Introduction

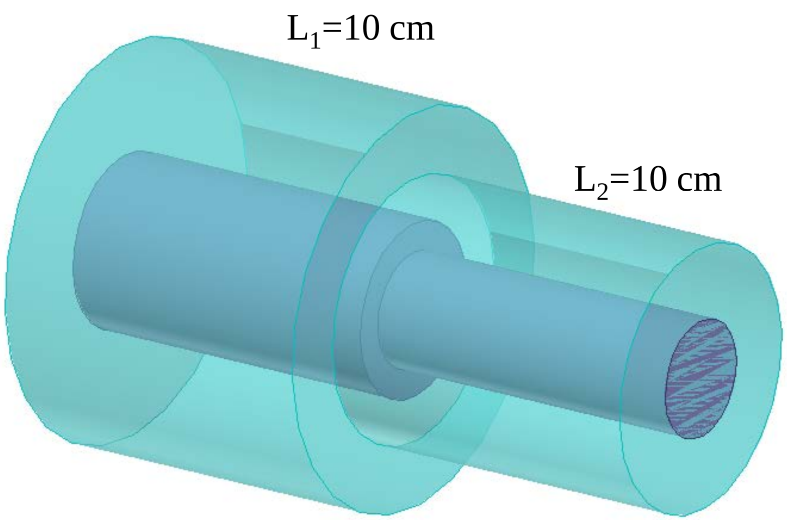
A microwave circuit is an interconnection of components whose size is comparable with the wavelength at the operation frequency

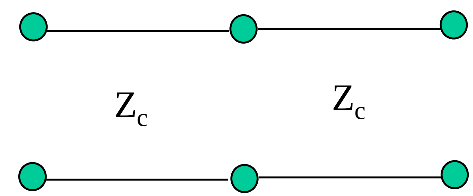
Type of Components:

- Interconnection: it is not an ideal connection (zero resistance) as in the case of lumped-element networks
- Pseudo-lumped components (they present, in a first approximation, the same frequency behavior of the lumped components, having however a finite size)
- Distributed components (transmission line sections or stubs arbitrarily terminated)

In microwave circuits the junction between two (or more) component do not correspond to an ideal node (as in lumped circuits). The junction must be suitably represented (typically by means of its scattering parameters) in order to take into account the phenomena which always arise at a discontinuity in a transmission line.

Discontinuity between two transmission lines



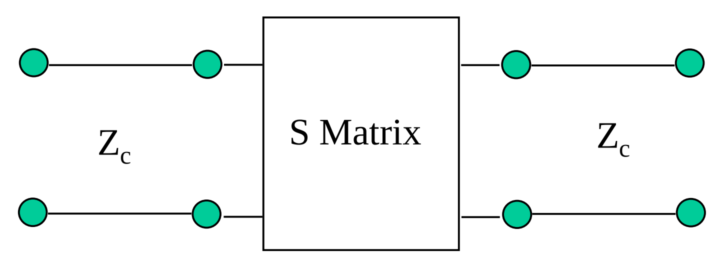


No good!

Coax 1: R_1=5 cm, r_1=2.17 cm

Coax 2: R_2=3 cm, r_1=1.3 cm

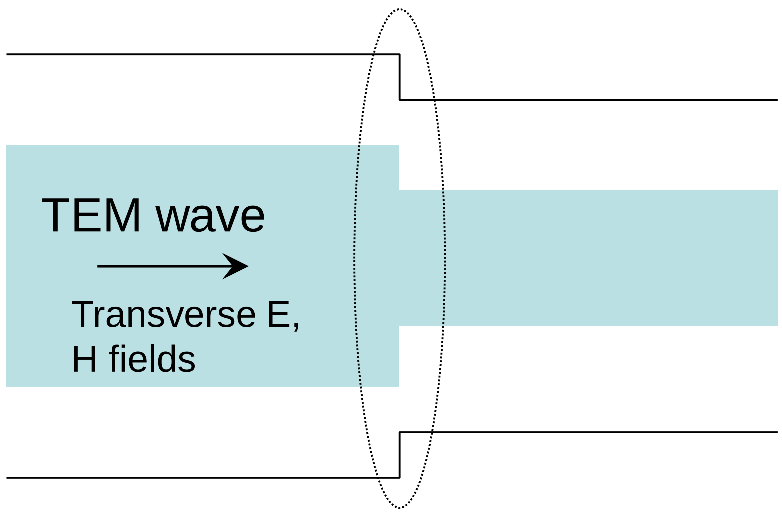
$$Z_c = 60\cdot\ln\left(\frac{R_1}{r_1}\right) = 60\cdot\ln\left(\frac{R_2}{r_2}\right) = 50\ \Omega$$

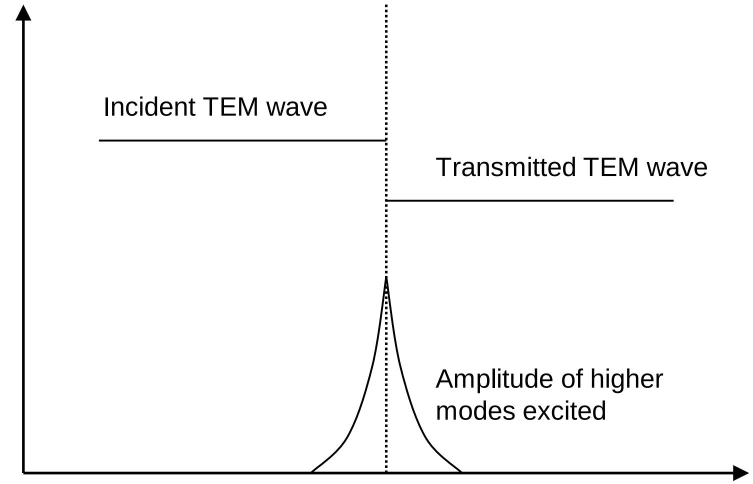


Discontinuity

Correct model. Why?

Higher modes excitation





At the discontinuity the E field cannot be transverse

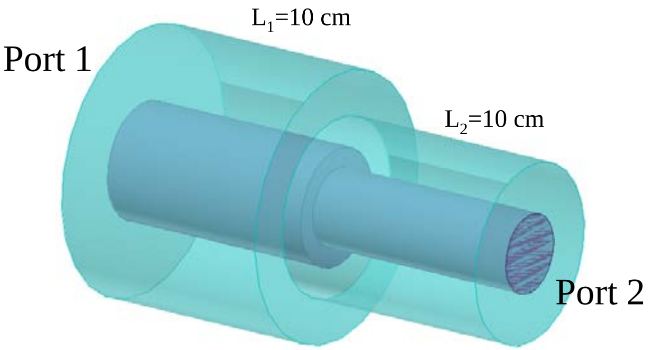
Higher modes are then generated, allowing the fulfillment of the continuity conditions. These modes however cannot propagate (they are below cutoff)

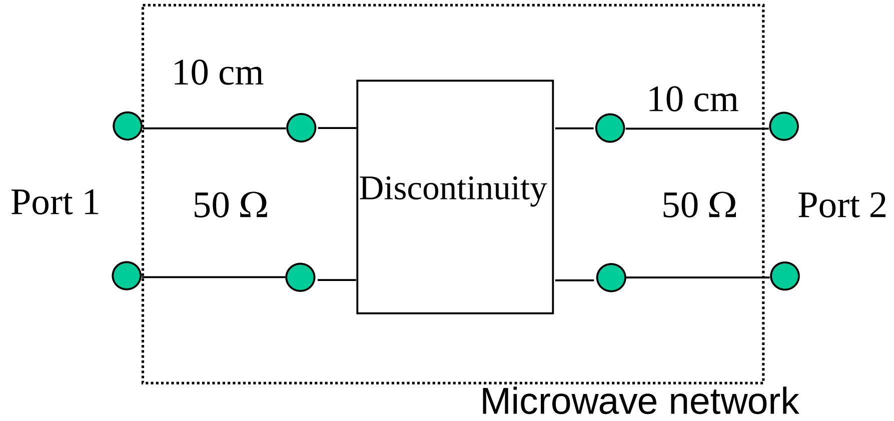
The energy of the em field associated to the higher modes is confined closely to the discontinuity, and it affects the incident wave like a lumped reactance connected at the junction of the two coaxial lines

Scattering Matrix of the discontinuity

The scattering parameters of the following configuration can be evaluated by means of an em simulator (numerical solution of the em fields). It has (f_0=1 GHz, Zc=50 Ω):

$$S_{11} = S_{22} = 0.1792\angle 20°, \qquad S_{12} = S_{21} = 0.9838\angle 109.61°$$



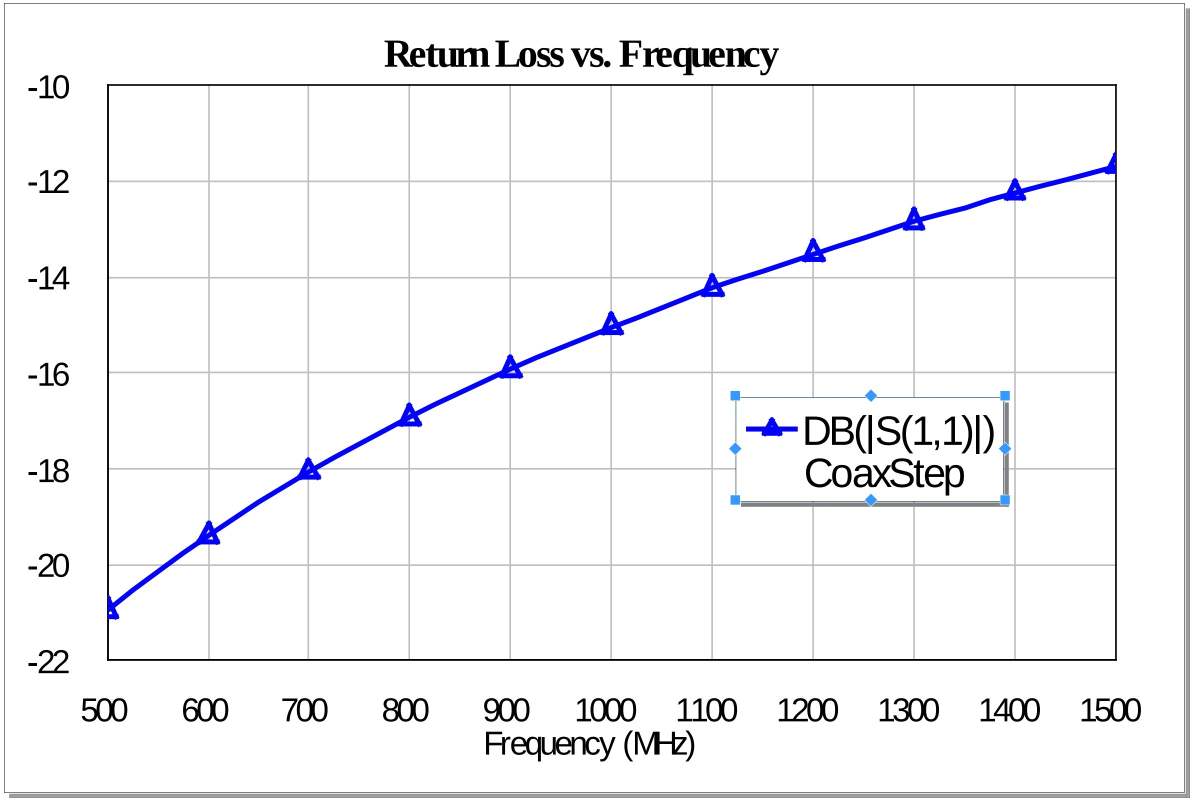


To get the S parameters of discontinuity we need to move the reference sections (ports 1 and 2) by 10 cm inwards:

$$\phi = -\beta \cdot 10\ cm = \frac{2\pi f}{v} \cdot 10\ cm = -2.094\ rad \quad (-120°)$$

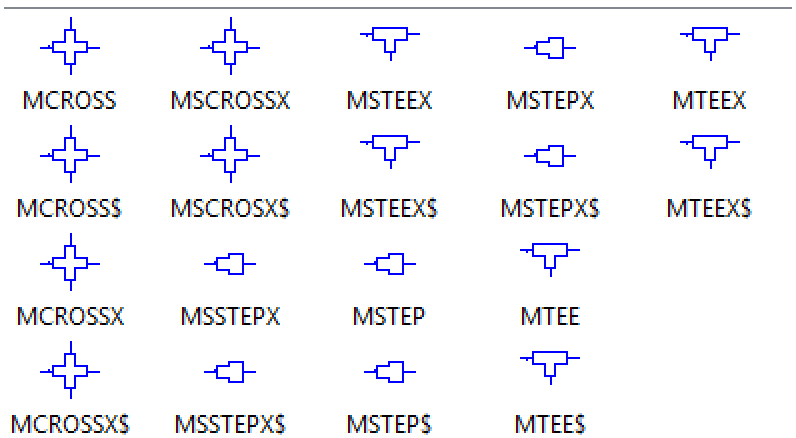
$$S'_{11} = S'_{22} = S_{11}\exp(-j2\phi) = 0.1792\angle -100°, \qquad S'_{12} = S'_{21} = S_{12}\exp(-j2\phi) = 0.9838\angle -10.39°$$

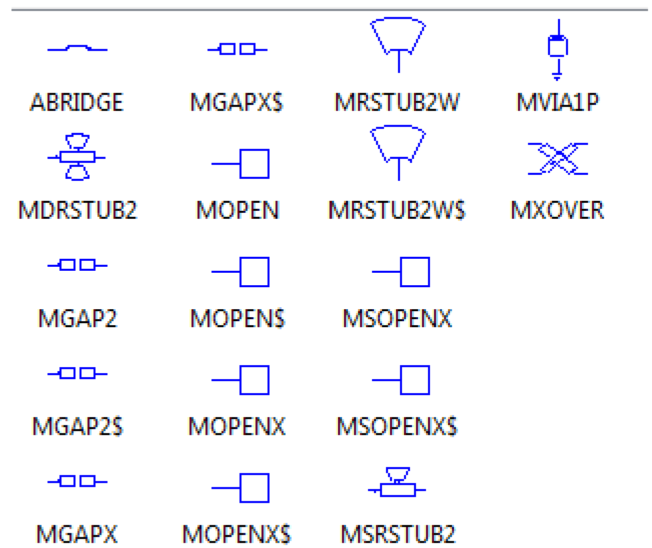
Frequency dependance

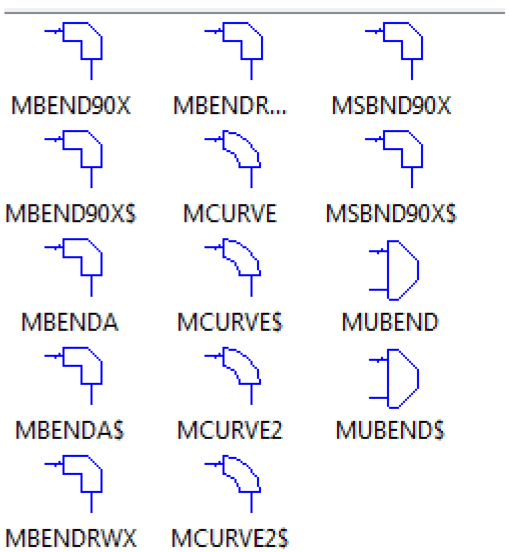


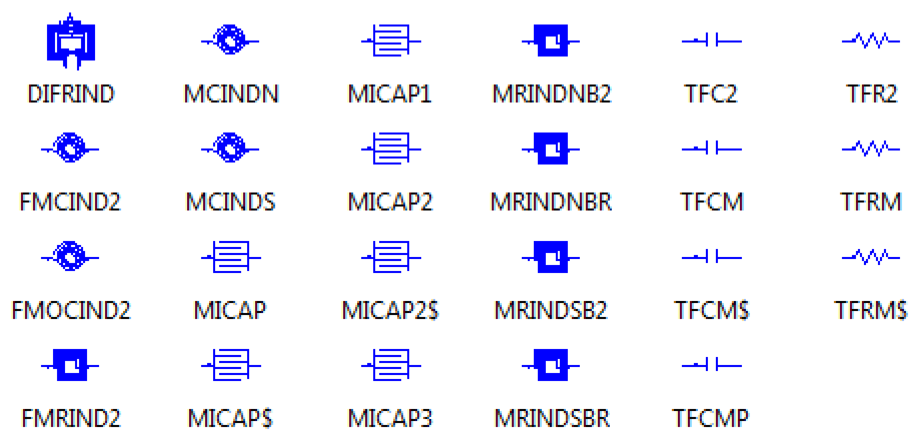
The effects of the discontinuities are generally frequency-dependent

List of some discontinuities and components in the library of MWOffice

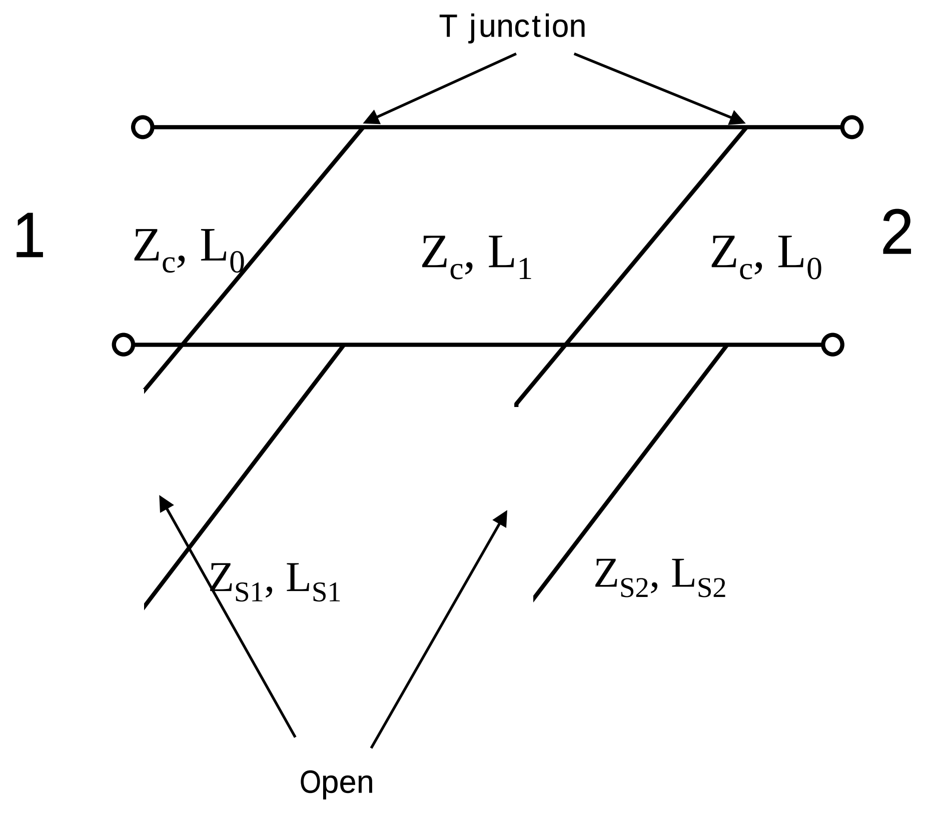






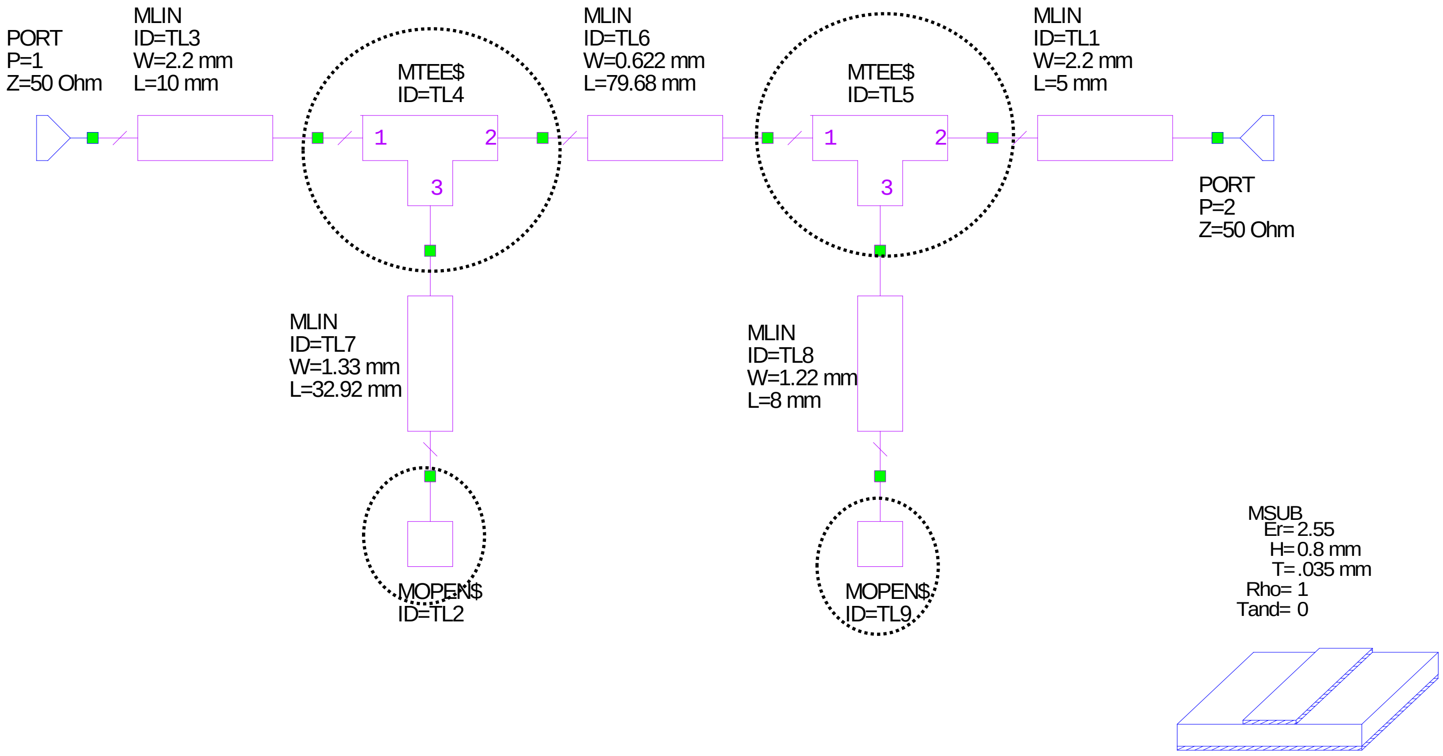


Example: microstrip implementation of a double stub matching network

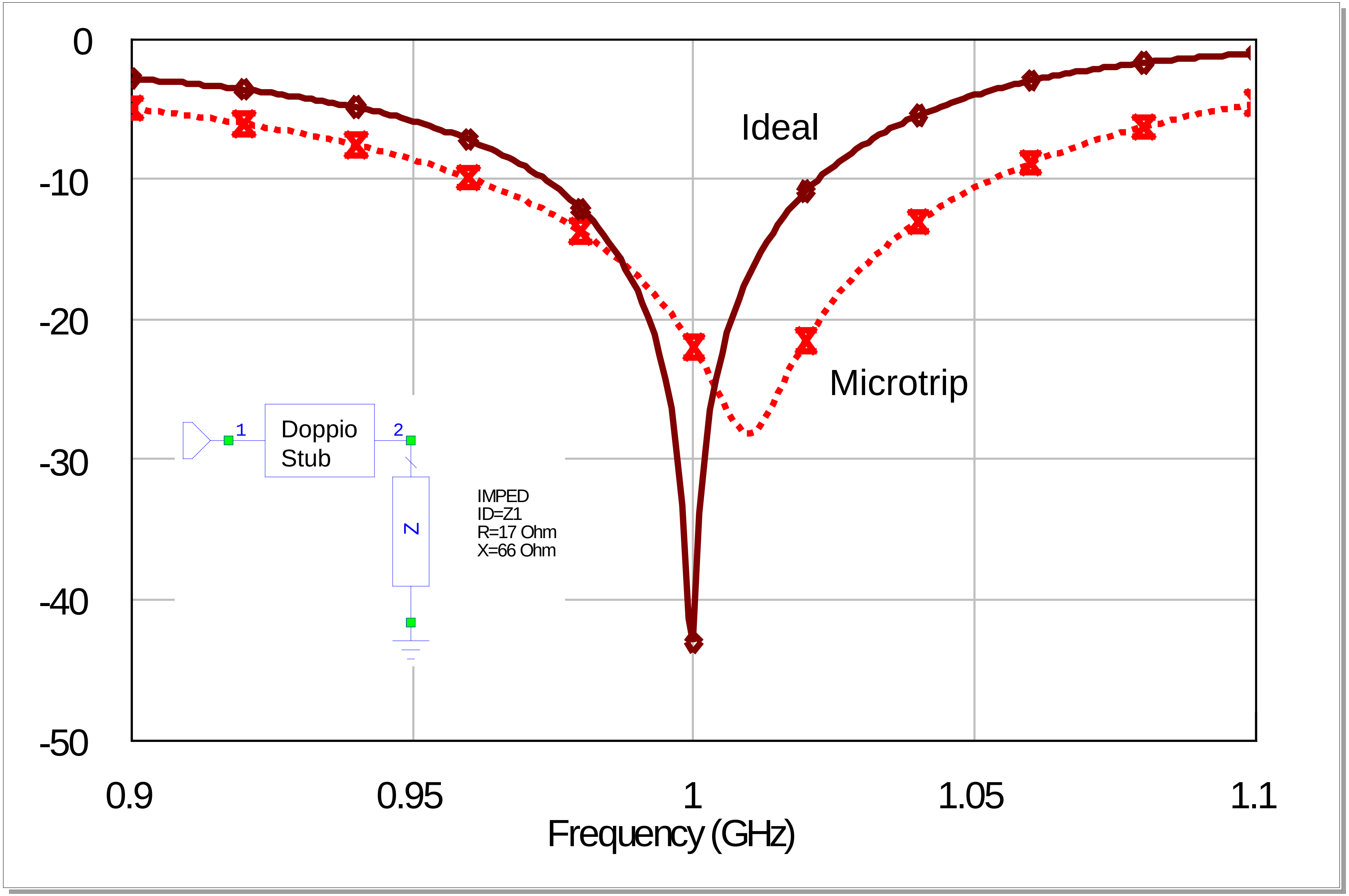


Ideal scheme

Discontinuities inclusion



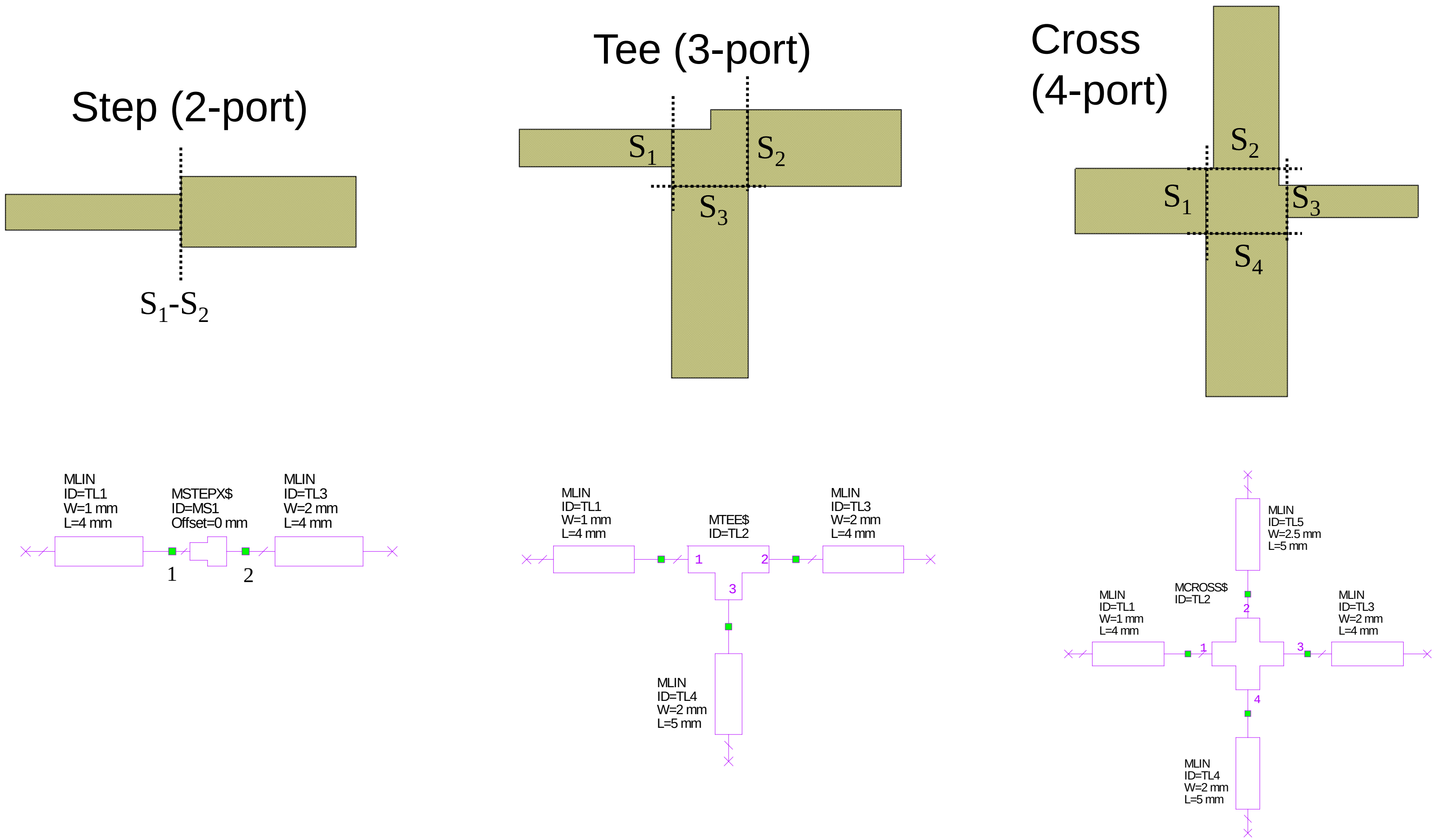
Comparison


0
-10
-20
-30
-40
-50
0.9
0.95
1
1.05
1.1
Frequency (GHz)
Ideal
Microtrip
1
Doppio
Stub
2
Z
IMPED
ID=Z1
R=17 Ohm
X=66 Ohm

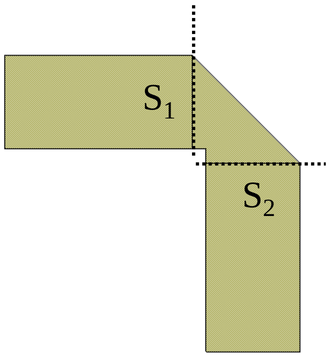
Computing the models of the discontinuities

- Equivalent circuits (lumped and/or distributed elements, numerically evaluated)
- Analytical formulas (simplest cases, low accuracy)
- Electromagnetic analysis (by means of em simulators, very time consuming)
- Different models for the same discontinuity are often available (high accuracy=long computation time)

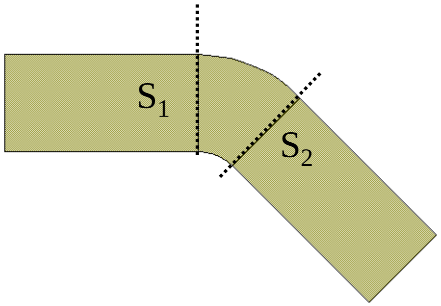
Microstrip discontinuities: junctions



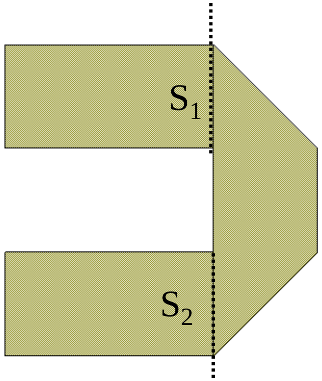
Bend (2-port)



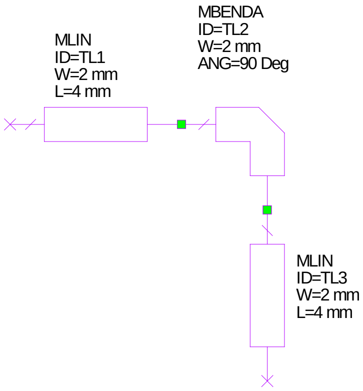
S_1
S_2



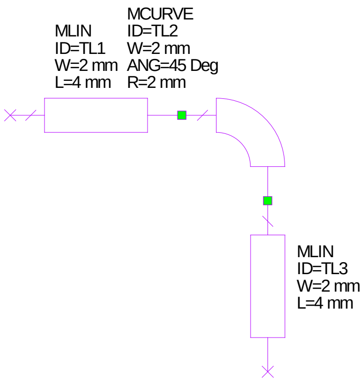
S_1
S_2



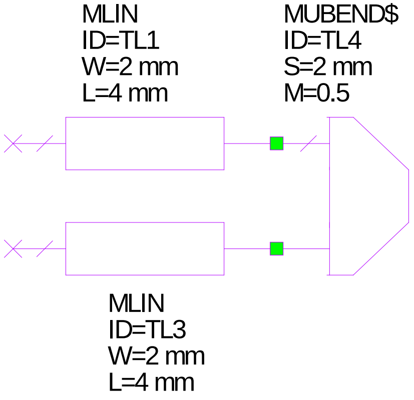
S_1
S_2



MLIN
ID=TL1
W=2 mm
L=4 mm
MBENDA
ID=TL2
W=2 mm
ANG=90 Deg
MLIN
ID=TL3
W=2 mm
L=4 mm



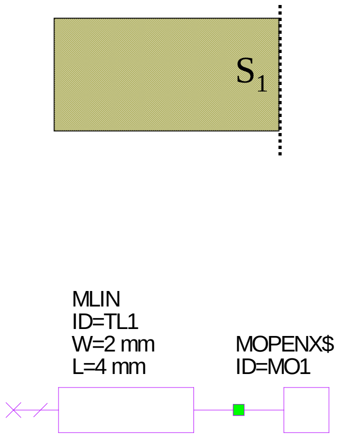
MLIN
ID=TL1
W=2 mm
L=4 mm
MCURVE
ID=TL2
W=2 mm
ANG=45 Deg
R=2 mm
MLIN
ID=TL3
W=2 mm
L=4 mm



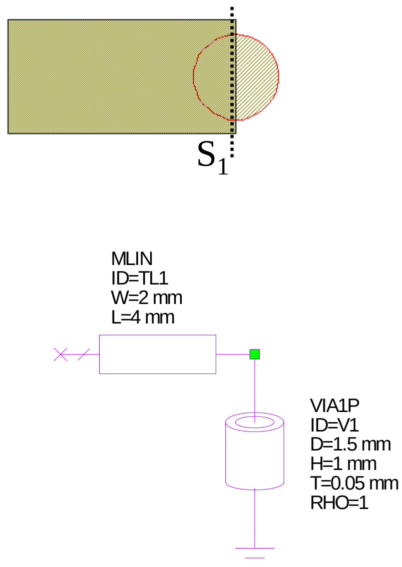
MLIN
ID=TL1
W=2 mm
L=4 mm
MUBEND$
ID=TL4
S=2 mm
M=0.5
MLIN
ID=TL3
W=2 mm
L=4 mm

Terminations (1-port)

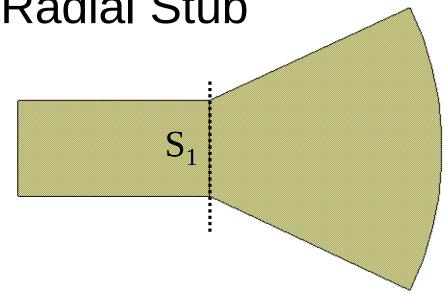
Open end

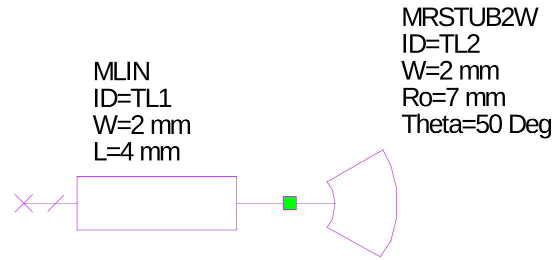


Via hole



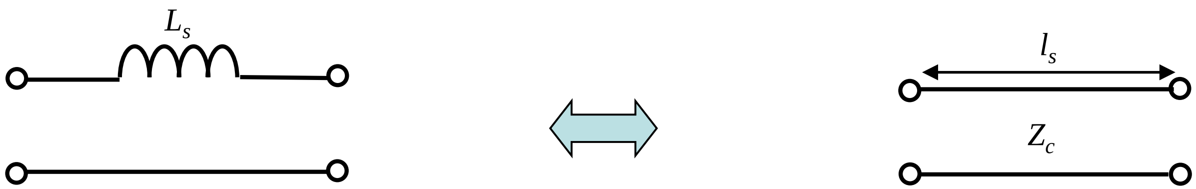
Radial Stub





Pseudo-lumped components

Series inductor



$$Y_L = \left| \begin{matrix} \dfrac{1}{j\omega L_s} & -\dfrac{1}{j\omega L_s} \\ -\dfrac{1}{j\omega L_s} & \dfrac{1}{j\omega L_s} \end{matrix} \right|$$

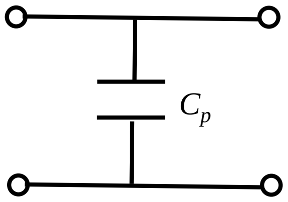
$$Y_S = \left| \begin{matrix} \dfrac{1}{jZ_c \tan(\beta l_s)} & -\dfrac{1}{jZ_c \sin(\beta l_s)} \\ -\dfrac{1}{jZ_c \sin(\beta l_s)} & \dfrac{1}{jZ_c \tan(\beta l_s)} \end{matrix} \right|$$

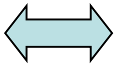
$$Y_L \cong Y_S \quad \Rightarrow \quad \beta l_s \cong 0 \quad \Rightarrow \quad \tan(\beta l_s) \simeq \sin(\beta l_s) \simeq \beta l_s$$

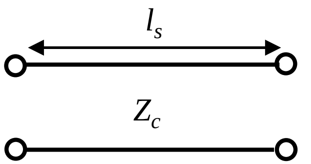
$$\omega L_s \cong Z_c \beta l_s$$

$$\boxed{L_s \cong \frac{Z_c l_s}{v}}$$

Shunt capacitor:







$$Z_C = \begin{vmatrix} \frac{1}{j\omega C_p} & \frac{1}{j\omega C_p} \\ \frac{1}{j\omega C_p} & \frac{1}{j\omega C_p} \end{vmatrix}$$

$$Z_S = \begin{vmatrix} \frac{1}{jY_c \tan(\beta l_s)} & \frac{1}{jY_c \sin(\beta l_s)} \\ \frac{1}{jY_c \sin(\beta l_s)} & \frac{1}{jY_c \tan(\beta l_s)} \end{vmatrix}$$

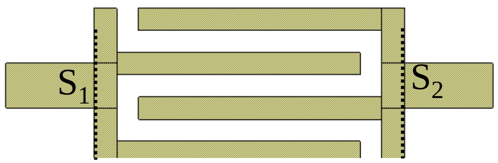
$$Z_L \cong Z_S \quad \Rightarrow \quad \beta l_s \cong 0 \quad \Rightarrow \quad \tan(\beta l_s) \simeq \sin(\beta l_s) \simeq \beta l_s$$

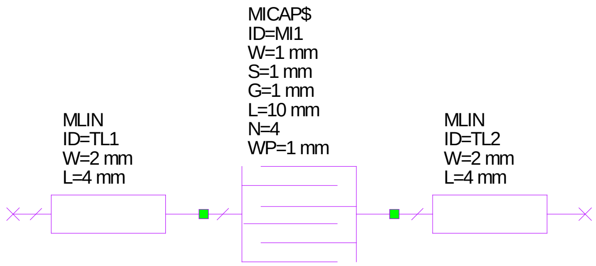
$$\omega C_p \cong Y_c \beta l_s$$

$$\boxed{C_p \cong \frac{Y_c l_s}{v}}$$

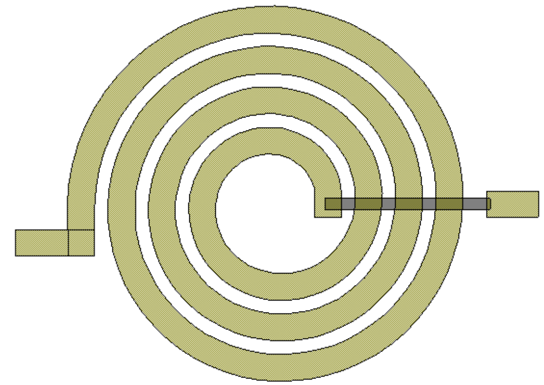
Other components

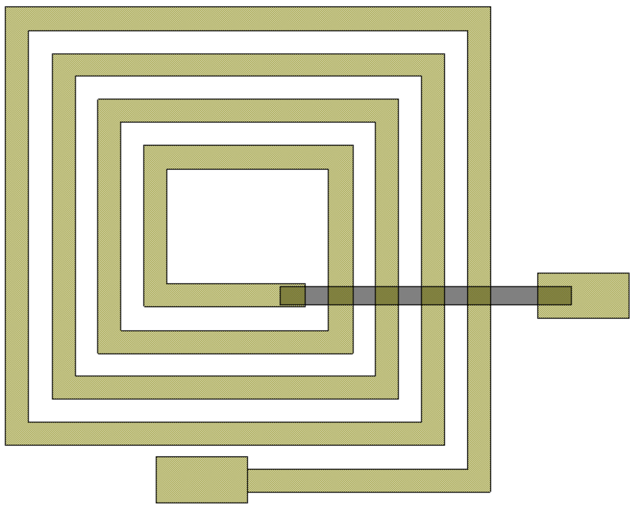
Interdigital capacitor

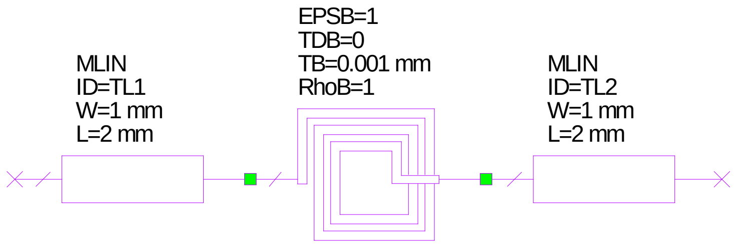




Spiral Inductors







Two-port circuits

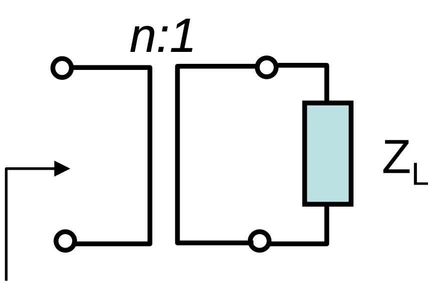
- S matrix for reciprocal, lossless networks:

$$\mathbf{S} \cdot \tilde{\mathbf{S}}^{*} = \mathbf{U} \quad \Rightarrow \quad \begin{aligned} &|S_{11}| = |S_{22}| \\ &|S_{12}|^2 = 1 - |S_{11}|^2 \\ &\phi_{11} + \phi_{22} - 2\phi_{21} = \pm\pi \end{aligned}$$

Only three real parameters are needed for defining S (for instance $|S_{11}|$, ϕ_{11}, ϕ_{22})

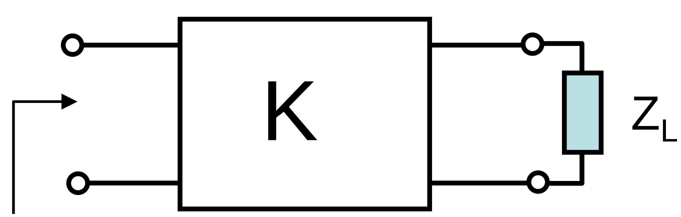
How to get a transformer at microwaves?

- The transformer is a 2-port widely used in many circuital applications
- The ideal component is reciprocal and lossless.
- At microwave frequencies it is however very difficult to be realized. In those application where the goal is the impedance (admittance) scaling, the component employed is the <u>impedance (admittance) inverter</u>:



$$Z_{in} = n^2 Z_L$$

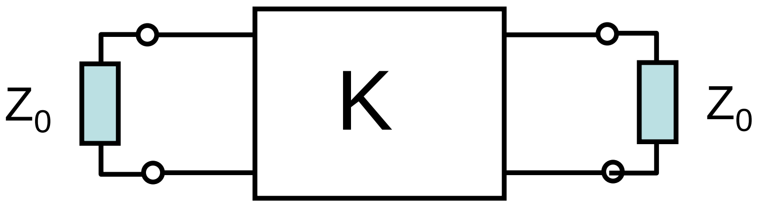
Ideal Transformer



$$Z_{in} = \frac{K^2}{Z_L}$$

Ideal Impedance inverter

S parameters of the impedance inverter



$$S_{11} = S_{22} = \frac{\left(K^2 / Z_0\right) - Z_0}{\left(K^2 / Z_0\right) + Z_0} = \frac{K^2 - Z_0^2}{K^2 + Z_0^2}$$

$$\phi_{11} = \phi_{22} = 0 \ (\pi)$$

$$S_{12} = S_{21} = \pm j2 \frac{K \cdot Z_0}{K^2 + Z_0^2}$$

$$\phi_{12} = \phi_{21} = \pm \frac{\pi}{2}$$

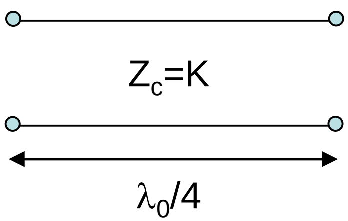
S_{12} has been obtained by imposing the lossless conditions:

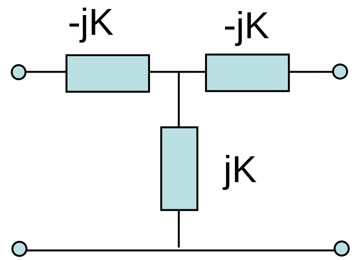
$$\left|S_{12}\right|^2 = 1 - \left|S_{11}\right|^2 = \left[2 \frac{K \cdot Z_0}{K^2 + Z_0^2}\right]^2$$

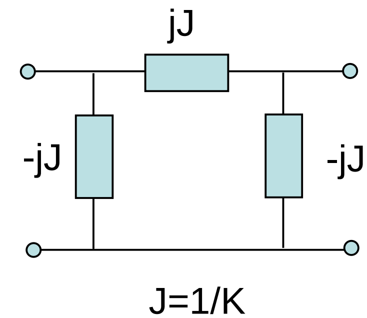
$$\phi_{11} + \phi_{22} - 2\phi_{12} = \pm\pi \quad \Rightarrow \quad \phi_{12} = \phi_{11} \pm \frac{\pi}{2} = \pm \frac{\pi}{2}$$

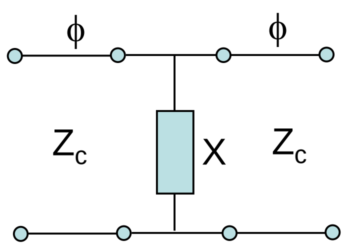
Practical implementation of inverters

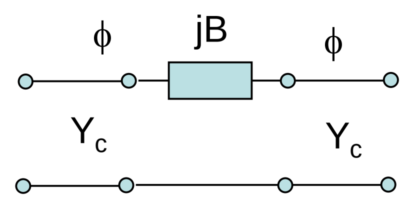
<u>Note</u>: the parameter K of the ideal inverter is independent on frequency. The physical implementation however can only approximate this condition in a limited frequency band









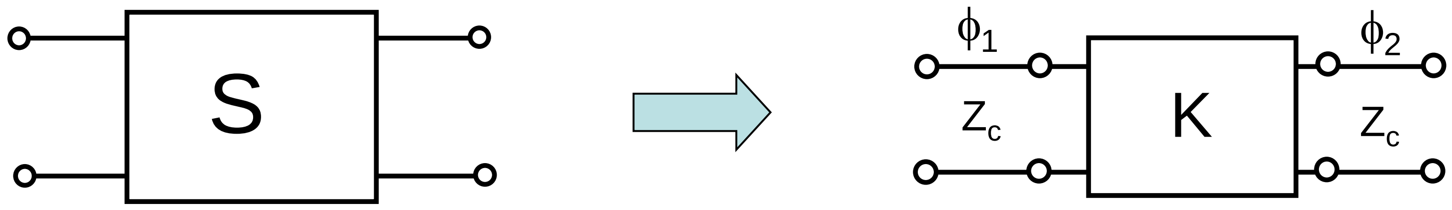


$$\phi = -\frac{1}{2}\tan^{-1}\left(\frac{2X}{Z_c}\right), \qquad X = \frac{K}{1-\left(K/Z_c\right)^2}$$

$$\phi = -\frac{1}{2}\tan^{-1}\left(\frac{2B}{Y_c}\right), \qquad B = \frac{J}{1-\left(J/Y_c\right)^2}$$

<u>Note</u>: for X (B) positive, ϕ is <0 and K/Z_c (J/Y_c) is <1.

Equivalent model for a 2-port lossless circuit



Choice 1 ($|K|<Z_c$):

$$|K| = Z_c \sqrt{\frac{1-|S_{11}|}{1+|S_{11}|}} \qquad \phi_1 = \frac{\pi - \angle S_{11}}{2}, \qquad \phi_2 = \frac{\pi - \angle S_{22}}{2}$$

Choice 2 ($|K|>Z_c$):

$$|K| = Z_c \sqrt{\frac{1+|S_{11}|}{1-|S_{11}|}} \qquad \phi_1 = -\frac{\angle S_{11}}{2}, \qquad \phi_2 = -\frac{\angle S_{22}}{2}$$

$$\angle K = \angle S_{12} - \frac{\pi}{2} + \phi_1 + \phi_2$$ (must be 0 or π)