RF Systems under large signals excitation

- In electric circuits excited by signals with not negligible amplitude, non-linear distortion is produced

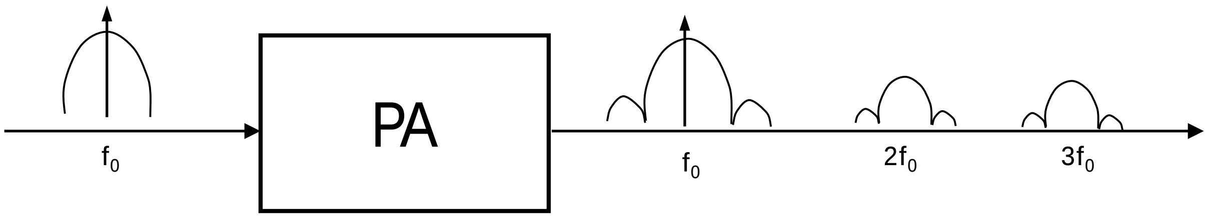
- In power amplifiers (PA) this kind of distortion increases quickly with the power delivered to the load; specs concerning the delivered power must be always accompanied with linearity requirements

- A further issue to be considered when working with this kind of amplifiers is the DC→AC conversion: the higher is the linearity, the lower the efficiency

Non linear networks and RF Signals

Let assume a non linear 2-port (for instance, a PA) excited by a RF signal. The non-linearity produces:

- Deformation and spreading of the output signal spectrum
- Generation of spurious signals at harmonics of the carrier frequency (f_0)



Analysis of non linear network/systems

- For these networks the superposition principle cannot be applied. The analysis in the frequency domain becomes meaningless.
- In fact, a sinusoidal excitation produces a response which may contain, in general, all the harmonics of the input sinusoid.
- The solution for this class of networks is obtained by integrating in the time domain the system of differential equations which describe the relationships between voltages and currents imposed by the components (efficient numerical methods are employed in commercial software like SPICE)
- 2-port circuits can no longer be characterized by the concept of Transfer Function. In case of memoryless networks, an input-output characteristic function can be used for modeling the device (very coarse model of the physical reality because memory is discarded)

2-port Memoryless Network

In a memoryless network the output voltage at time t_0 depends only on the input voltage at the same time (in networks with memory also depends on the time before t_0).

In this case, the input-output relationship can be analytically described by means of a power series expansion:

$$v_o = a_0 + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + ...$$

NOTE: The memoryless model is an ideal abstraction. The real networks including reactive components (capacitors, inductors, transmission lines) have always memory.

Average power and Peak power

Signal composed by *n* tones of same amplitude and different frequencies (not harmonically related):

$$V = A\cos(\omega_1 t) + A\cos(\omega_2 t) + A\cos(\omega_3 t) + ... + A\cos(\omega_N t)$$

Let assume P_T as the power in each tone (proportional to A^2)

The <u>average power</u> is the sum of the power of the tones:

$$P_a = N \cdot kA^2 = N \cdot P_T$$

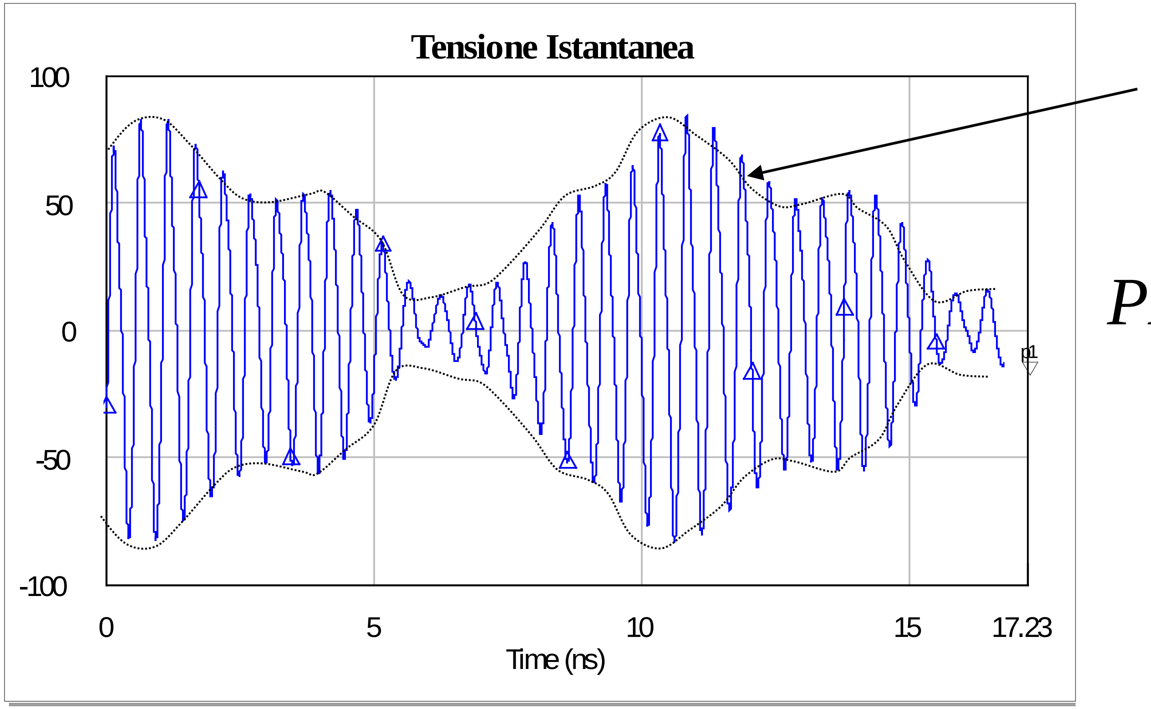
The <u>peak power</u> is obtained at the time all the sinusoids reach their maximum (all phasors aligned). It is then proportional to $(N \cdot A)^2$:

$$P_p = k(N \cdot A)^2 = N^2 \cdot P_T$$

The ratio $F = P_p/P_a$ is defined <u>peak factor</u> of the signal (it is equal to N if the tones have the same amplitude)

Peak Envelope Power (PEP)

It is the instantaneous power delivered at the peak of the modulated signal:



$$PEP \doteq \frac{1}{2}\left|\max\left(V_{env}(t)\right)\right|^2$$

Sinusoidal excitation of a memoryless 2-port

$v_i = A\cos(\omega t)$ is the network excitation; at output it is obtained:

$$v_o = b_0 + b_1\cos(\omega t) + b_2\cos(2\omega t) + b_3\cos(3\omega t) + \ldots.$$

With:

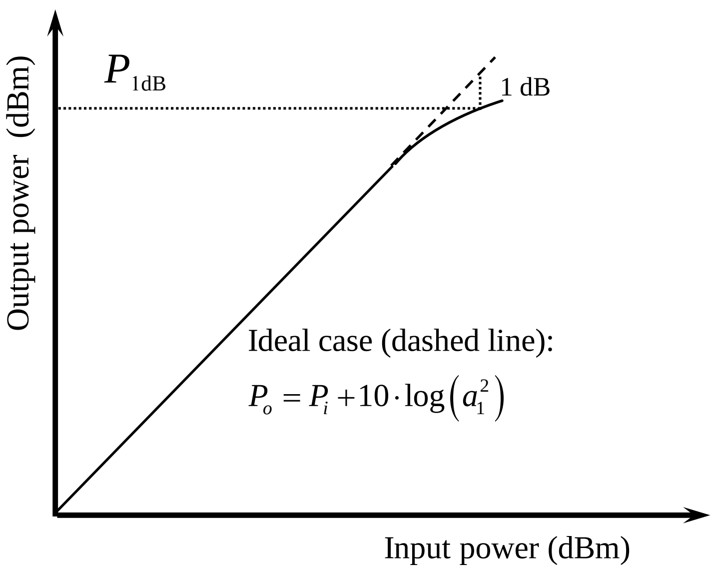
$$b_0 = \frac{1}{2}a_2A^2, \qquad b_1 = a_1A + \frac{3}{4}a_3A^3, \qquad b_2 = \frac{1}{2}a_2A^2, \qquad b_3 = \frac{1}{4}a_3A^3$$

b_1 represents the amplitude of the output signal at ω. As can be seen, it does not depend linearly on A (there is the non-linear term $(3/4)a_3A^3$). In case of the amplifiers a_3 è typically negative, then there a **compression** of the output signal.

1 dB compression power (P_{1dB})

Let consider a PA excited with a sinusoid at frequency f_0.
Assuming the memoryless model defined by the power series expansion seen before, the power in dBm at input and output of the PA at the frequency f_0 are given by :

$$P_{in,dBm} = 10 \cdot \log\left\{\left(\frac{A}{\sqrt{2}}\right)^2 \frac{1000}{R}\right\}, \qquad P_{o,dBm} = 10 \cdot \log\left\{\left(\frac{a_1 A + (3/4) a_3 A^3}{\sqrt{2}}\right)^2 \frac{1000}{R}\right\}$$



At P_{1dB} the output power is reduced of 1 dB with respect the one given by the linear term.
With R=50Ω it has:

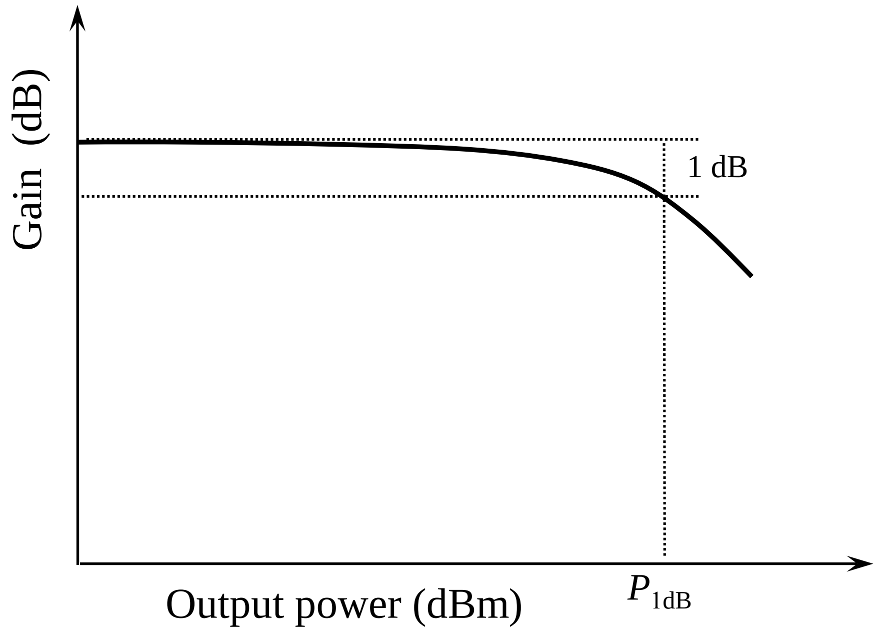
$$P_{1dB} = 10 \log \frac{a_1^3}{|a_3|} + 0.62 \ \text{dBm}$$

Gain vs. output power

The gain of the PA referred to the fundamental frequency is a function of A^2:

$$G = P_{o,dBm} - P_{in,dBm} = 20 \cdot \log\left\{a_1 + (3/4) a_3 A^2\right\}$$

In practice G is often graphically represented as a function of the output power $P_{o,dBm}$:



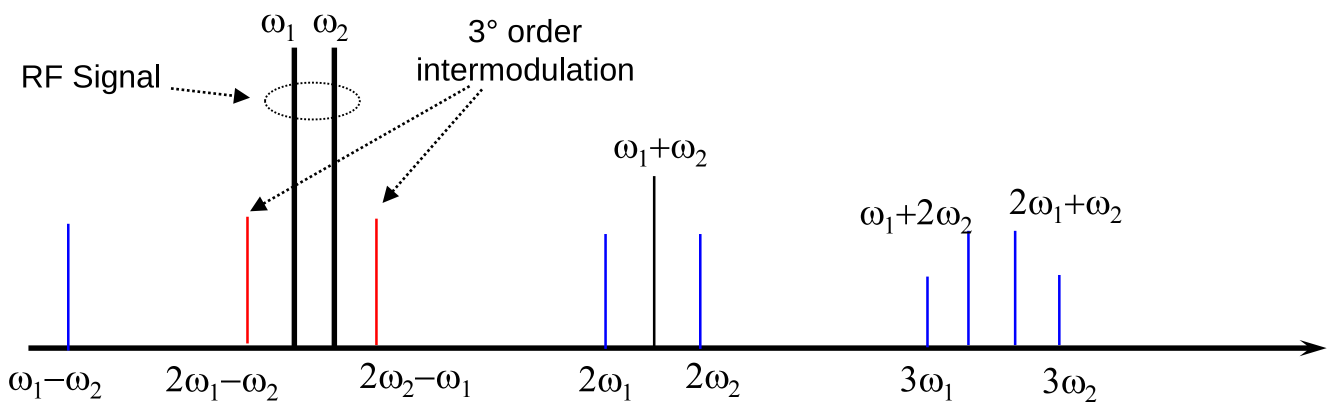
AM-to-AM Conversion

Two-tone Excitation

When a non-linear device is excited by several tones the phenomenon of **intermodulation** is produced. As a results, in addition to the harmonics of each tone, also new tones at the frequencies $|\pm n\omega_1 \pm m\omega_2 \pm k\omega_3 \pm \ldots|$ are generated at output. In case of two tones with the same amplitude A and frequencies f_1 and f_2, the following tones are generated by a non-linearity of 3° order :

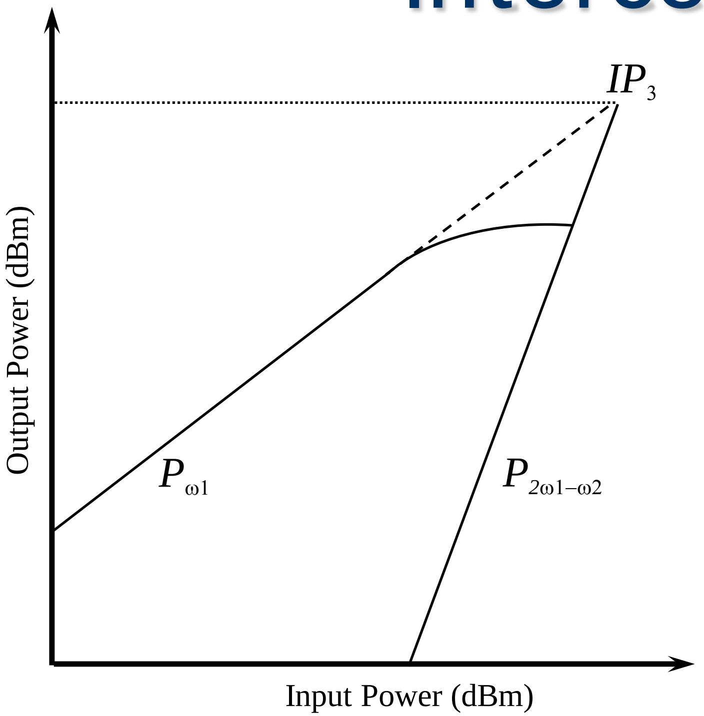
Frequency	Order	Expression
f_1, f_2	1	$a_1A+(9/4)a_3A^3$
$(f_1-f_2), (f_1+f_2)$	2	a_2A^2
$(2f_1-f_2), (2f_2-f_1)$	3	$(3/4)a_3A^3$
$(2f_1+f_2), (2f_2+f_1)$	3	$(3/4)a_3A^3$
$(2f_1, 2f_2)$	2	$(1/2)a_2A^2$
$(3f_1, 3f_2)$	3	$(1/4)a_3A^3$

Spectrum



Two tones very close in frequency are the most simple model of a RF signal (the envelope is periodic with period $2/\Delta f$). The intermodulation tones closest to the RF signal (and then the most harmful ones) are the 3th order tones $(2\omega_1-\omega_2)$, $(2\omega_2-\omega_1)$; their amplitude is an index of the non-linearity of the amplifier.

Intercept Point (IP_3)



$$P_{\omega_1} = 10\cdot\log\left\{\left(\frac{a_1A+(9/4)a_3A^3}{\sqrt{2}}\right)^2\frac{1000}{R}\right\} \cong 10\cdot\log\left\{\left(\frac{a_1A}{\sqrt{2}}\right)^2\frac{1000}{R}\right\}$$

$$P_{2\omega_1-\omega_2} = 10\cdot\log\left\{\left(\frac{(3/4)a_3A^3}{\sqrt{2}}\right)^2\frac{1000}{R}\right\}$$

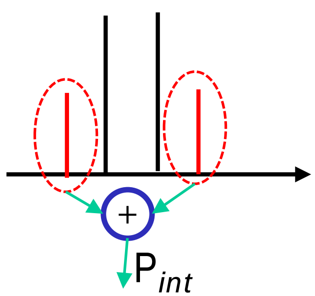
$$IP_3 = 10\cdot\log\left\{\left(\frac{2}{3}\frac{a_1^3}{|a_3|}\right)\frac{1000}{R}\right\} \quad \rightarrow$$

$$\rightarrow \quad IP_3 = 10\cdot\log\frac{a_1^3}{|a_3|}+11.25 \ \text{dBm} \quad (R=50\Omega)$$

$$\boxed{P_{2\omega_2-\omega_1} \simeq 3P_{\omega_1} - 2IP_3} \qquad P_{\omega_1} << IP_3$$

$$\boxed{P_{int} \simeq 3P_m - 2IP_3 - 6}$$

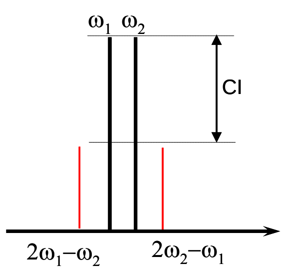
P_{int} =($P_{2\omega 2-\omega 1}$+3) is the mean power in the intermodulation lines



Assuming P_m =($P_{\omega 1}$+3) the average power of the two-tone signal and CI the ratio between the signal power and intermodulation power, from the previous equation it is obtained:

$$P_{\omega_1} - P_{2\omega_2-\omega_1} = -2P_{\omega_1} + 2IP_3 = 2IP_3 - 2P_m + 6$$

$$CI_{dB} \simeq 2IP_3 - 2P_m + 6$$



Relationship between P_{1dB} e IP_3

IP_3 e P_{1dB} can be expressed, with R=50 Ohm, as:

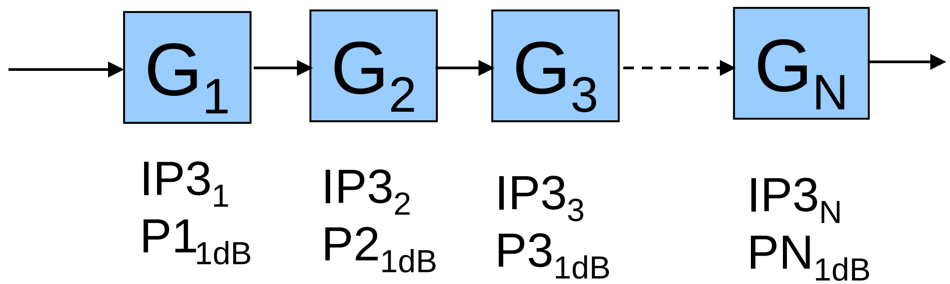
$$IP_3 = 10 \cdot \log \frac{a_1^3}{|a_3|} + 11.25 \;\; \text{dBm}, \quad P_{1dB} = 10 \log \frac{a_1^3}{|a_3|} + 0.62 \;\; \text{dBm}$$

Combining the two relations:

$$IP_3 = P_{1dB} + \Delta_p \qquad (\Delta_p = 10.63 \text{ dB})$$

The reported value of Δ_p (10.63) refers to an ideal device with a 3th order non-linearity. In real devices its value may be more or less different (typically in the range 9-11 dBm)

P_{1dB} and IP_3 of cascaded of 2-ports



$$\left(\frac{1}{IP3}\right)^2 = \left(\frac{1}{G_2G_3\cdots G_N IP3_1}\right)^2 + \left(\frac{1}{G_3G_4\cdots G_N IP3_2}\right)^2 + \left(\frac{1}{G_4G_5\cdots G_N IP3_3}\right)^2 + \ldots\left(\frac{1}{IP3_n}\right)^2$$

$$\left(\frac{1}{P_{1dB}}\right)^2 = \left(\frac{1}{G_2G_3\cdots G_N P1_{1dB}}\right)^2 + \left(\frac{1}{G_3G_4\cdots G_N P2_{1dB}}\right)^2 + \left(\frac{1}{G_4G_5\cdots G_N P3_{1dB}}\right)^2 + \ldots\left(\frac{1}{PN_{1dB}}\right)^2$$

These formulas have been computed by summing the power of the distortion products. The sum of the voltages gives a similar expression without the square on all the terms

Specifying the output power of a PA

When assigning the output power of a PA, also the corresponding level of nonlinear distortion it must be specified.
The 2-tone signal is often using as reference excitation. In this case it is usual to specify the maximum PEP power for which the **CI is equal to 30 dB**:

$$PEP = P_m + 3 \qquad \text{(2-tone signal)}$$

$$30 = 2IP_3 - 2(PEP - 3) + 6 \qquad \Rightarrow \qquad PEP = IP_3 - 9$$

IP_3 and P_{1dB} are related by: $IP_3 = P_{1dB} + \Delta_p$

Then:

$$PEP = P_{1dB} + (\Delta_P - 9) \qquad \text{for CI=30 dB}$$

NOTE: Δ_p is 10.63 dB for a 3° order characteristic. However real microwave semiconductor devices (MESFET, HEMT, LDMOS) do not fit exactly this assumption. A more accurate value for Δ_p is 9 dB, from which $PEP \approx P_{1dB}$ for CI=30 dB.

Backoff definition

The backoff of a PA is the difference in dB between the output power at P_{1dB} (1 dB compression point) and the average output power. From the previous equations, it can be obtained:

$$BO \approx \frac{CI|}{2} - \Delta_p - 3$$

(2 tone signal)

For instance, for CI=30 dB and Δ_p=10.63 dB, it is obtained: BO≈1.4 dB.

NOTE: As said before, microwave semiconductor devices do not present exactly a 3° order characteristic. With Δ_p = 9 dB, it has BO ≈ 3 dB for CI=30 dB.

PEP and Average Power relationship

N-tone signal

$$P_m = N \cdot k \cdot V^2, \qquad PEP = k \cdot \left(N \cdot V\right)^2$$

$$PF = \frac{PEP}{P_m} = N$$

Modulated carriers

PF depends on the type of modulation. In the practice it is statistically defined (Probability to exceed a given value of PF).

In case of several modulated carriers grouped together (OFDM), the value of PF computed with the previous formula is a rough estimation of the real one.

For a given P_m, IMD increases with PF

DC-AC Efficiency Conversion

A PA can be considered as a converter of electric energy from DC to RF. The figure of merit to be utilized for characterizing this conversion is the **Power Added Efficiency (PAE)**:

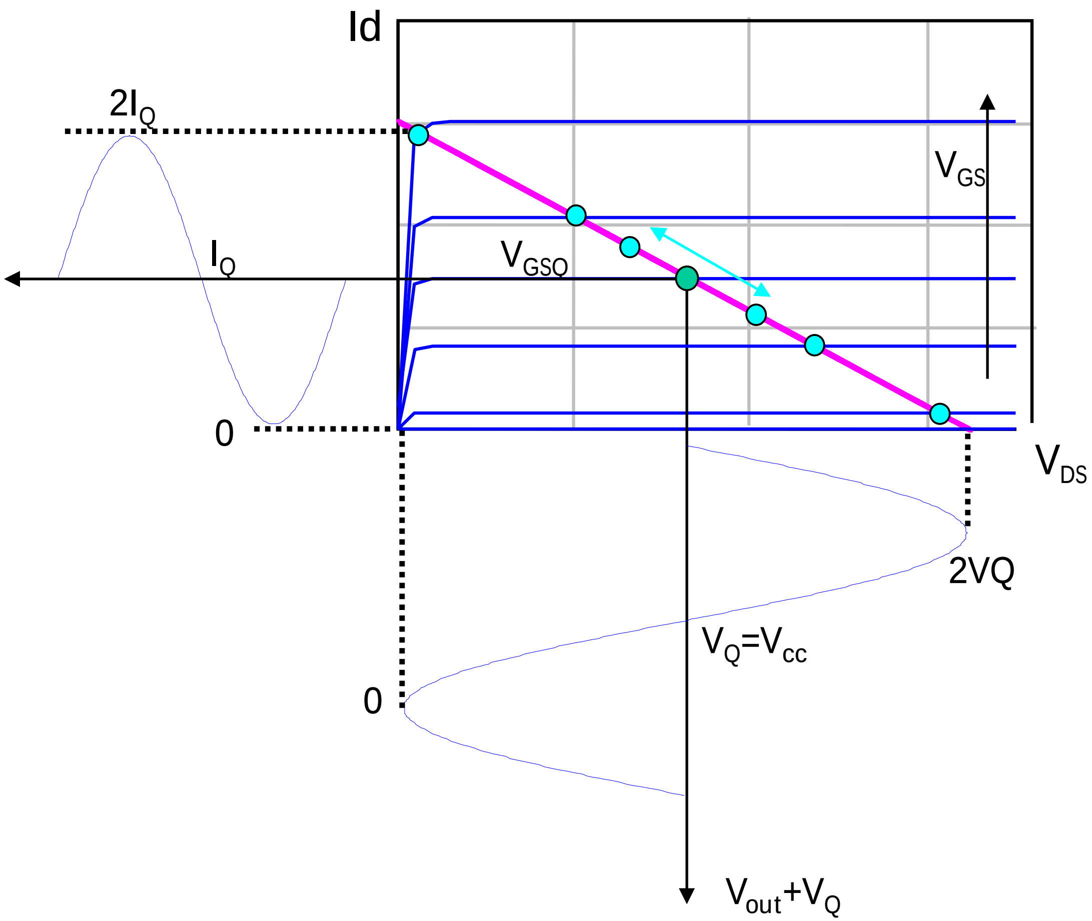
$$\mathrm{PAE} = 100\,\frac{(\text{RF Power at load -RF Power at input})}{\text{DC Power}}\ \%$$

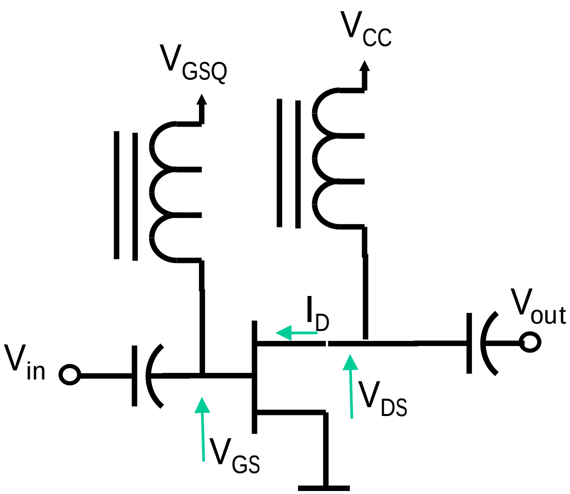
The DC power is delivered by the biasing sources of PA (mostly the drain bias current).

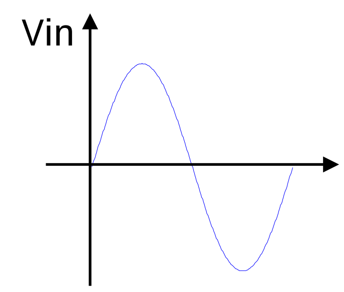
The PAE is strictly related to the operating class of the PA, then to its linearity features:

To increase the linearity , the PAE must be decreased

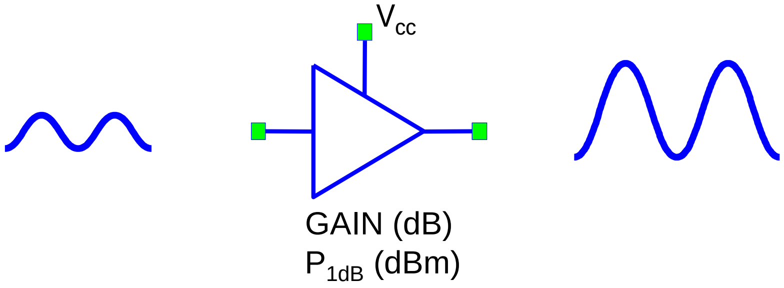
Classe A Bias







PAE for Class A Power Amplifiers



Without signal:

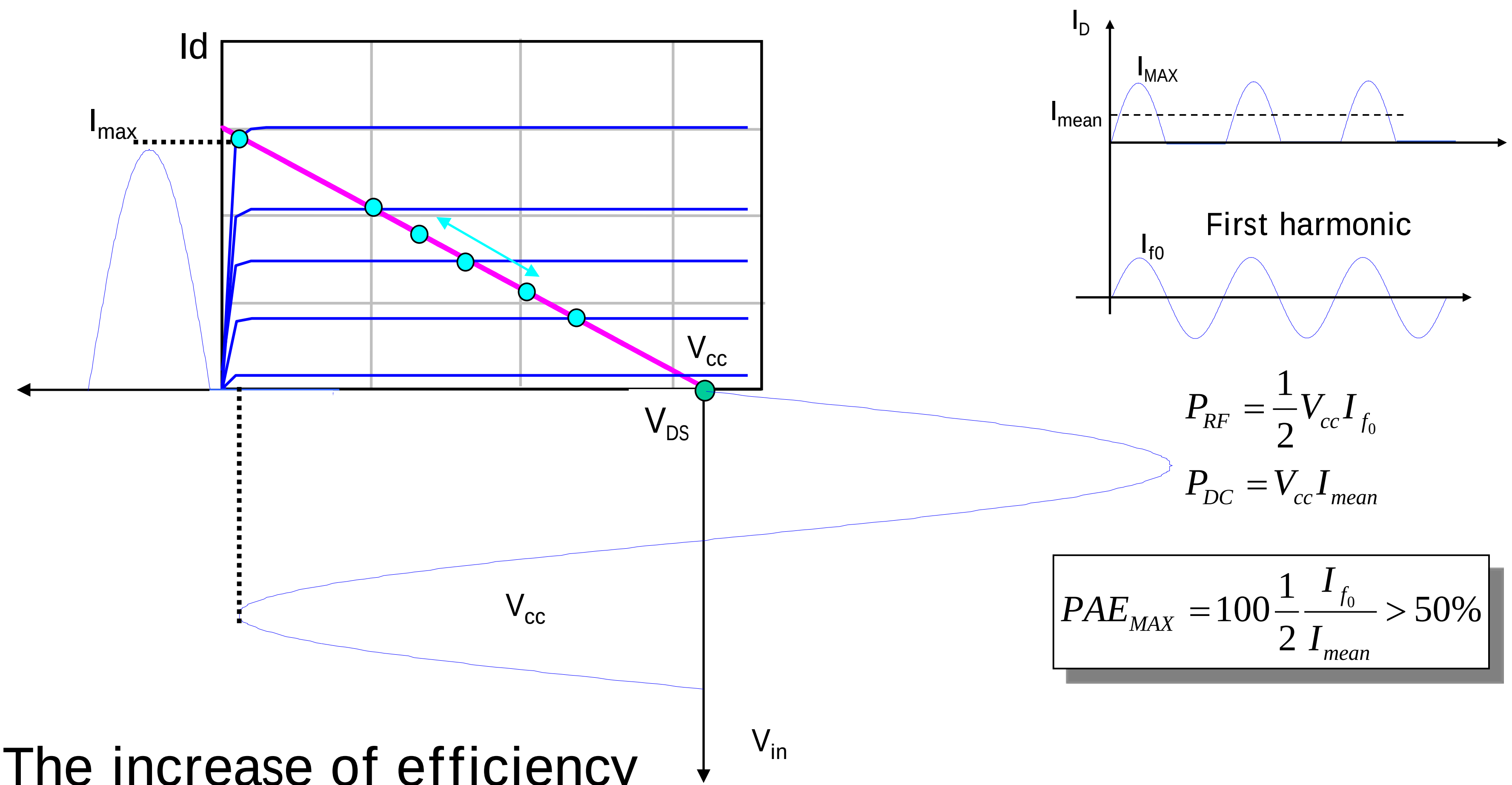
$V_{DS}=V_{cc}$, $I_D = I_Q$

Assuming to bias the active device at the centre of the linear zone, the maximum variations of voltage and current at the output of PA result $\pm V_{cc}$ and $\pm I_Q$; then:

$$P_{RF} = \frac{1}{2} V_{cc} I_Q \quad , \quad P_{DC} = V_{cc} I_Q \quad \Rightarrow \quad PAE_{MAX} = 100 \frac{1}{2} = 50\%$$

In real cases, the maximum range of di V e I at output must be much less than V_{cc} e I_Q in order to remain in the linear zone; PAE is then much less than 50% (typical values with 1 tone excitation are in the order of 25-35%).

Classe B Bias



$$P_{RF} = \frac{1}{2} V_{cc} I_{f_0}$$

$$P_{DC} = V_{cc} I_{mean}$$

$$PAE_{MAX} = 100 \frac{1}{2} \frac{I_{f_0}}{I_{mean}} > 50\%$$

The increase of efficiency is paid with the increase of distortion

Practical RF PA

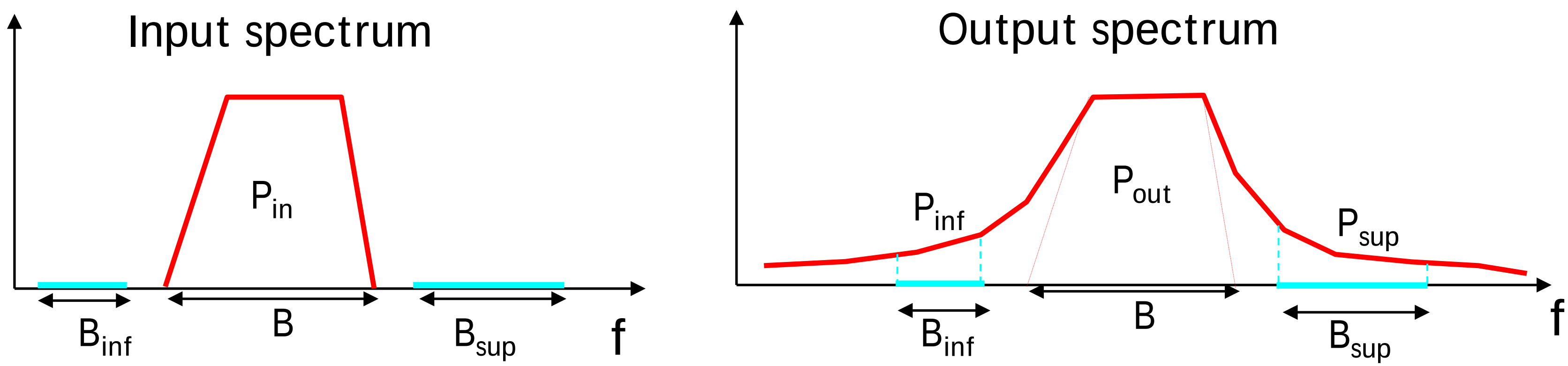
The power amplifier used in RF communication systems works in a intermediate class between A and B (called ***class AB***).
This allows to get the best compromise between distortion and efficiency.

Note: The efficiency, like the distortion, depends on the specific signal. The results shown before refer to sinusoids.
In general, <u>increasing the peak factor, the efficiency descreases</u> (for the same distortion)

Real world signals

- The 2-tone test is useful to characterize the non linearity of a device but cannot be used for predicting the actual distortion obtained when a non linear device is excited by an arbitrary signal
- In order to characterize the effect of the non linearity under arbitrary excitation further figures of merit have been introduced
- The exciting signal is in general defined by its spectrum, which is assumed to be limited to the band B
- The effect of distortion is to generate new frequency components, both in the band of the signal and outside the band
- We can represent the effects of distortion by comparing the power of the generated distortion components with the power of the amplified signal

Adjacent channel power ratio (ACPR)



$$ACPR_{inf} = \frac{P_{out}}{P_{inf}} \qquad ACPR_{sup} = \frac{P_{out}}{P_{sup}}$$

ACPR is a measure of the degree of signal spreading into adjacent channels, caused by nonlinearities of the device.

It represents a constraint on the “degradation” produced by the non linear device operating in channel B on the signals occupying the adjacent channels.

NOTE: The position of adjacent channels must be specified together the ACPR requirements

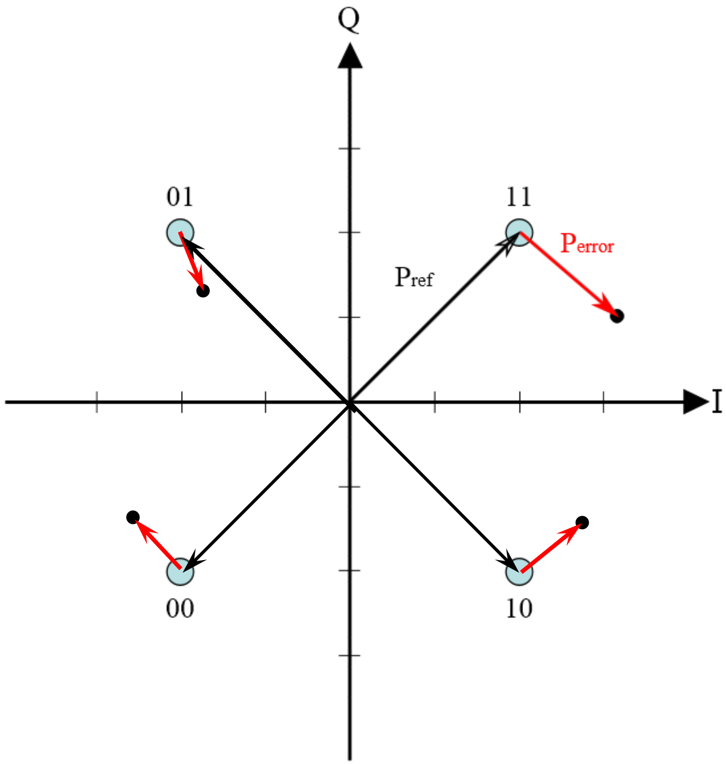
Degradation of digital signals quality

In digital communications the received signal can be represented by a vector which may assume a limited number of positions. The set of possible positions is defined the ideal constellation points.

Whatever disturb overlapping the signal produces a displacement between the ideal constellation and the actual received signal. Such displacement then represents an indicator of the signal quality degradation due to various sources (including the non linearity).

The parameter used for quantifying the above displacement is the **error vector magnitude (EVM)**

Error Vector Magnitude (EVM)



Constellation of 4-QAM modulation

Given the error vectors between the ideal constellation and the degraded signal, EVM is defined as follows:

$$EVM(dB) = 10\log_{10}\frac{P_{error}}{P_{reference}}$$

$$EVM(\%) = 100\cdot\sqrt{\frac{P_{error}}{P_{reference}}}$$

with:

P_{error}=average power of the error vectors

$P_{reference}$ = average power of the reference vectors

Non-linear systems with memory

In non linear systems, memory means that the response to an excitation at a given time *t* depends also on the time history prior to *t*. As a result a simple polynomial model is no more adequate to obtain even a rough estimation of the response.
In case of sinusoidal excitation, the observation of the phase of the output sinusoid at the same frequency vs. the output power can be used to point out the presence of memory. This dependence defines a good model for representing weak memory effects (**AM to PM conversion**)

Amplitude and phase response vs. power

Consider a RF signal with amplitude modulation only:

$$s_{RF}(t) = V_M(t)\cos(\omega_0 t)$$

Passing this signal through a memoryless non linear system, the output response at the fundamental frequency ω_0 is amplitude-modulated by the non-linearity $f(\cdot)$. The phase however doesn't depend on V_M:

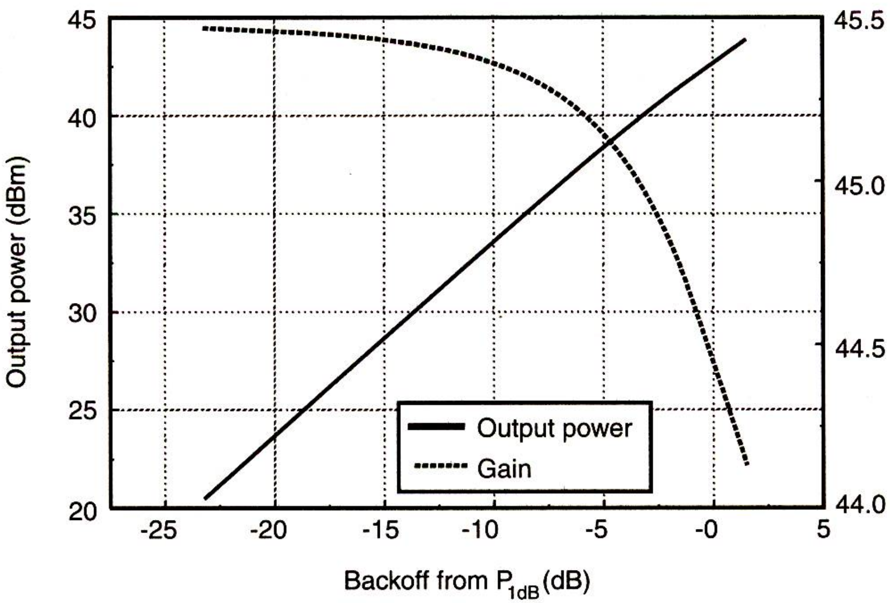
$$S_{out}(\omega_0) \equiv f\left(V_M(t)\right)\cos(\omega_0 t + \phi_0)$$

Real systems exhibit instead a dependence of the phase on the exciting amplitude V_M.

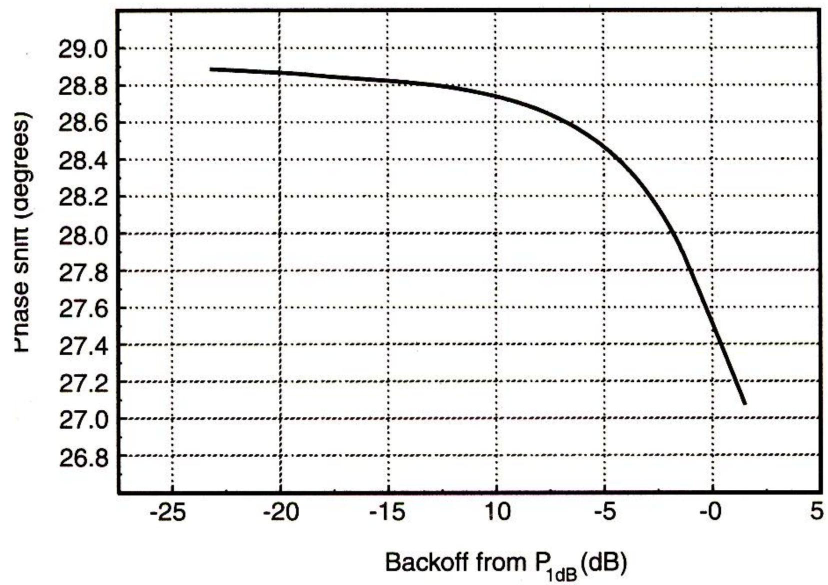
This dependence is generally described by the graph of the output phase vs. the output power (AM-to-PM conversion):

$$\phi_o(\omega_0) \equiv g(P_{out})$$

Example of AM-AM and AM-PM curves of a real class A Power Amplifier



AM-AM Conversion

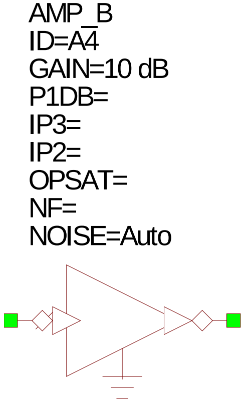


AM-PM Conversion

The x-axis reports the output power as the backoff from P_{1dB}

Behavioral models of real amplifiers

All CAD tools (both systems and circuital) offer various kind of behavioral model for amplifiers. These models allow to represent the effect of noise (through the Noise Figure) and the non linearity (by means P_{1dB} and IP_3). The most sophisticated models represent the effect of output saturation (above P1db) and the weak memory effect (AM-PM conversion).



VSS Model of the real amplifier

The nonlinearity is approximated using a fifth-order polynomial. P_{1dB}, IP_2 and IP_3 are implemented. Also the saturation power is specified (above this point P_{out} remains constant). AM-PM conversion is not implemented in the basic model

Non linear behavior of Mixers

Also practical mixers exhibit a non-linear behavior. Under large RF input signal, the spectrum of the converted signal at output is distorted (in a similar way to what happens in amplifiers).

A polynomial model can be used for characterization, like the one for memory-less PA:

$$v_o = a_0 + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + a_4 v_i^4 ....$$

In this case the excitation v_i is the sum of RF signal + the LO: $V_{RF}\cos(\omega_{RF}t)+V_{OL}\cos(\omega_{OL}t)$.
It can be verified that a pure 2-order characteristic does not produce distortion at $f_{RF} \pm f_{OL}$, which is instead obtained from the even terms of order ≥4

Characterization of Mixers under large signal excitation

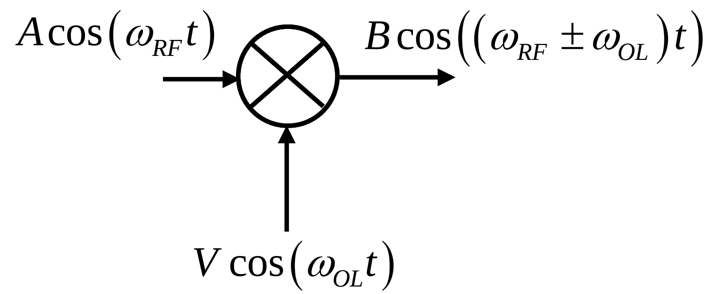
Characterization is made with the same parameters used for PA (**P_{1dB} and IP_3**), with some differences.

First, the input/output model refers to different frequencies (f_{RF} at input and $f_{(RF \pm OL)}$ at output)

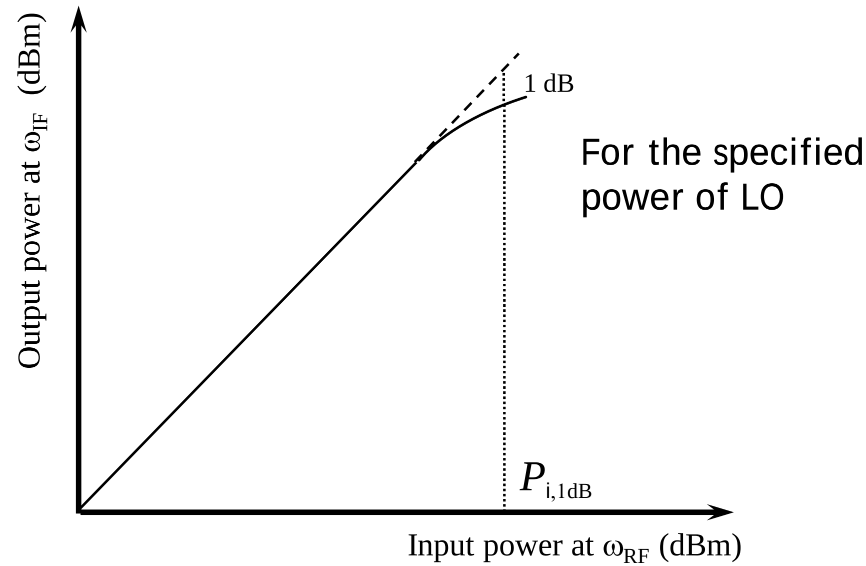
Second, these parameters depend on the power of local oscillator (P_{OL}): typically there is a level of P_{OL} above which P_{1dB} and IP_3 remain constant.

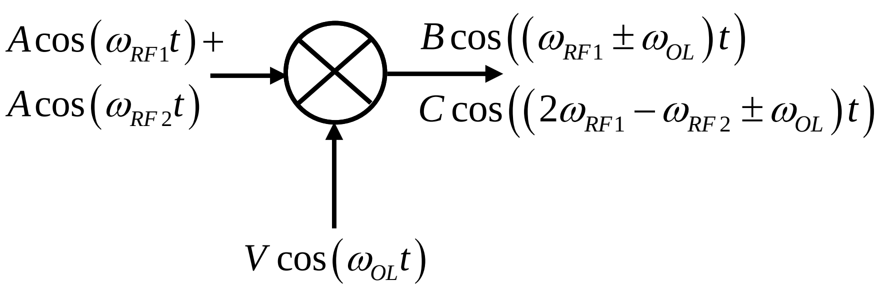
Finally, being the most used mixers at microwave frequencies passive devices, the value of P_{1dB} and IP_3 are **referred at input** (and not at output as in PA):

$$P_{i,1dB} = P_{1dB} - G_{dB}, \qquad IIP_3 = IP_3 - G_{dB} \qquad\qquad G_{dB} = \text{Linear Gain}$$

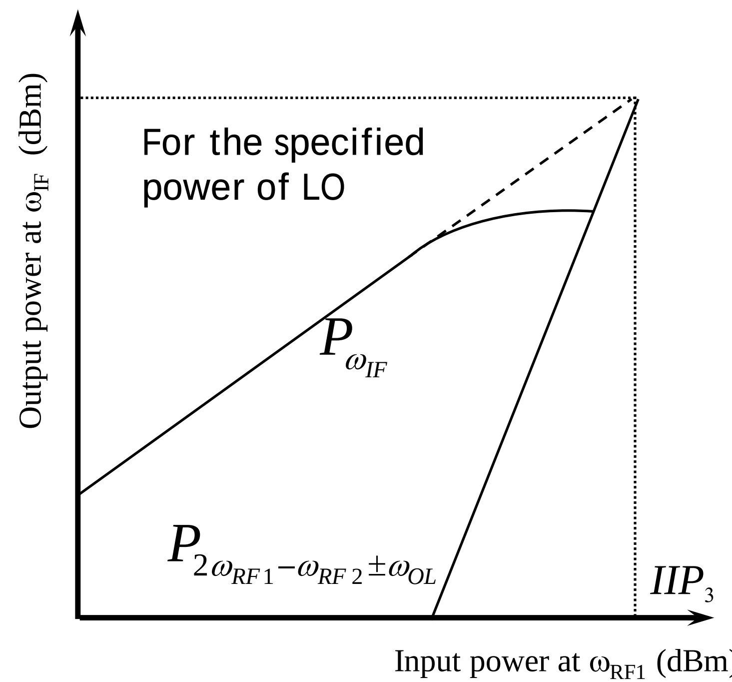


Single Tone





Two-Tone



Harmonic mixing

The non linear characteristic of a real mixer determines at output all the frequencies resulting from the following equation:

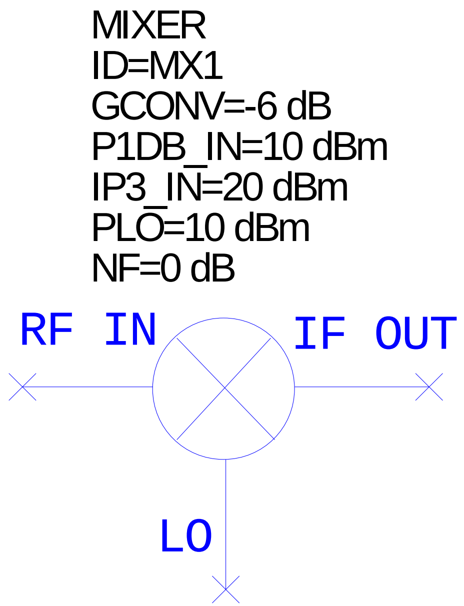
$$f_{out} = \left| \pm m \cdot f_{RF} \pm n \cdot f_{OL} \right| \qquad n, m = 1, 2, \ldots.$$

In the practice, suitable solutions are devised in order to reduce as much as possible the unwanted products (taking into account that only those with |m|+|n|<~5 are really relevant).

Some particular type of mixers (called **sub-harmonic mixers**) exploit one of the harmonic of f_{OL} for obtaining the desired f_{out}:

$$f_{out} = f_{RF} \pm n \cdot f_{OL} \qquad \text{n}=3-5$$

Behavioral models of mixers



Systems CAD require the translated frequency (IF) to be specified.

Main parameters:
P1dB (1 dB Compression Point referred to RF IN)
IP3 (3° order intercept power referred to RF IN)
GCONV (small signal gain conversion)
PLO (LO power for specified conversion gain)
NF (SSB Noise Figure) or TSSB (SSB equivalent temperature)