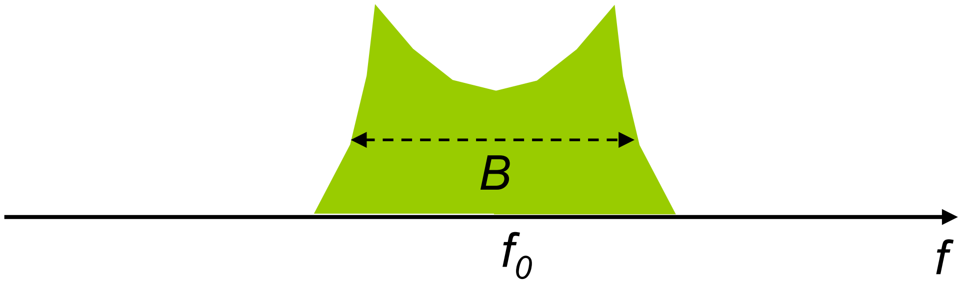
RF Signal



$$\frac{B}{f_0} << 1$$

Analytic Formulation:

$$s_{RF}(t) = V_M(t)\cos\left(\omega_0 t + \Phi(t)\right) = V_I(t)\cos\left(\omega_0 t\right) + V_Q(t)\sin\left(\omega_0 t\right)$$

$V_M(t)$, $\Phi(t)$: Magnitude and phase (instantaneous)
$V_I(t)$, $V_Q(t)$: In phase and Quadrature signal components

Information associated to RF Signal is carried by these signals only (carrier does not convey any information)

Complex envelop representation

$$s_{RF}(t) = V_I(t)\cos(\omega_0 t) + V_Q(t)\sin(\omega_0 t) = V_M(t)\cos(\omega_0 t + \Phi(t))$$

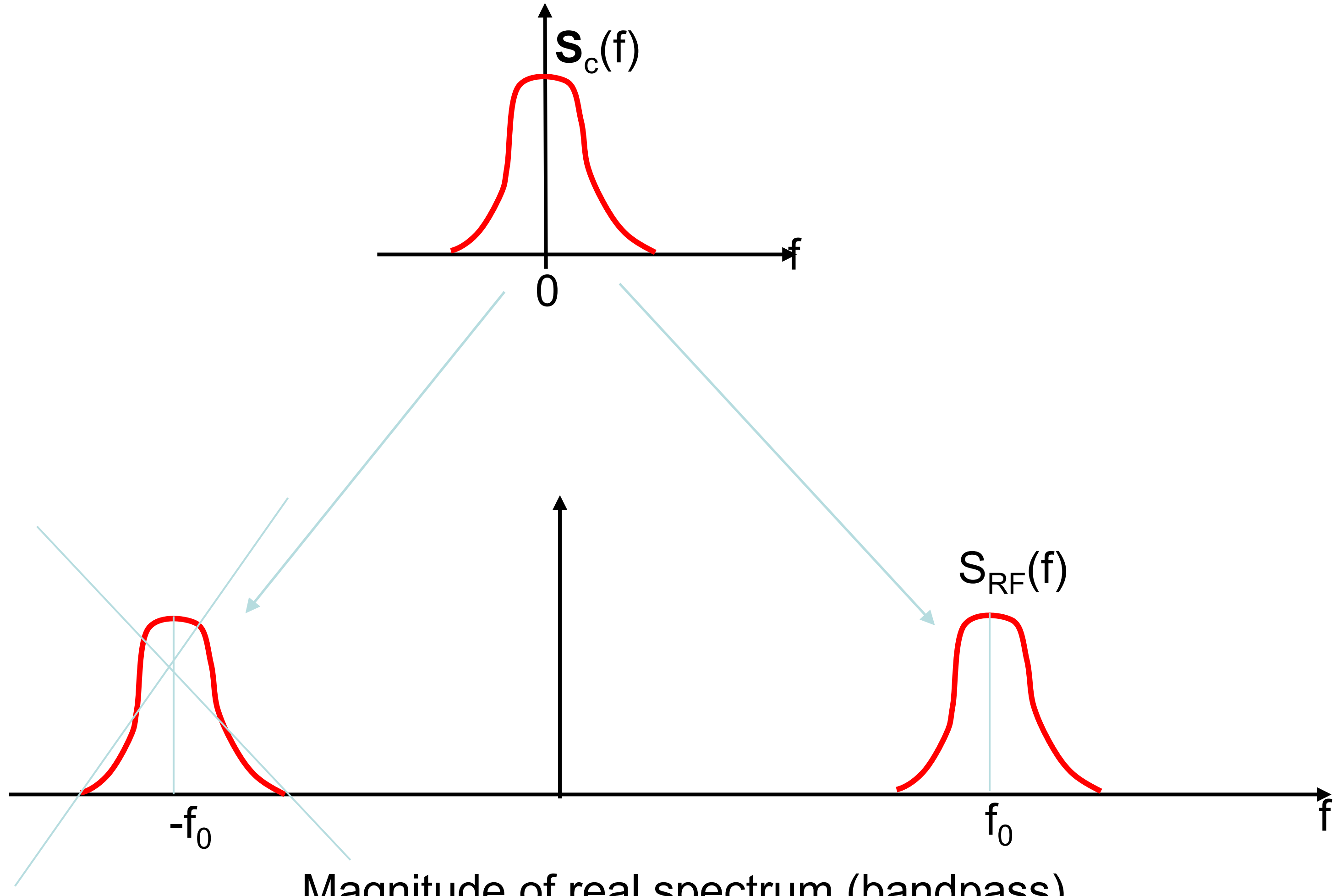
Phasor notation:

$$s_{RF}(t) = \mathrm{Re}\left\{V_M(t)\, e^{j\Phi(t)} e^{j\omega_0 t}\right\} = \mathrm{Re}\left\{\mathbf{V}_C(t)\, e^{j\omega_0 t}\right\}$$

$\mathbf{V}_c(t)$ is a complex function of time called complex envelope.
In case of RF signals the spectrum of $\mathbf{V}_c(t)$ is around 0 (base band) and contains all the information carried by the modulated signal.
The spectrum of $\mathbf{V}_c(t)$ is related to the spectrum $S_{RF}(f)$ of the real signal ($s_{RF}(t)$) as follows:

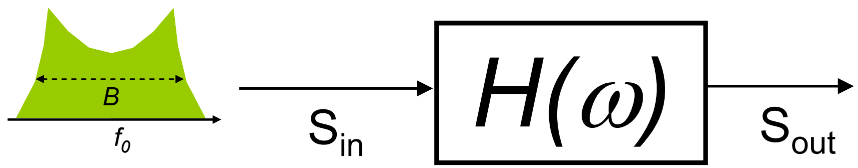
$$S_{RF}(f) = \mathbb{F}\left\{\mathrm{Re}\left\{\mathbf{V}_C(t)\, e^{j\omega_0 t}\right\}\right\} = \mathbf{S}_C(f - f_0) + \mathbf{S}_C^*(f + f_0)$$

Magnitude of complex envelope spectrum (lowpass)



Magnitude of real spectrum (bandpass)

RF Signal across a linear 2-port: no distortion condition



Given the input RF signal in the frequency domain (S_{in}), the output signal is given by $S_{out}=H(\omega)\cdot S_{in}$.
To avoid a degradation of the output signal (=loss of information), the transfer function must satisfy the following conditions:

$$\left|H(\omega)\right| = \mathrm{cost}, \quad \angle H(\omega) \doteq \omega$$

These conditions are requested in the signal band (bandpass characteristic).

Baseband equivalent 2-port

The response of a 2-port to a RF signal excitation can be obtained from the complex envelope signal. Defining the baseband as $\Omega=\omega-\omega_0$ and $C_{in}(\Omega)$ the spectrum of the complex signal, we have:

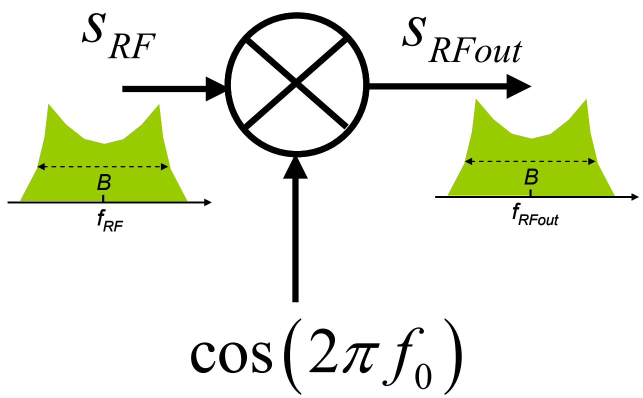
$$C_{out}(\Omega) = H(\omega-\omega_0)C_{in}(\Omega)$$

Note that $H(\omega-\omega_0)$ is in this case a lowpass function (it is centered at zero frequency).
Note also that the use of complex envelope equivalence is meaningful when the transient response is exhausted (steady-state condition)

Advantage of using the complex envelope representation

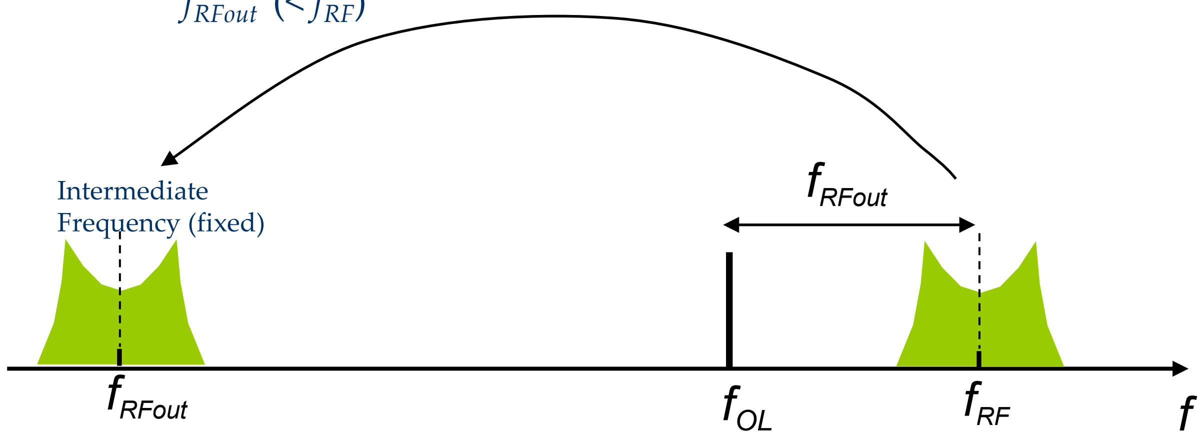
- In a RF signal all the information is associated to the complex envelope while the carrier don’t carry information.
- Moreover the minimum time step associated to the carrier is several order of magnitude smaller than the one associated to the envelope.
- Using CAD techniques for analyzing RF systems, a huge computation time is requested if the analysis is performed directly in the bandpass domain
- Using equivalent lowpass networks and complex envelope equivalent signals the computation time is strongly reduced without affecting the accuracy of the steady-state response

Principle of Frequency Conversion



$$f_{RFout} = \left| f_{RF} \pm f_0 \right|$$

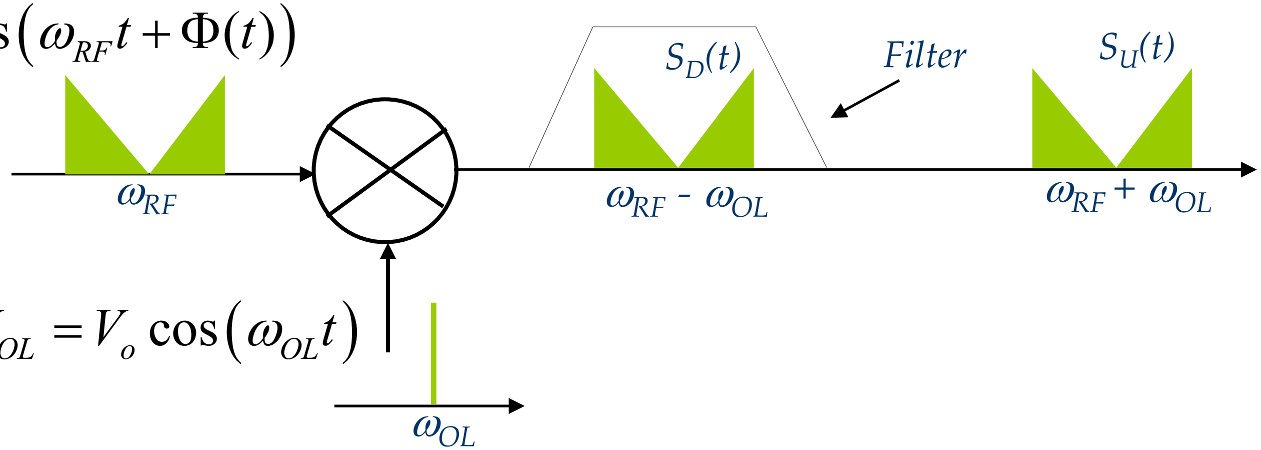
Frequency Down Conversion (from f_{RF} to f_{RFout} ($< f_{RF}$))



How can we get the frequency conversion?

Ideally: By multiplying the RF signal by a sinusoidal tone

$$S_{RF} = V_M\left(t\right)\cos\left(\omega_{RF}t + \Phi(t)\right)$$

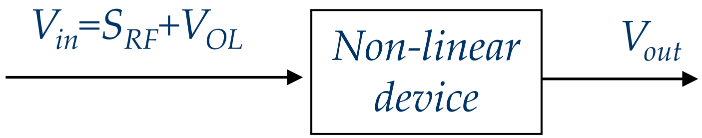


$$V_{OL} = V_o\cos\left(\omega_{OL}t\right)$$

$$S_{OUT} = S_{RF}\cdot V_{OL} = V_M\left(t\right)V_0\cos\left(\omega_{RF}t + \phi(t)\right)\cos\left(\omega_{OL}t\right) = S_U\left(t\right) + S_D\left(t\right)$$

$$\left\{\frac{V_0\cdot V_M\left(t\right)}{2}\cos\left(\left(\omega_{RF} - \omega_{OL}\right)t + \phi(t)\right)\right\} + \left\{\frac{V_0\cdot V_M\left(t\right)}{2}V_M\left(t\right)\cos\left(\left(\omega_{RF} + \omega_{OL}\right)t + \phi(t)\right)\right\}$$

In RF practice: exploiting the non-linearity of a device

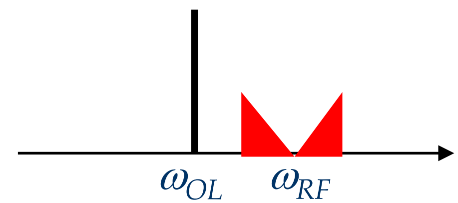


$$V_{out} = a_1 V_{in} + a_2 V_{in}^2 + \ldots$$

$$S_{RF} = V_M(t)\cos(\omega_{RF} t)$$

$$V_{OL} = V_o \cos(\omega_{OL} t)$$

$$V_{in} = V_M \cos(\omega_{RF} t) + V_o \cos(\omega_{OL} t)$$



$$V_{out} = \frac{a_2}{2}\left(V_0 + V_M\right) + a_1\left(V_0 \cos(\omega_{OL} t) + V_M \cos(\omega_{RF} t)\right) +$$

$$+\frac{a_2}{2}\left(V_0^2 \cos(2\omega_{OL} t) + V_M^2 \cos(2\omega_{RF} t)\right) + a_2\left(2V_0 V_M \cos(\omega_{OL} t)\cdot\cos(\omega_{RF} t)\right) + \ldots$$

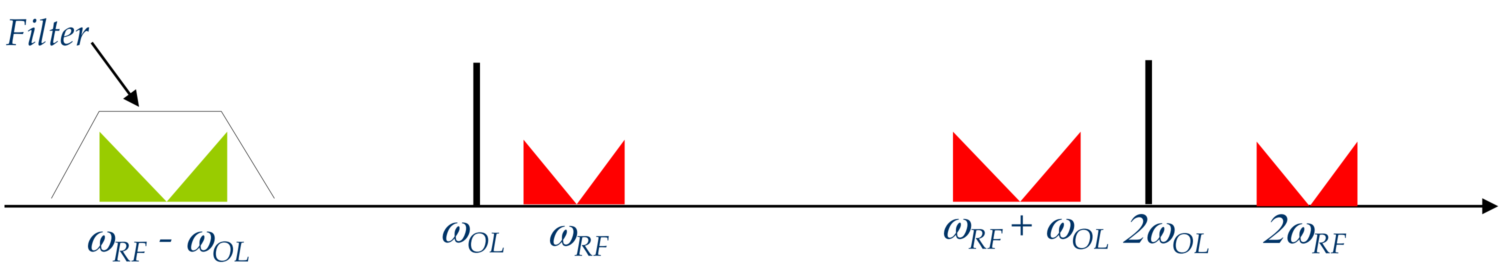
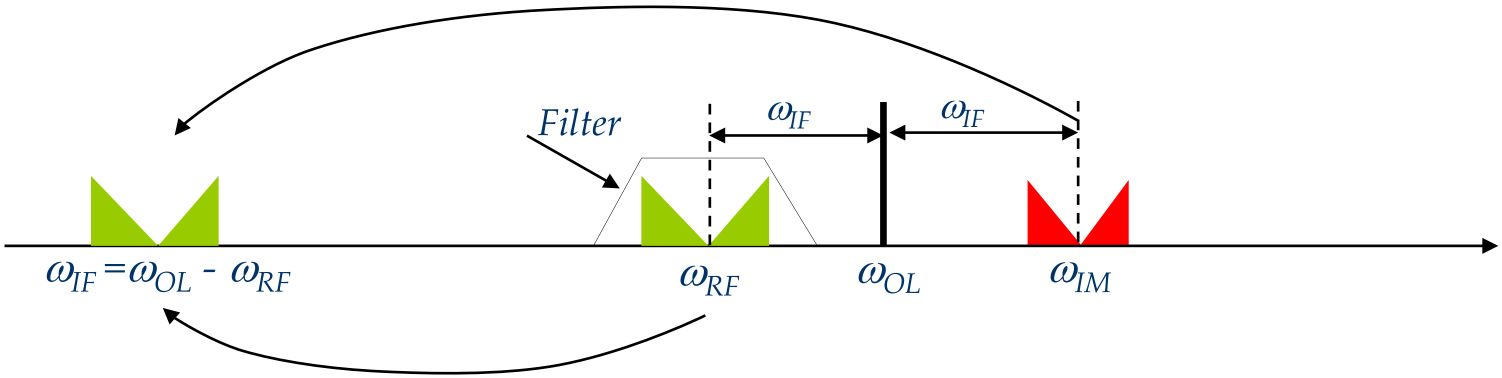


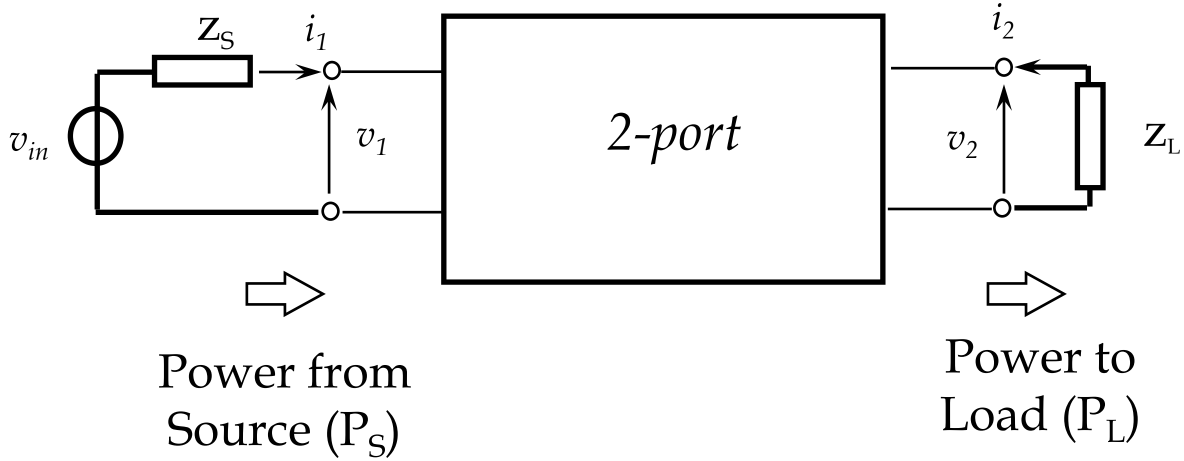
Image Frequency



The image frequency is separated 2ω_{IF} from ω_{RF}

The passband filter at ω_{RF} is needed for eliminating the image band (signal and noise)

Power Transfer from Source to Load (Amplifier)



Problem: there are several ways to define P_S and P_L!

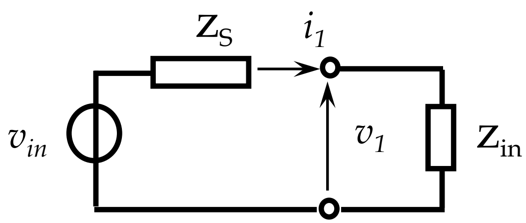
Definition of Power Gain G_P

$$P_S = P_{in} = \frac{1}{2}\mathrm{Re}\left(v_1 \cdot i_1^*\right) = \text{Power Entering the 2-port}$$

$$P_L = P_{out} = \text{Power delivered to Load} = -\frac{1}{2}\mathrm{Re}\left(v_2 \cdot i_2^*\right)$$

$$\boxed{Gain = G_P = \frac{P_{out}}{P_{in}}}$$

BUT: P_{in} may be very small, making G_p unreasonable large. In fact:

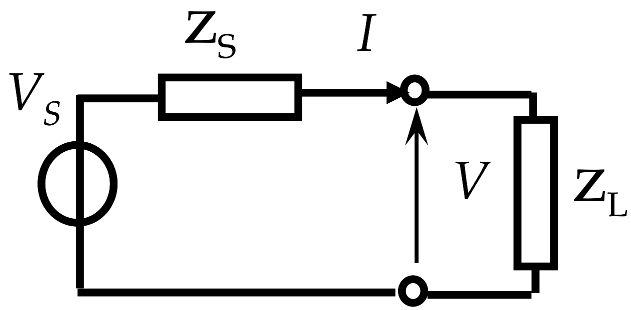


$$v_1 = v_{in}\frac{Z_{in}}{Z_{in}+Z_S} \quad \Rightarrow \quad v_1 \text{ goes to 0 for } Z_{in} << Z_S$$

$$i_1 = \frac{v_{in}}{Z_{in}+Z_S} \quad \Rightarrow \quad i_1 \text{ goes to 0 for } Z_{in} >> Z_S$$

G_p tends to infinite when the 2-port presents a very small (or very large) input impedance (with respect to Z_s)!

Available power from a source and conjugate matching



The Available power of a source is the maximum power the source can deliver to a load. This happens when the load impedance is equal to the conjugate of the source impedance (**conjugate matching**)

$$P_{av} = \frac{1}{8} \frac{\left|V_S\right|^2}{\mathrm{Re}\left(Z_S\right)}$$

$$Z_L = Z_S^* \quad \Rightarrow \quad P_L = P_{av}$$

Definition of Transducer Power Gain G_T

$$P_S = \text{ Available Power from Source} = P_{AV,in} = \frac{1}{8}\frac{|v_{in}|^2}{\mathrm{Re}(Z_S)}$$

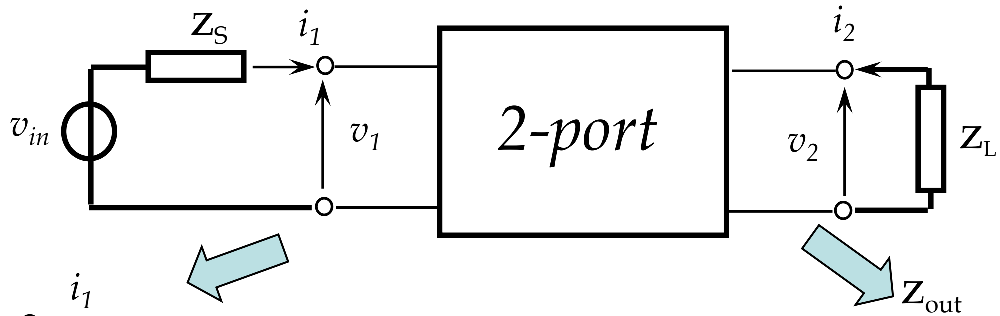
$$P_L = \text{ Power delivered to Load} = P_{out} = \frac{1}{2}\frac{|v_2|^2}{\mathrm{Re}(Z_L)}$$

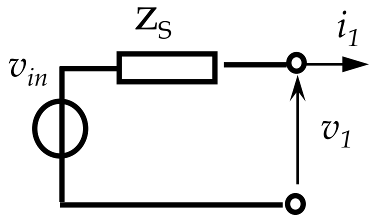
$$\boxed{Transducer\ \ Power\ \ Gain = G_T = \frac{P_{out}}{P_{AV,in}} = 4\frac{\mathrm{Re}(Z_S)}{\mathrm{Re}(Z_L)}\left|\frac{v_2}{v_{in}}\right|^2}$$

<u>Meaning of G_T</u>: It represents the power delivered to the load normalized to the maximum power deliverable from the source.

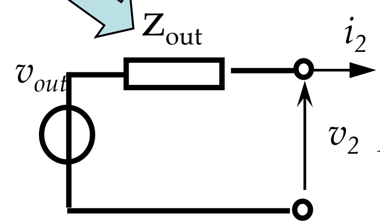
G_T is the power gain definition more suitable for describing the attitude of a device to transfer power to a load (with respect to the use of a passive matching network connecting the source to the load)

Another Power Gain definition: G_A





$$P_{AV,in} = \frac{1}{8}\frac{|v_{in}|^2}{\mathrm{Re}(Z_S)}$$



$$P_{AV,out} = \frac{1}{8}\frac{|v_{out}|^2}{\mathrm{Re}(Z_{out})}$$

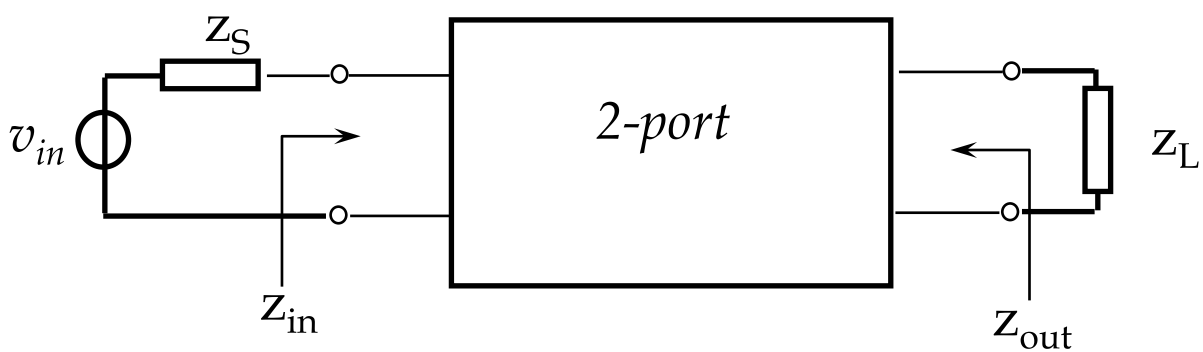
P_S=Available power from source = $P_{AV,\,in}$

P_L = Available power at output =$P_{AV,\,out}$

$$\boxed{Available\ Power\ Gain = G_A = \frac{P_{AV,out}}{P_{AV,in}} = \frac{\mathrm{Re}(Z_S)}{\mathrm{Re}(Z_{out})}\left|\frac{v_{out}}{v_{in}}\right|^2}$$

Specific features of Power Gain definitions

1. The value of G_T is always smaller than or equal to the value of G_P and G_A (for a given 2-port with assigned Z_L e Z_S)
2. There is a particular condition where all the three gains assume the same value: it happens when both input and output are conjugate matched:

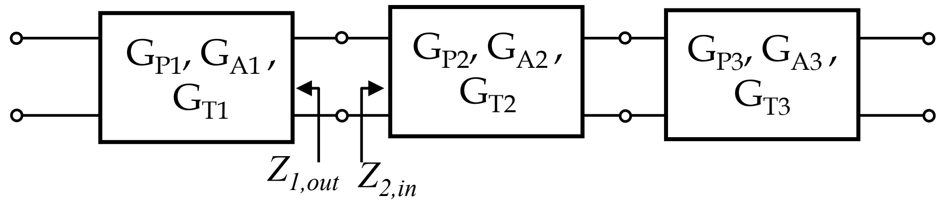


Conjugate Match at input: $Z_{in} = Z_S^* \Rightarrow G_T = G_P$

Conjugate Match at output: $Z_{out} = Z_L^* \Rightarrow G_T = G_A$

If both these conditions are verified: $G_T = G_P = G_A$

Cascade of 2-ports



The overall G_P and G_A is simply the product of the 2-ports gains:

$$G_P = G_{P1}G_{P2}G_{P3}, \qquad G_A = G_{A1}G_{A2}G_{A3}$$

This does not apply, in general, for the overall transducer G_T (it depends on the in/out matching of the 2-ports).
However, if all the 2-ports are matched, also the total G_T is given by the product of the 2-ports G_{Tk}.

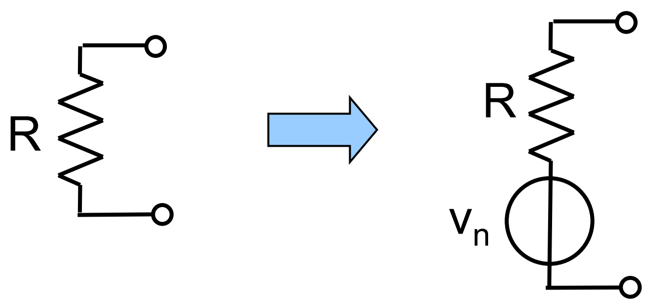
$$G_T = G_{T1}G_{T2}G_{T3} \quad \text{for} \quad Z_{k+1,\text{in}} = Z^*_{k,out}$$

Noise generated by a resistor at T (°K)

The **Available Noise Power** produced by a resistor is given by:

$$P_N = K \cdot T \cdot \Delta F$$

K= Boltzman's Constant, T=Temperature (°K), ΔF: Bandwidth



$$v_n = \sqrt{4K \cdot T \cdot R \cdot \Delta F}$$

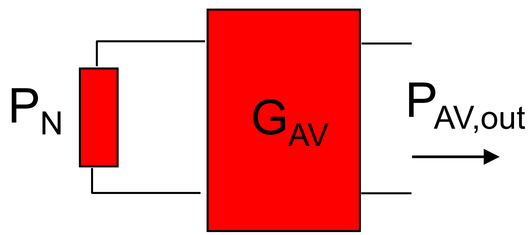
RMS noise voltage

$$P_N = \frac{1}{4}\frac{v_n^2}{R} = K \cdot T \cdot \Delta F$$

Available Noise Power: Maximum power delivered to a load

Characterization of Noisy 2-ports

Available noise power at the output of a noiseless two-port:

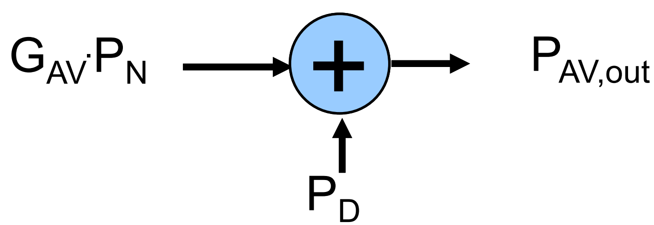


$$P_{AV,out}=G_{AV}\cdot P_{AV,in}=G_{AV}\cdot P_N=G_{AV}\cdot K\cdot T\cdot \Delta F$$

G_{AV}=<u>Available Power Gain</u>

If the 2-port produces noise (P_D), the output power becomes

$$P_{AV,out}= G_{AV}P_N + P_D=K\cdot T\cdot \Delta F\cdot G_{AV}+P_D$$

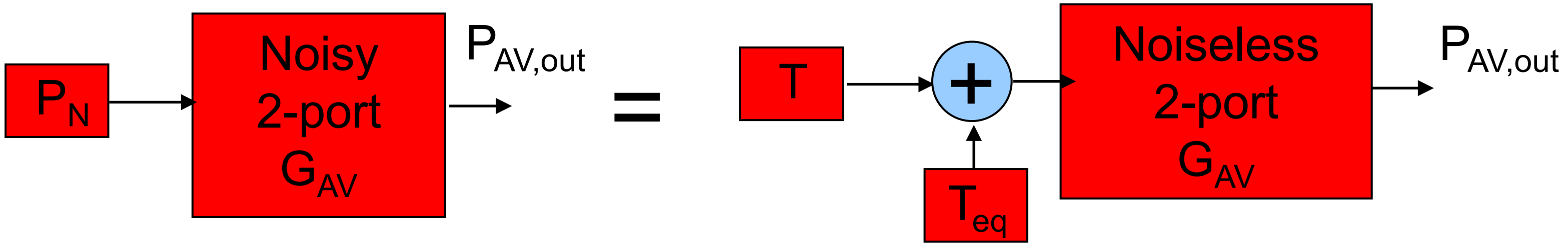


Equivalent Noise Temperature at input

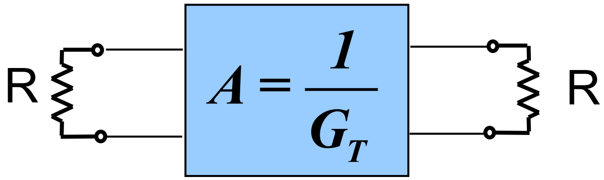
$$\boxed{P_D = K \cdot T_{eq} \cdot \Delta F \cdot G_{AV}}$$

$$P_{AV,out} = K \cdot T \cdot \Delta F \cdot G_{AV} + P_D = K \cdot (T + T_{eq}) \cdot \Delta F \cdot G_{AV}$$

T_{eq}: Equivalent Noise Temperature at the input of the 2-port

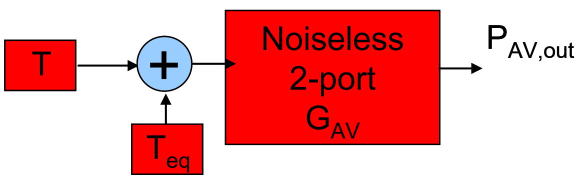


T_{eq} of a dissipative Attenuator



The 2-port is matched at input and output: $G_{AV}=G_T<1$.

Assuming T_0 the physical temperature (°K), the noise power in the resistor at the output port must be $P_{out}=KT_0\Delta F$. This power must be equal to the available power at the output port obtained with the equivalent model:



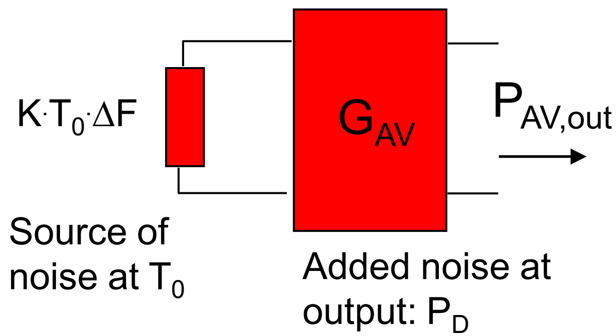
$$P_{AV,out} = K\cdot(T_0+T_{eq})\cdot\Delta F\cdot(G_T) = P_{out} = KT_0\Delta F$$

The equivalent temperature T_{eq} is then given by:

$$\boxed{T_{eq}=(1/G_T-1)T_0 =(A-1)T_0}$$

In the practice is more used the transducer attenuation $A=1/G_T>1$

Noise Figure NF of a 2-port



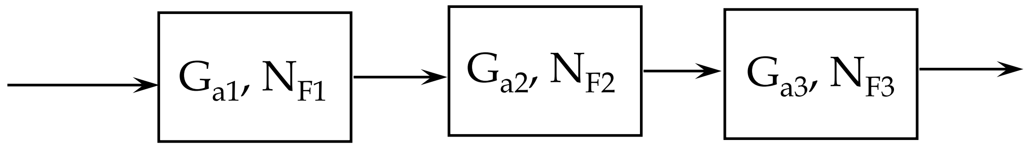
$$P_D = G_{AV} K \cdot T_{eq} \Delta F$$

$$NF = \frac{\text{Noise Power at output}}{\text{Noise Power at output with noiseless 2-port}}$$

$$NF = \frac{G_{AV} K T_0 \Delta F + P_D}{G_{AV} K T_0 \Delta F} = 1 + \frac{P_D}{G_{AV} K T_0 \Delta F} = 1 + \frac{T_{eq}}{T_0}$$

Note that NF depends on the temperature T_0, which is conventionally assumed 293 °K (20 °C)

Noise Figure for cascaded 2-ports



$$(NF)_{TOT} = NF_1 + \frac{NF_2 - 1}{G_{a1}} + \frac{NF_3 - 1}{G_{a1} G_{a2}} + \cdots$$

The noise figure is mainly determined by the first stage

Relationship between T_{eq} and Noise Figure NF

From $P_D = G_{AV} K \cdot T_{eq} \Delta F$ it is obtained:

$$\boxed{NF = 1 + \frac{T_{eq}}{T_0}}$$

$$\boxed{T_{eq} = \left(NF - 1\right) T_0}$$

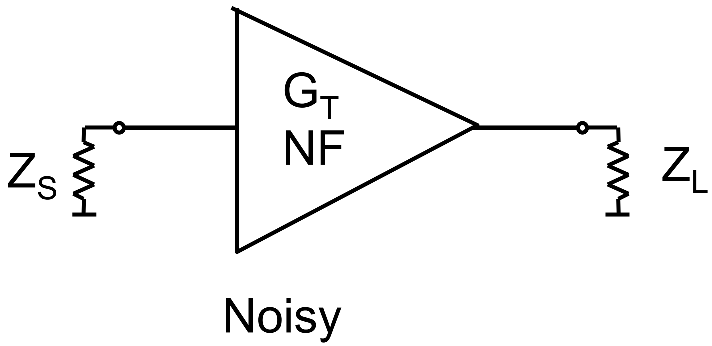
T_0 = 290 °K

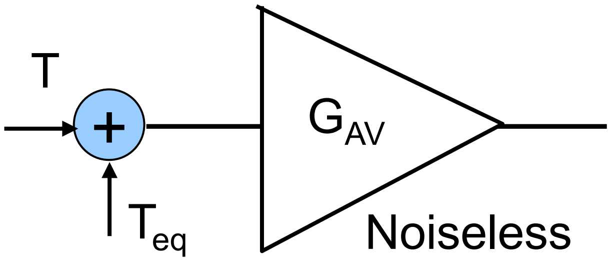
Note that NF is a multiplicative factor while T_{eq} is additive. The generation of noise is however an additive phenomenon, so the model based on the equivalent temperature is more close to the physical reality. The Noise Figure has been introduced because it is more easy to be measured than T_{eq}. It has in fact:

$$\boxed{NF = \frac{(S/N)_{in}}{(S/N)_{out}}}$$

NF is the degradation of the S/N ratio from the input to the output of a 2-port (input noise $KT_0\Delta F$)

Noisy RF Amplifier

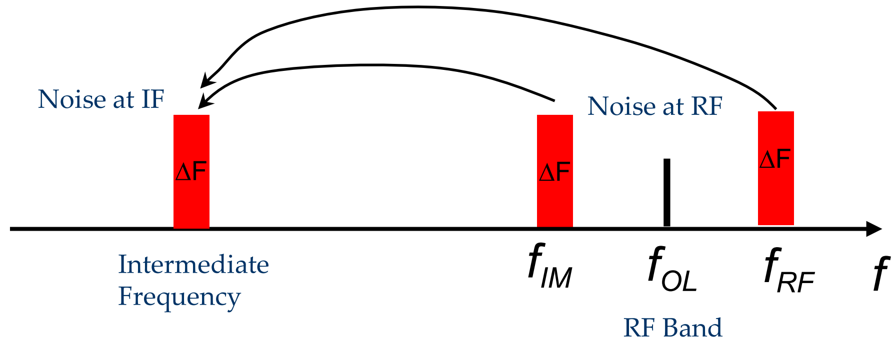




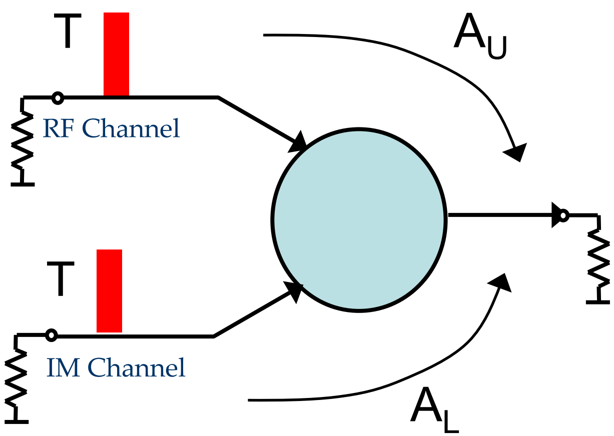
$$T_{eq} = \left(NF - 1\right)T_0$$

NF is in general dependent on the source impedance Z_S

Noise in Frequency Conversion (DBS)

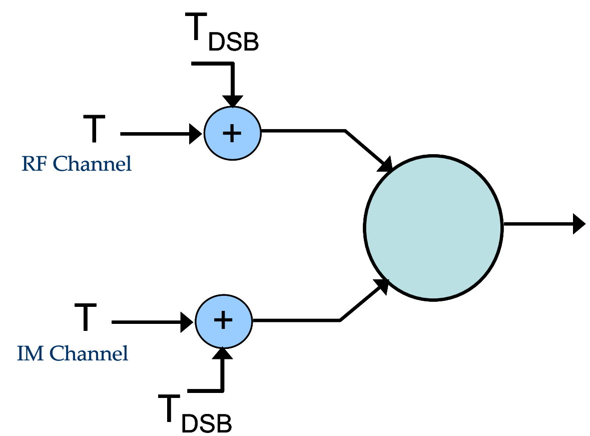


The mixer can be considered as a 3-port attenuator:



=>

$A_U \approx A_L = A_C$



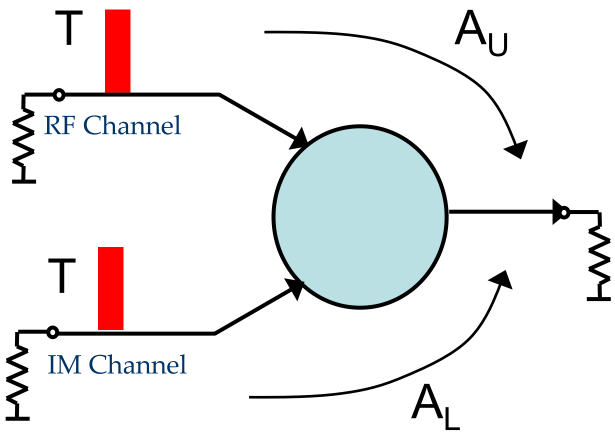
$$\frac{P_{AV,out}}{\Delta F} = \frac{2K\left(T_{DSB} + T_0\right)}{A_C} = KT_0$$

$$\boxed{T_{DSB} = T_0'\left(\frac{A_C}{2} - 1\right)}$$

T'_0 is slightly larger than T_0 to take into account the noise generated by the mixer directly at intermediate frequency

SSB Noise model

In this case we have only one equivalent noise source at input:



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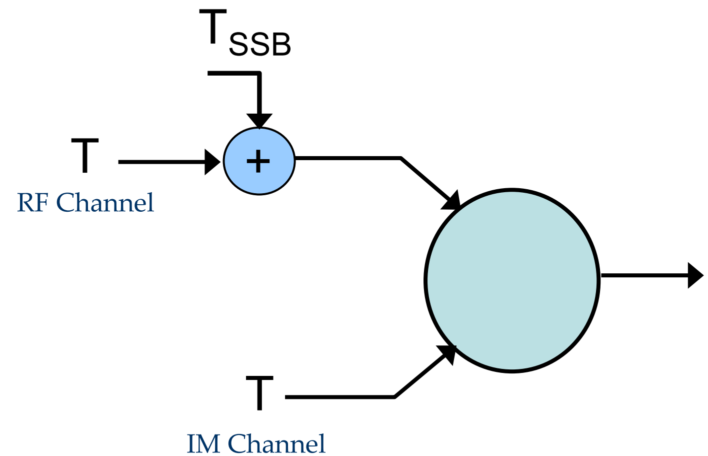
$A_U \approx A_L = A_C$



$$\frac{P_{AV,out}}{\Delta F} = \frac{K\left(T_{SSB} + 2T\right)}{A_C} = KT$$

$$\boxed{T_{SSB} = T_0'\left(A_C - 2\right)} = 2\,T_{DSB}$$

Noise Figure of a Mixer

The NF of a mixer can be expressed as the degradation of the signal-to-noise ratio from input to output.
We assume P_S the power of the signal (RF channel); T_{SSB} the equivalent noise temperature of the mixer (SSB); $KT_0\Delta F$ the noise power in the RF channel at the mixer input. It has:

$$\left(\frac{S}{N}\right)_{in} = \frac{P_S}{KT_0'\Delta F}, \qquad \left(\frac{S}{N}\right)_{out} = \frac{P_S / A_c}{K\left(2T_0' + T_{SSB}\right)\Delta F / A_c}$$

$$\boxed{NF = \frac{\left(\frac{S}{N}\right)_{in}}{\left(\frac{S}{N}\right)_{out}} = 2 + \frac{T_{SSB}}{T_0'}}$$

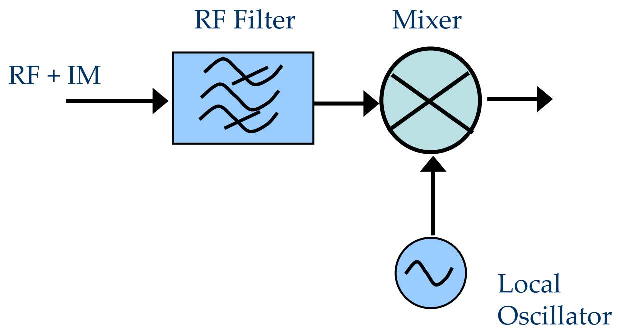
IEEE Noise Figure definition

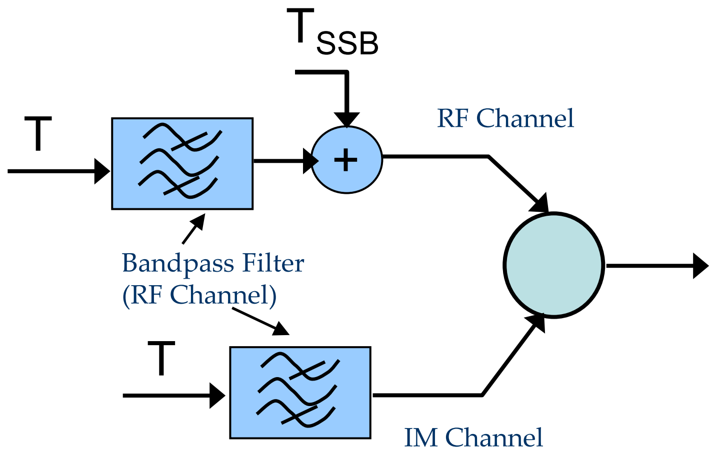
The previous expression says that a noiseless mixer (T_{SSB}=0) would present NF=2 (3dB), which sounds strange (it should be NF=1).
This result is due to the inclusion of the noise coming from the image channel in the mixer noise at output. If this contribute is not included (because for instance a sharp filter is added in that channel), the expression of the noise figure become the following:

$$\left(\frac{S}{N}\right)_{in} = \frac{P_S}{KT_0\Delta F}, \qquad \left(\frac{S}{N}\right)_{out} = \frac{P_S/A_c}{K\left(T_0 + T_{SSB}\right)\Delta F/A_c}$$

$$\boxed{NF_{IEEE} = \left(\frac{S}{N}\right)_{in} \Bigg/ \left(\frac{S}{N}\right)_{out} = 1 + \frac{T_{SSB}}{T_0'}}$$

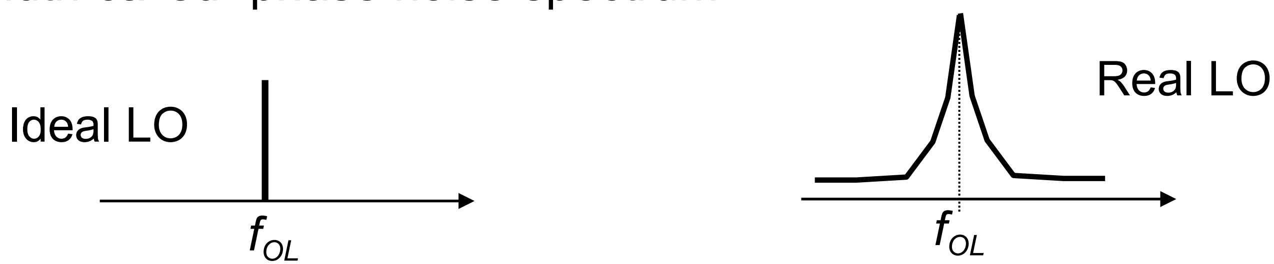
<u>IEEE Definition</u>





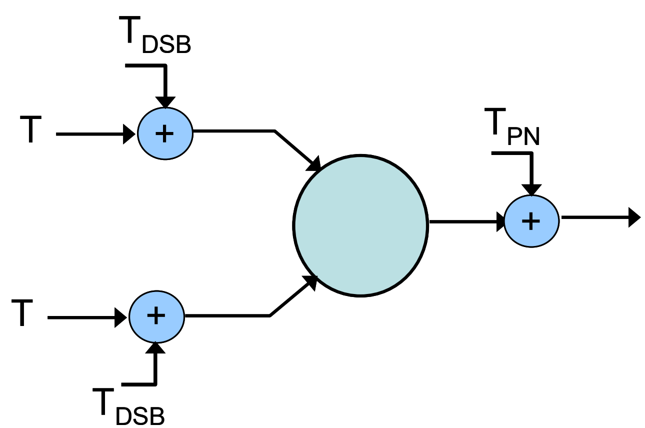
Noise from local oscillator

The spectrum produced by a real oscillator is different from the ideal one (a single tone at f_{OL}). The tone actually spreads out in in a spectrum with not zero width called "phase noise spectrum"



During the mixing process, the phase noise spectrum is converted to IF as it would be located in the RF band. It then contributes to worsen the overall noise behavior of the receiver.

From an equivalent point of view, the contribution of the phase noise can represented by an equivalent temperature (T_{PN}) at the mixer output



General configuration of a RF Link