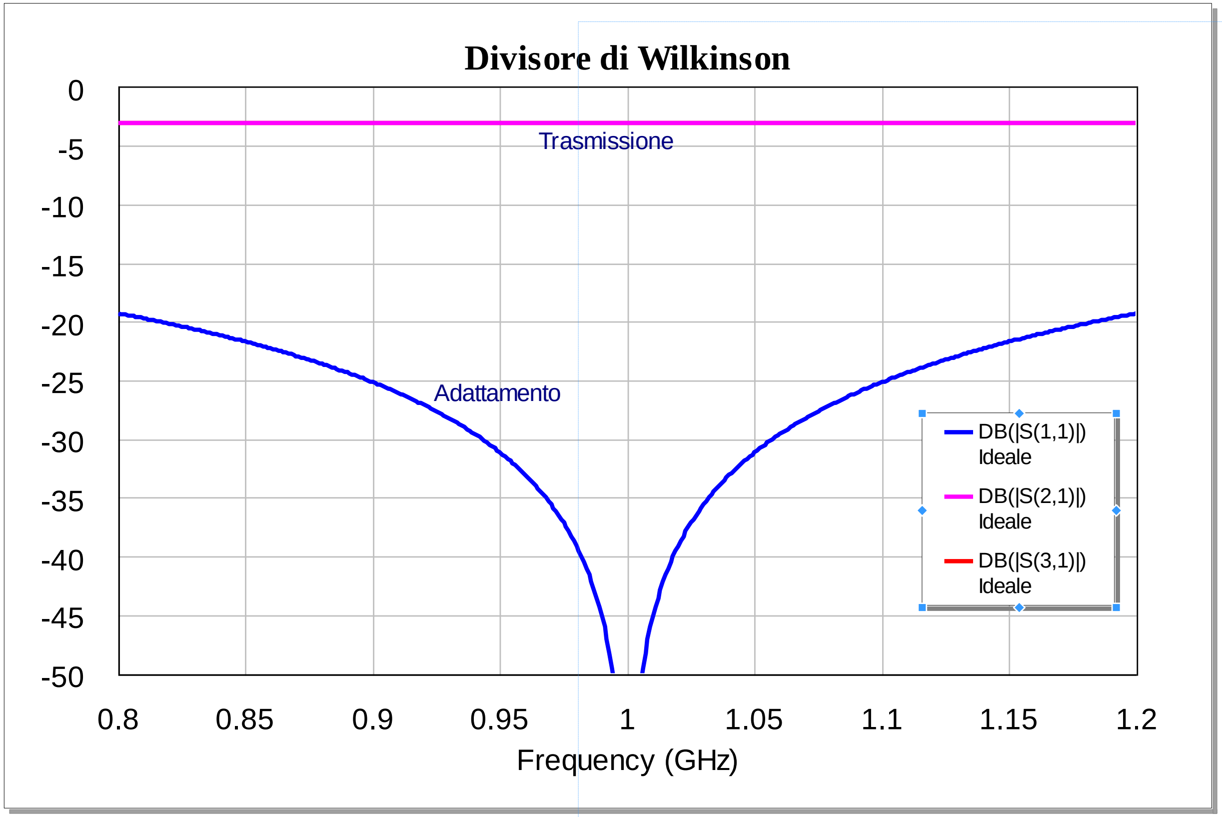
Evaluation of S_{21}= S_{31}

Power entering port 1 is not reflected and there is there is no dissipation in R_W. So the power is all transferred to ports 2 e 3; for the symmetry, the power exiting at each port is the half of Pin:

$$|S_{21}|^2 = |S_{31}|^2 = 0.5$$

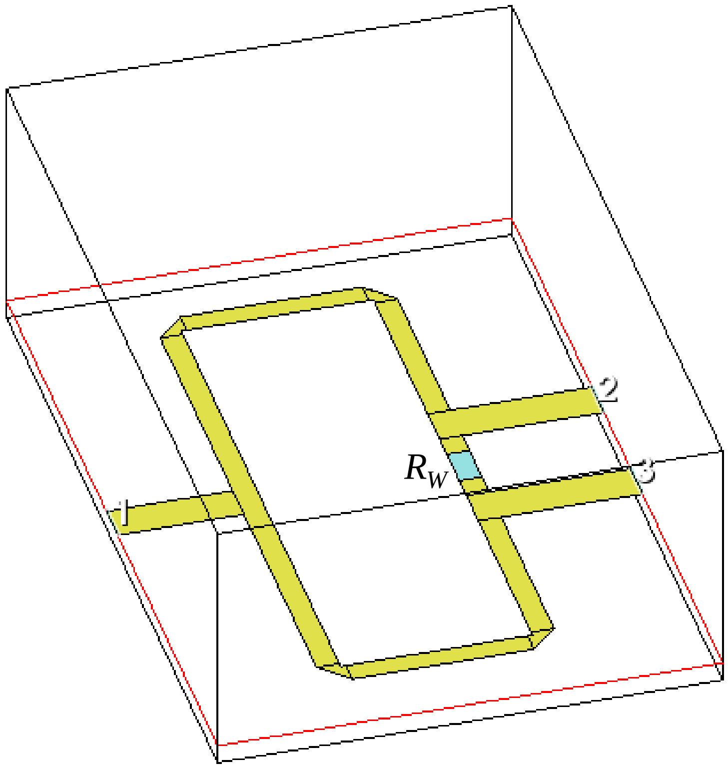
The phase for both parameters is -90°.

Frequency response



- The bandwidth for RL=20 dB is about 40% of f_0
- Transmission ($|S_{21}|=|S_{31}|$) is independent on frequency
- Dissipation in R_W is zero provided that the load at ports 2 and 3 is the same

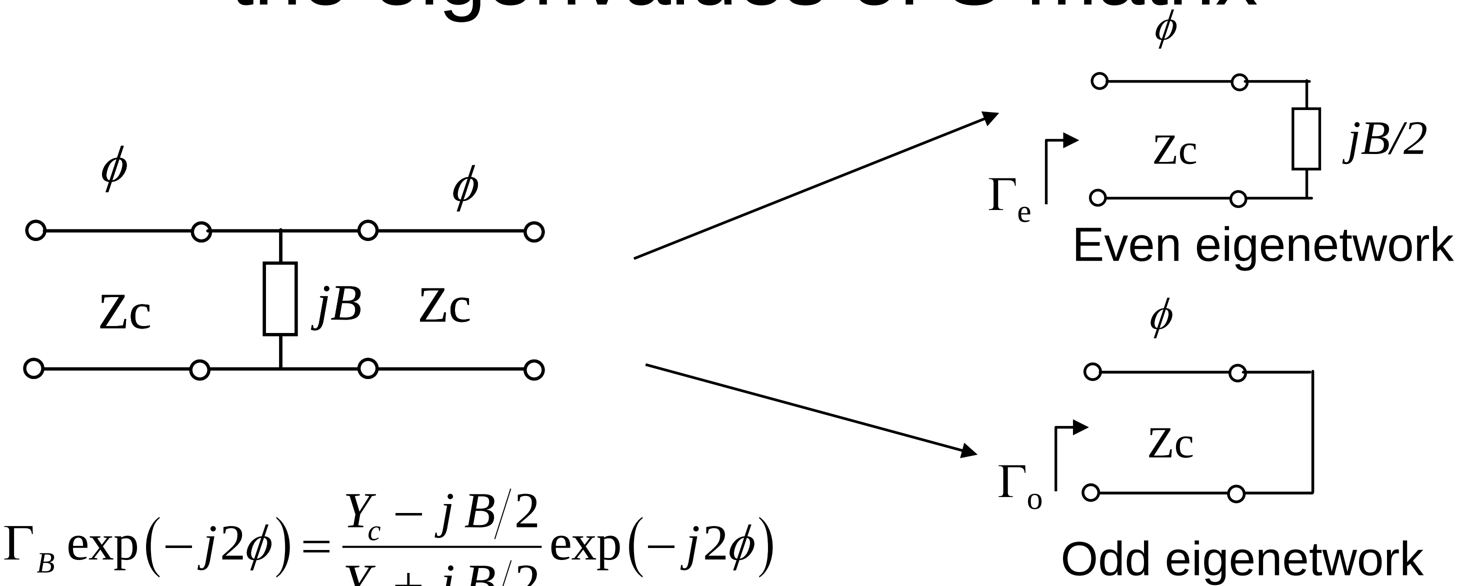
Microstrip implementation



The very small size of Rw (pseudo lumped component) must be accounted for. The two output lines must be enough close to allow the connection of Rw.

On the other hand is not advisable to have the output lines too close each other because an unwanted coupling may arise. So diverging lines are often used.

Example of derivation of S parameters from the eigenvalues of S matrix



$$\Gamma_e = \Gamma_B \exp(-j2\phi) = \frac{Y_c - j\,B/2}{Y_c + j\,B/2}\exp(-j2\phi)$$

$$\Gamma_o = \Gamma_{cc}\exp(-j2\phi) = -\exp(-j2\phi)$$

$$S_{11} = S_{22} = \frac{\Gamma_e + \Gamma_o}{2} = \frac{1}{2}\exp(-j2\phi)\left\{\frac{Y_c - j\,B/2}{Y_c + j\,B/2} - 1\right\} = -\exp(-j2\phi)\left\{\frac{jB}{2Y_c + jB}\right\}$$

$$S_{12} = S_{21} = \frac{\Gamma_e - \Gamma_o}{2} = \frac{1}{2}\exp(-j2\phi)\left\{\frac{Y_c - j\,B/2}{Y_c + j\,B/2} + 1\right\} = \exp(-j2\phi)\left\{\frac{2Y_c}{2Y_c + jB}\right\}$$