Antenna Exercise

The directivity function of an antenna is given by:

$$f(\theta) = \cos^{100}(\theta)$$

1) Evaluate the exact value of the maximum directivity D_{MAX} [it is reminded that $\int f^n(x) \cdot f'(x) = \frac{f^{n+1}(x)}{n+1}$]

2) Assuming the radiation impedance is 50Ω and the equivalent loss resistance is 2Ω evaluate the efficiency factor η and the gain G of the antenna

3) Evaluate the exact value of the 3dB beam width

Solution:

1) $D_M = \frac{4\pi}{\Delta\Omega}$, $\Delta\Omega = \int_0^{\pi}\int_0^{2\pi} f(\theta,\varphi)\sin(\theta)\,d\theta\,d\varphi$

$$\Delta\Omega = \int_0^{2\pi} d\varphi \int_0^{\pi} \cos^{100}(\theta) \cdot \sin(\theta)\,d\theta = -2\pi \cdot \left[\frac{\cos^{101}(\theta)}{101}\right]_0^{\pi} =$$

$$= 2\pi \cdot \frac{2}{101} = \frac{4\pi}{101}$$

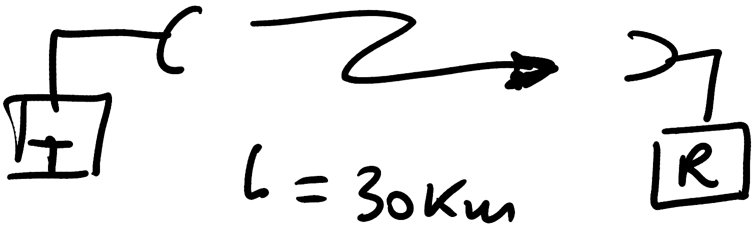
$$D_{MAX} = \frac{4\pi}{4\pi} \cdot 101 = 101$$

2) $\eta = \frac{50}{50+2} = 0,9615$ $\quad G = \eta \cdot D_{MAX} = 97.1 \;(19.87\,dB)$

3) $f(\bar{\theta}) = 0.5 = \cos^{100}(\bar{\theta})$ $\quad \bar{\theta} = \cos^{-1}(0.5^{\frac{1}{100}}) = 6.738°$

Terrestrial link

Consider a link operating at 6.5 GHz with the following data:



Signal: 64 QAM

$\alpha = 0.2$

$R = 140$ Mbit/sec

Antennas: $G_T = G_R = 30$ dB, $T_a = 200\,°K$

Receiving Front-end:

$A_f = 2$ dB

NF = 2 dB

G = 10 dB

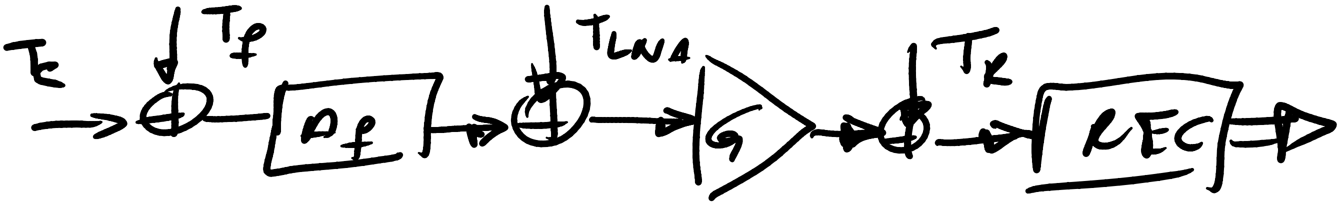
$E_b/N_0 = 15$ dB for $P_{in} = -80$ dBm

Question: Find the power to be transmitted to get $(SNR)_{sys} = 30$ dB at the receiver.

Solution

a) We know that $SNR_{sys} = \frac{P_r}{K T_{sys} B}$ with P_r power at the output of the receiving antenna, $B = (1+\alpha) R / \log_2 64 = 28$ Hz (signal bandwidth)

T_{sys} = System Noise Temperature of the receiver.



$$T_{sys} = T_a + T_f + T_{LNA} \cdot A_f + \frac{T_R \cdot A_f}{G}$$

Evaluation of T_R

$$\frac{E_b}{N_0} = \frac{P/R}{N_0} = \frac{P}{R K T_R} \Rightarrow T_R = \frac{P}{K R \frac{E_b}{N_0}} = 163.7\,°K$$

$$T_{sys} = 290 + 290(10^{0.2}-1) + 290(10^{0.2}-1)\,10^{0.2} + \frac{167.3 \cdot 10^{0.2}}{10}$$

$$= 664.4\,°K$$

$$P_r = SNR_{sys} \cdot K \cdot T_{sys} \cdot B = 2.57 \cdot 10^{-10}\,W\ (-65.9\,dBm)$$

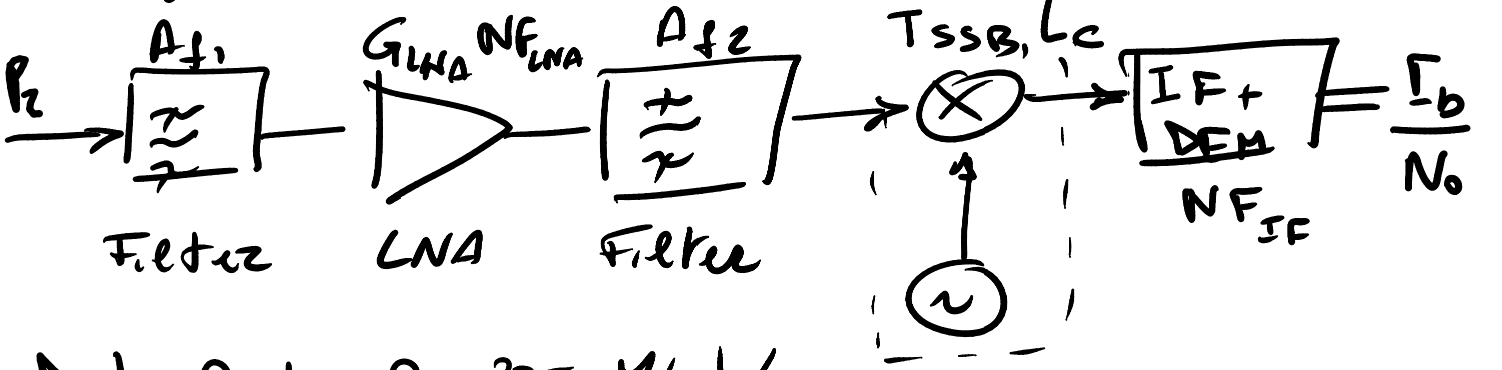
$$P_r = P_t - L_f + 2\,G_A \Rightarrow P_t = -65.9 + 138.2 - 60 = 12.34\ dBm$$

$$L_f = 20 \log\left(\frac{4\pi L}{\lambda}\right) = 138.2\,dB$$

$$\lambda_0 = \frac{3 \cdot 10^8}{65 \cdot 10^9} = 4.6 \cdot 10^{-2}\,m$$

Receiver Budget Evaluation

The scheme of a digital receiver at 18 GHz is given in the following figure:



Data Rate $R = 225$ Mbit/sec (128-QAM, $\alpha = 0.25$)

$A_{f1} = 0.5$ dB $A_{f2} = 1.5$ dB $NF_{IF} = 3$ dB

$L_c = 6$ dB, $T_{SSB} = 350\,°K$

Question: Find possible values for G_{LNA} and NF_{LNA} in order to get $\frac{E_b}{N_0} = 25$ dB with $P_r = -60$ dB. Assume the input noise at room temperature

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$KB = 6.28 \cdot 10^{-16}$ $\frac{R}{B} = \frac{\log_2 128}{1+\alpha} = 5.6$

$$SNR_{sys} = \left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right) = 32.48 \text{ dB}$$

$$B = \frac{R}{5.6} = 45.54 \text{ MHz}$$

Solution

Evaluation of T_{sys} :

$$T_{sys} = T_0 + T_{f1} + T_{LNA} \cdot A_{f1} + \frac{T_{f2} A_{f1}}{G_{LNA}} + \frac{T_{SSB} \cdot A_{f1} A_{f2}}{G_{LNA}} +$$

$$+ \frac{T_{IF} \cdot L_C A_{f1} \cdot A_{f2}}{G_{LNA}}$$

$$= 290(\not{1} + A^{0.05} - 1) + T_{LNA}(10^{0.05}) + \frac{10^{0.05}}{G_{LNA}}\left(290(10^{0.15} - 1) + \right.$$

$$\left. + 350 \cdot 10^{0.15} + 290(10^{0.3} - 1) \cdot 10^{0.6} \cdot 10^{0.15}\right.$$

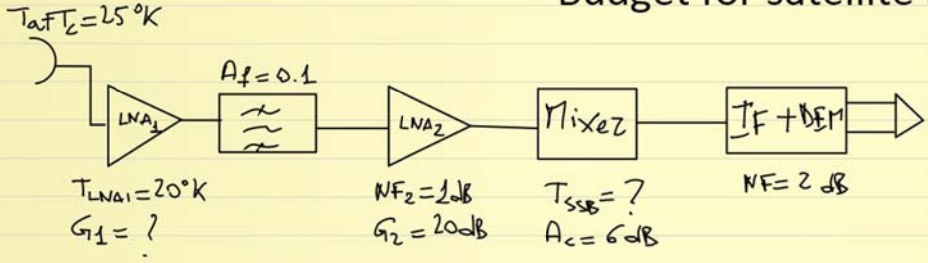
$$T_{sys} = 325.8 + 1,12 T_{LNA} + \frac{2510,1}{G_{LNA}}$$

$$T_{sys} = \frac{P_2}{SNR_{sys} \, k B} = 899.6$$

$$\frac{2510.1}{G_{LNA}} + 1.12 T_{LNA} = 573.8$$

Assigning $G_{LNA} = 10 \rightarrow T_{LNA} = 288.2 \ ^\circ K$ (NF= ~ 3dB)

Budget for satellite receiver frontend



This figure represents the frontend of a receiver for a satellite ground station.
The first amplifier is cooled in order to reduce the noise
The goal is to get the system SNR of at least 20 dB assuming a minimum power level at antenna output of -85 dBm. The signal bandwidth is 35 MHz.
Evaluate the gain of LNA1 and TSSB of the mixer in order to fit the goal
Find also the maximum rate R which is allowed for a digital signal when the ratio Eb/No is equal to 15 dB

SOLUTION

a) The system SNR is given by

$$SNR_{sys} = \frac{P_{rec}}{K T_{sys} B}$$

$$T_{sys} = T_a + T_c + T_{LNA1} + \frac{T_{f1}}{G_1} + 2\,\frac{T_{LNA2}\cdot A_{f1}}{G_1} + \frac{T_{SSB}\cdot A_{f1}}{G_1\cdot G_2} + \frac{T_{IF}\cdot A_{f1}\,A_c}{G_1\cdot G_2}$$

b) $$P_{Noise} = K T_{sys} B = \frac{P_{rec}}{SNR_{sys}} = \frac{10^{-11.5}}{100} = 3.162\cdot 10^{-14}\ W$$

$$T_{sys} = \frac{P_{Noise}}{KB} = 65.5\ °K$$

c) Budget (T_{sys} equation)

$$65.5 = 45 + \left\{290\,(10^{0.01}-1) + 2\cdot 290\,(10^{0.1}-1)\cdot 10^{0.01} + T_{SSB}\cdot 10^{0.01}/100 + 10^{0.61}\,290\,(10^{0.2}-1)/100\right\}\frac{1}{G_1}$$

$$20.5 = \frac{0.01\,T_{SSB} + 167.3}{G_1} \Rightarrow G_1 = \frac{0.01\,T_{SSB} + 167.3}{20.5}$$

Assigning $T_{SSB} = 200\ °K \Rightarrow G_1 = 8.26$ (~ 9.2 dB)

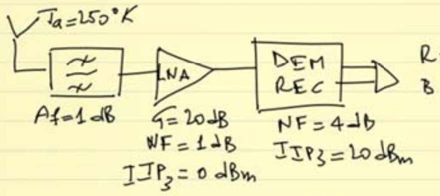
d) The system SNR can be written as:

$$(SNR)_{sys} = \left(\frac{E_b}{N}\right)\frac{R}{B} \Rightarrow 20 = 15 + \left(\frac{R}{B}\right)_{dB} \Rightarrow \frac{R}{B} = 10^{0.5} = 3.16$$

$$R = 35\cdot 3.16 = 110.7\ Mbit/sec$$

Dynamic Range of a Receiver

A direct conversion digital receiver is characterized by the following block diagram:

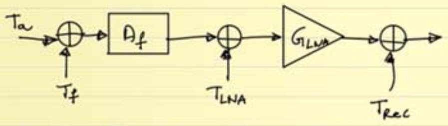


With the received signal equal to the receiver sensitivity, the E_b/N_0 at baseband is equal to 15 dB.

Evaluate the dynamic range of the receiver and the maximum power of the received signal.

Solution

Evaluation of the system noise temperature T_{sys}:



$T_a = 250\,°K$

$T_f = 290\,(10^{0.1} - 1) = 75.1\,°K$

$T_{LNA} = 75.1\,°K$

$T_{REC} = 290\,(10^{0.4} - 1) = 438.4\,°K$

$$T_{sys} = T_a + T_f + A_f T_{LNA} + \frac{A_f T_{Rec}}{G_{LNA}} = 425.16\,°K$$

The SNR is given by:

$$SNR_{sys} = \frac{S}{K \cdot T_{sys} B} = \left(\frac{E_b}{N_0}\right) \cdot \left(\frac{R}{B}\right)$$

Then:

$$S = \frac{E_b}{N_0}\left(\frac{R}{B}\right) \cdot K T_{sys} B = +15 + 6 - 115.32 = -94.32\ \text{dBm}$$

The dynamic range is the obtained with:

$$DR = \frac{2}{3}\left(IIP3 + 3 - S\right)$$

where the system IIP3 is expressed as:

$$\left(\frac{A_f}{IIP3}\right)^2 = \left(\frac{1}{IIP3_{LNA}}\right)^2 + \left(\frac{G_{LNA}}{IIP3_{Rec}}\right)^2 \Rightarrow IIP3 = -0.5\ \text{dBm}$$

Substituting:

$$DR = \frac{2}{3}(-0.5 + 3 + 94.3) = 64.5 \text{ dB}$$

$$P_{max} = S + DR = -29.75 \text{ dBm}$$