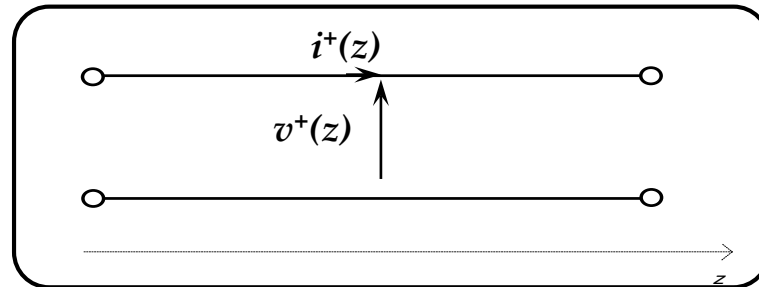


Transmission lines: wave propagation

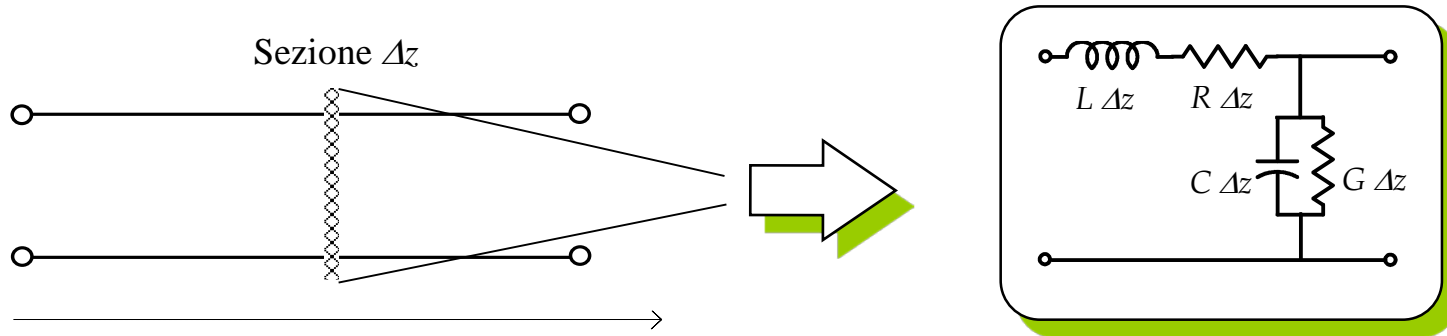


A transmission line is a physical structure with the cross-section (x-y plane) of definite shape and the developing indefinitely along the normal direction (z).

At each coordinate z the voltage $V(z)$ and the current $I(z)$ are represented as a combination of waves propagating along the positive and negative direction of z:

$$v(z) = v^+(z) + v^-(z), \quad i(z) = i^+(z) + i^-(z)$$

Physical parameters of the line



R, L, C, G , are the **physical parameters** of a transmission line. They depend on the geometry of the transmission line and on the materials used for the conductors and dielectric medium .

Through the physical parameters and the Kirckoff laws the Telegraphist Equations can be derived, whose solution is the voltage (current) wave propagating along the line:

$$v^+(z) = (V_0 e^{j\omega_0 t}) \cdot e^{-\gamma \cdot z}$$

Meaning of propagation constant γ

Given $\gamma = \alpha + j\beta$, it has:

$$v^+(z) = \left[(V_0 e^{j\omega t}) \cdot e^{-\alpha \cdot z} \right] \cdot e^{-j\beta \cdot z}$$

- Attenuation constant α : it defines the rate of reduction (in space) of the wave amplitude. It is measured in Neper/m or dB/m (1Np = 8.686 dB)
- Phase constant β : it is the rate of variation of the phase of the wave along the z coordinate (for t=cost). It is related to the wavelength and to radian frequency: $\beta = 2\pi/\lambda_0 = \omega/v$ (v represents the phase velocity determined by the medium). β is measured in rad/m (or °/m).

Characteristic impedance

- In general there are two waves in a transmission line, one propagating toward positive z and the other toward negative z
- The amplitude of the two waves depend on the impedance terminating the line
- There is a particular value of this impedance for which there is only the wave in positive z direction
- This impedance depends on the line and it is called Characteristic Impedance (Z_c). It is a real number for sufficiently high frequencies (>10 MHz)

Secondary Parameters

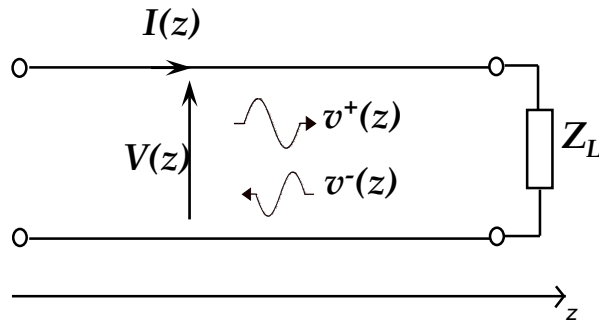
- Characteristic Impedance (Z_c): it is defined as the input impedance of a line with infinite length (i.e. there is only the wave propagating along the positive z direction)
- Propagation constant $\gamma = \alpha + j\beta$

Formulas relating physical and secondary parameters:

$$Z_c = \sqrt{\frac{L}{C}}, \quad \alpha = \frac{1}{2} \frac{R}{Z_c} + \frac{1}{2} G \cdot Z_c, \quad \beta = \omega \cdot \sqrt{L \cdot C}$$

These relations are valid for $\omega \gg R/L, G/C$; transmission lines used at microwave frequencies always satisfy these conditions

Voltage and current on the line



v^+, i^+ : Incident waves

$$V(z) = v^+(z) + v^-(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

$$I(z) = i^+(z) + i^-(z) = \frac{V_0^+}{Z_c} e^{-j\beta z} - \frac{V_0^-}{Z_c} e^{+j\beta z}$$

v^-, i^- : Reflected waves

The sign of the reflected wave of voltage is the same of the incident wave; the sign of reflected wave of current is instead opposite to that of the incident wave. In any case (incident or reflected), the ratio between the wave of voltage and the wave of current is equal to Z_c in magnitude. A reflected wave is produced by a discontinuity in the transmission line structure (as for instance the termination with a load). The reflected wave is eliminated by assigning **Z_L equal Z_c** .

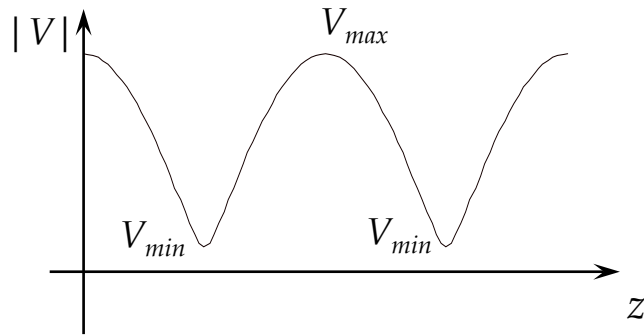
Reflection Coefficient

$$\Gamma(z) = \frac{\text{Reflected Wave}}{\text{Incident wave}} = \frac{V_0^- e^{+j\beta z}}{V_0^+ e^{-j\beta z}} = \frac{V_0^-}{V_0^+} e^{+j2\beta z} = \Gamma_0 e^{+j2\beta z} = \Gamma_0 e^{+j2\pi \frac{z}{\lambda/2}}$$

Properties of $\Gamma(z)$ with $\alpha=0$:

- ❑ The magnitude does not depend on z (it is constant along the line)
- ❑ The phase is periodic in z (period equal to $\lambda/2$)
- ❑ The magnitude is always less than 1 with a passive load (the reflected power must be less than the incident one).

$|V(z)|$ as a function of z



$$V(z) = v^+(z) + v^-(z) = v^+(z) \cdot [1 + \Gamma(z)]$$

$$|V(z)| = |V_0^+| |1 + \Gamma_0 \cdot e^{+j2\beta z}| =$$

$$|V_0^+| \left\{ 1 + 2|\Gamma_0| \cdot \cos(2\beta z) + |\Gamma_0|^2 \right\}^{1/2}$$

The magnitude of V is periodic (same period of Γ) with maxima and minima given by:

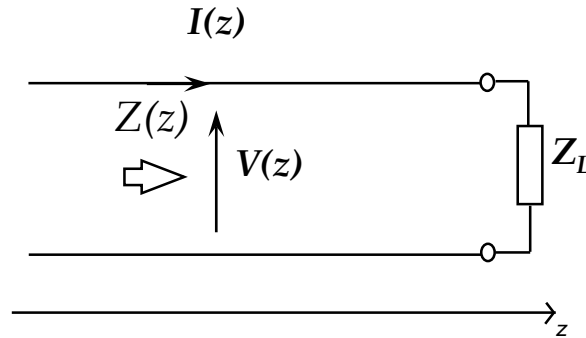
$$V_{max} \div 1 + |\Gamma| \quad , \quad V_{min} \div 1 - |\Gamma|$$

The Voltage Standing Wave Ratio (**VSWR**) is defined as the ratio of these voltages:

$$VSWR = \frac{V_{max}}{V_{min}}$$

\nearrow = 1 for perfect matched line ($\Gamma=0$)
 \searrow = ∞ for completely mismatched line ($\Gamma=1$)

Impedance along the line



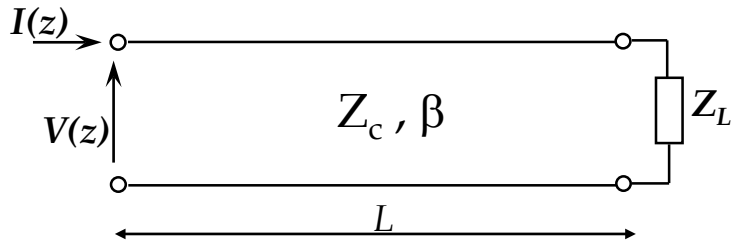
There is a biunivocal relationship between the reflection coefficient and the impedance (normalized to Z_c) seen along the line (in the load direction):

$$\frac{Z(z)}{Z_c} = \frac{1}{Z_c} \frac{V(z)}{I(z)} = \frac{V^+(z) + V^-(z)}{V^+(z) - V^-(z)} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

Inverse relation:

$$\Gamma(z) = \frac{Z(z)/Z_c - 1}{Z(z)/Z_c + 1}$$

Impedance as a function of the load



$$Z_{in} = \frac{V_{in}}{I_{in}} = Z_c \frac{Z_L + jZ_c \tan(\beta L)}{Z_c + jZ_L \tan(\beta L)}$$

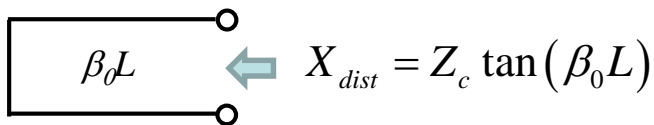
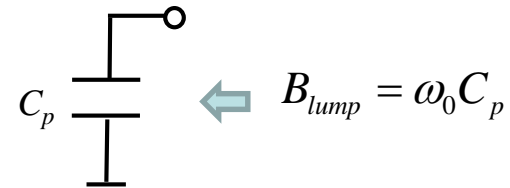
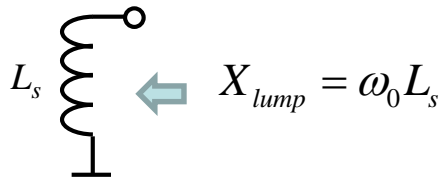
Particular cases (stubs):

$Z_L = 0$ (short circuit) $\Rightarrow Z_{in} = \frac{V_{in}}{I_{in}} = jZ_c \tan(\beta \cdot L) = jZ_c \tan(2\pi \frac{L}{\lambda_0})$

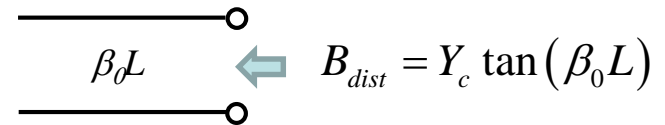
$Z_L = \infty$ (open circuit) $\Rightarrow Z_{in} = \frac{V_{in}}{I_{in}} = -jZ_c \cot(\beta \cdot L) = -jZ_c \cot(2\pi \frac{L}{\lambda_0})$

Stub as a circuit component

At high frequencies it is difficult to realize lumped components like inductors and capacitors. It is much easier to use stubs which realize the same reactance (susceptance) of the lumped component:



$$L < \lambda_0 / 4$$

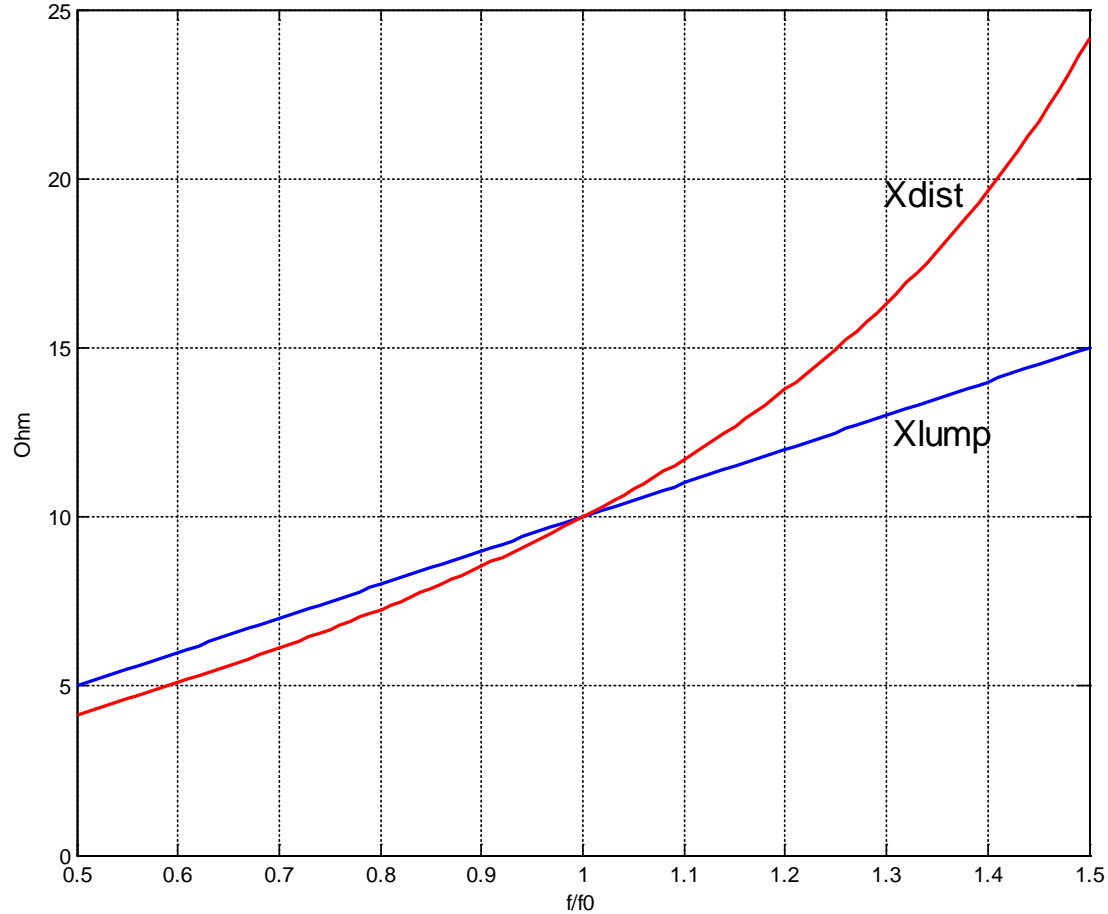


$$Z_c = \frac{X_{lump}}{\tan(\beta_0 L)} = \frac{\omega_0 L_s}{\tan\left(\frac{\omega_0}{v} L\right)}$$

$$Y_c = \frac{B_{lump}}{\tan(\beta_0 L)} = \frac{\omega_0 C_p}{\tan\left(\frac{\omega_0}{v} L\right)}$$

Note: The equivalence is correct at fixed frequency. When the frequency varies the two components behavior is greatly different (lumped: monotonic, distributed: periodic)

Example



$$X_{lump}(f_0) = X_{dist}(f_0) = 10 \Omega \rightarrow \beta_0 L = 45^\circ, Z_c = 10 \Omega$$

$$X_{lump}(f) = 10 \cdot (f/f_0), \quad X_{dist}(f) = 10 \cdot \tan(45^\circ \cdot f/f_0)$$

Transmission Lines Classification

☞ TEM Lines (Transverse Electric Magnetic)

- **Coaxial**
- **Stripline**

☞ non-TEM Lines

- **Rectangular waveguide**
- **Circular waveguide**
- **Fin-line**

☞ quasi_TEM Lines

- **Microstrip**
- **Suspended Stripline**
- **Inverted Stripline**
- **Coplanar Lines**

TEM Lines

- They are constituted by two (at least) independent conductors (a voltage can be applied among them), embedded in a homogeneous medium
- Electric and magnetic fields of the propagating wave are orthogonal each other and do not have components in the direction of propagation
- A TEM transmission line is characterized by:
 - A univocal and frequency independent characteristic impedance
 - A constant phase velocity lower than or equal to the light velocity c :

$$v = \frac{c}{\sqrt{\epsilon_r}}$$

with ϵ_r relative dielectric constant of the medium.

non-TEM Lines

- The electromagnetic field of a generic transmission line can assume specific configurations called **modes**. The modes are characterized by a minimum frequency (**cutoff frequency**), below of which they cannot propagate. The mode with the smallest cutoff frequency is called **dominant mode** and it is the most used in the practice. For line supporting TEM mode the cutoff frequency on the dominant mode (also called principal mode) is zero
- Non-TEM lines are constituted by a single hollow conductor having a section of arbitrary shape, generally called **waveguide**
- The modes in non-TEM lines are generally classified in:
 - **TE modes**: the electric field has no components in the propagation direction
 - **TM modes**: the magnetic field has no components in the propagation direction
- The characteristic impedance of non-TEM mode is not univocally defined
- Also voltage and current are not univocally defined

quasi-TEM Lines

- These lines consist of two (or more) conductors, surrounded by a non homogeneous medium
- As a consequence there is at least one component of E or H field in the direction of wave propagation. To difference of non-TEM modes, the dominant mode of quasi-TEM lines has the cutoff frequency equal to zero
- The rigorous study of this kind of lines is rather complex; has been then developed an approximate (much easier to compute) model, based on the concept of **equivalent TEM line**.
- The equivalent TEM model of a quasi-TEM line differs from a ideal TEM line in the fact that the characteristic impedance and the phase velocity **depend on the frequency**

Attenuation in transmission lines

Power along a matched line (positive z direction):

$$P(z) = \frac{1}{2} \frac{|v^+(z)|^2}{Z_c} = \frac{1}{2} \frac{|v_0^+|^2}{Z_c} e^{-2\alpha z}$$

The power lost per unit length is then:

$$\frac{\partial P}{\partial z} = -2\alpha \left[\frac{1}{2} \frac{|v_0^+|^2}{Z_c} e^{-2\alpha z} \right] = -2\alpha P(z) = -P_{diss}$$

The definition of α then results:

$$\alpha = \frac{P_{diss}}{2P} = \frac{\text{Dissipated power p.u.l}}{\text{Carried power}}$$

Sources of attenuation in transmission lines

- Attenuation α_J due to losses in the conductors
- Attenuation α_D due to losses in the medium (dielectric)

The overall attenuation α is the sum of the above contributes:

$$\alpha = \alpha_J + \alpha_D$$

Attenuation in the conductors α_J

- It is caused by the finite conductivity of the conductor material employed. In TEM lines α_J increases with frequency as the square root of f (skin effect).
- The actual conductivity of a material depends, other than its physical properties, also on the surface roughness determined by the fabrication process adopted. Without a suitable processing of the surfaces (polishing, lapping and plating), the degradation of conductivity due to the surface roughness may be even less 50 % of the ideal value

Attenuation in the dielectric medium α_D

- In a real dielectric medium under sinusoidal excitation, some energy must be supplied for aligning the elementary dipoles of the material along the electric field direction. As a consequence the dielectric constant of the medium become a **complex number**
- A parameter called ***tan* δ** is introduced for characterizing dielectric losses, which represents the ratio between the imaginary and real part of the dielectric constant. This parameter is practically independent on the frequency (at microwave frequencies).
- Expressions of α_D as function of *tan* δ :

$$\alpha_d = \frac{\pi}{\lambda_0} \tan\delta \quad (TEM \text{ Lines})$$

$$\alpha_d = \frac{\pi}{\lambda_0} \frac{\tan\delta}{\sqrt{1 - \frac{f_c^2}{f^2}}} \quad (non-TEM \text{ Lines})$$

General expression of Z_c and α for TEM lines

General definition of the characteristic impedance of a TEM line:

$$Z_c = \frac{\Delta V}{I} = \frac{\int_{P_1}^{P_2} E_t \cdot dl}{\oint_{cont.} H_t ds} = Z_w \cdot F_z \quad Z_w = \frac{377}{\sqrt{\epsilon_r}} \Omega$$

$F_z =$ Form Factor of the line (depends on the line geometry)

Attenuation of a TEM line:

$$\alpha_J = \left(\frac{R_s}{2Z_w} \right) F_J \quad F_J = \text{Attenuation Factor of the line (depends on the line geometry)}$$

Wheeler's rule:

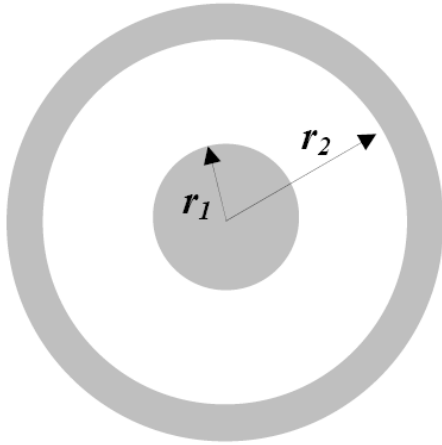
$$F_J = \frac{1}{F_z} \frac{\partial F_z}{\partial n}$$

The direction n represents the direction orthogonal to the conductors surface (entering into)

High order modes in TEM lines

- Also in TEM lines may exist propagating waves with components of E or H in the propagating direction (higher order modes)
- These modes are characterized by their cutoff frequency and can propagate only if the frequency is higher than this
- In practical application it is requested that only one mode can propagate (the one with the lowest cutoff frequency)
- TEM line must be dimensioned in order that the first non-TEM mode is above the maximum operating frequency

Coaxial Line



$$F_z = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi}$$

$$\frac{1}{F_z} \frac{\partial F_z}{\partial n} = \frac{1}{F_z} \left[\frac{\partial F_z}{\partial r_2} - \frac{\partial F_z}{\partial r_1} \right] = \frac{2\pi}{\ln\left(\frac{r_2}{r_1}\right)} \left(\frac{1/r_2 + 1/r_1}{2\pi} \right)$$

$$F_J = \frac{1/r_2 + 1/r_1}{\ln\left(\frac{r_2}{r_1}\right)}$$

Coaxial Line dimensioning

Given Z_c :

$$\frac{r_2}{r_1} = \exp\left(2\pi \frac{Z_c}{Z_0}\right) \approx \exp\left(2\pi \sqrt{\epsilon_r} \frac{Z_c}{377}\right)$$

$Z_c = 50 \Omega$ for $r_2/r_1 \cong 2.3$ in air

Single-mode propagation up to f_{max} :

$$f_{max} = \frac{v}{\pi(r_2 + r_1)} \Rightarrow r_2 < \frac{v}{\pi f_{max}} \frac{r_2/r_1}{1 + r_2/r_1}$$

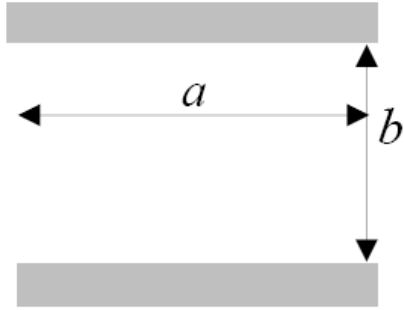
Minimum attenuation assigned the external radius: $F_J = (1/r_2) \frac{1 + r_2/r_1}{\ln(r_2/r_1)}$

$$\frac{\partial F_J}{\partial (r_2/r_1)} = \frac{1}{r_2} \frac{\partial}{\partial (r_2/r_1)} \frac{1 + r_2/r_1}{\ln\left(\frac{r_2}{r_1}\right)} = 0 \Rightarrow \frac{r_2}{r_1} \approx 3.6 \quad (Z_c \approx 76\Omega \text{ in air})$$

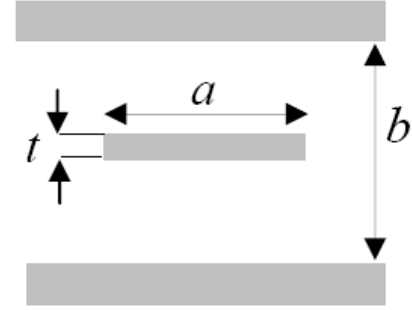
Minimum attenuation assigned f_{max} : $F_J = \left(\frac{1}{r_1 + r_2}\right) \frac{2 + r_2/r_1 + r_1/r_2}{\ln(r_2/r_1)}$

$$\frac{\partial F_J}{\partial (r_2/r_1)} = \frac{1}{r_1 + r_2} \frac{\partial}{\partial (r_2/r_1)} \frac{2 + r_2/r_1 + r_1/r_2}{\ln(r_2/r_1)} = 0 \Rightarrow \frac{r_2}{r_1} \approx 4.45 \quad (Z_c \approx 97\Omega \text{ in air})$$

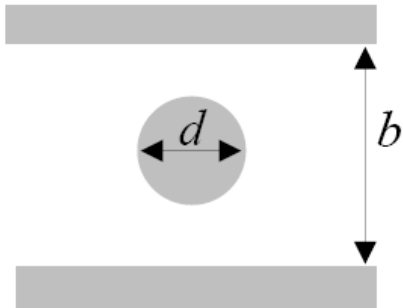
Other TEM lines



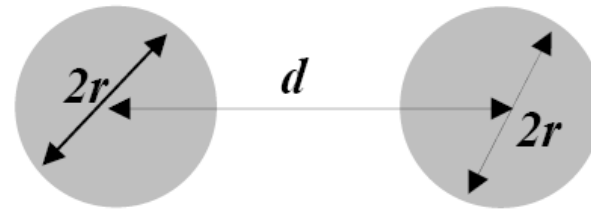
Parallel Plates



Stripline



Slabline



Bi-wires

Linea a strisce parallele (fig. 2.9)

$$F_z = \frac{1}{2\pi} \ln \left[1 + 2y^2 + y\sqrt{(2y)^2 + \pi^2} \right] \quad [2.54]$$

$$y = \frac{2b}{a'}, \quad a' = a + qt \quad [2.55]$$

$$q = \frac{1}{\pi} \ln \frac{4e}{\sqrt{\left(\frac{2t}{b}\right)^2 + \left(\frac{t/\pi}{a+1,1t}\right)^2}}$$

$$F_J = \frac{2}{b} \frac{\exp(-2\pi F_z)}{2\pi F_z} MN2y^2 \quad [2.56]$$

$$M = \left[\left[1 + \frac{\pi^2}{4y^2} \right]^{1/4} + \left[1 + \frac{\pi^2}{4y^2} \right]^{-1/4} \right]^2$$

$$N = 1 + y \left[\frac{\pi(q+1) - 1}{2\pi} + G \right],$$

$$G = \frac{t}{2b} \frac{4, 2r^3 - 1/(\pi)}{1 + r^2}, \quad r = b/[2\pi(a+1,1t)]$$

Linea a striscia schermata *stripline* (fig. 2.7)

$$F_z = \frac{K(k')}{4K(k)} \quad [2.38]$$

$$k = \tanh\left(\frac{2}{y}\right), \quad k' = \sqrt{1 - k^2}$$

$$F_z = \frac{1}{4\pi} \ln \left[1 + 2y^2 + \sqrt{4y^2 + 6,27} \right] \quad [2.42]$$

$$y = \frac{4}{\pi} \frac{b'}{a'}, \quad a' = a + qt, \quad b' = b - t \quad [2.43]$$

$$q \cong 1 \quad (a \gg t)$$

$$F_J \cong \frac{4}{b'} \frac{\exp[-4\pi F_z]}{4\pi F_z} 2y^2 M N \quad [2.45]$$

$$M = \left[\left[1 + \frac{6,27}{4y^2} \right]^{1/4} + \left[1 + \frac{6,27}{4y^2} \right]^{-1/4} \right]^2$$

$$N = 1 + \frac{\pi}{4} y \left[\frac{\pi(1+q) - 1}{2\pi} + G \right]$$

$$G = \frac{t}{2b-t} \frac{4, 2r^3 - 3/(2\pi)}{1 + r^2}$$

$$r = \frac{(2b-1)}{4\pi(a+1,1t)}$$

Slabline (fig. 2.6)

$$F_z = \frac{1}{2\pi} \ln \left[\frac{4b}{\pi d Q} \right] \quad [2.72]$$

$$Q \cong \exp \left[\frac{31}{630} \frac{(d/b)^3}{1 - (477/630)(d/b)^3} \right] \quad [2.73]$$

$$F_J = \frac{2}{d} \frac{1 + (d/b)}{\ln \left[\frac{4b}{\pi d Q} \right]} \left[1 + \frac{3 \ln(Q)}{1 - \frac{477}{630} \left(\frac{d}{b} \right)^3} \right] \quad [2.74]$$

Linea bifilare (fig. 2.21)

$$F_z = \frac{\arccos \left(\frac{d}{2r} \right)}{\pi} =$$

$$= \frac{1}{\pi} \ln \left[\frac{d}{2r} + \sqrt{\left(\frac{d}{2r} \right)^2 - 1} \right] \quad [2.36]$$

$$F_J = \frac{\frac{1}{r}}{\operatorname{arccosh} \left(\frac{d}{2r} \right)} \cdot \frac{1}{\sqrt{1 - \left(\frac{2r}{d} \right)^2}} \quad [2.37]$$

quasi-TEM Lines

A quasi-TEM line is obtained when a inhomogeneous medium is used in a TEM line. As a consequence the electromagnetic field is no more transverse with respect the direction of propagation.

Strictly speaking it is no possible in this case to define uniquely the voltages and currents. In the practice the quasi-TEM approximation is introduced. This consists in assuming an equivalent homogeneous medium characterized by an effective dielectric constant $\epsilon_{r,eff}$, defined as:

$$\epsilon_{r,eff} = C_m / C_0$$

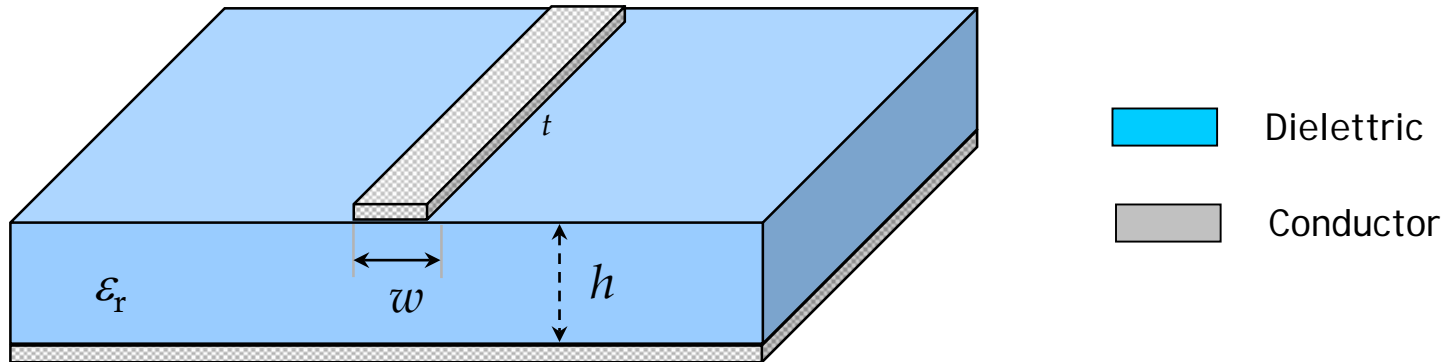
C_m : Capacitance p.u.l. of non-homogenous line
 C_0 : Capacitance p.u.l. of the line with $\epsilon_r=1$ (air everywhere)

Note that $\epsilon_{r,eff}$ is in general a function of the frequency. Also Z_c e v_f are then functions of frequency and can be expressed as:

$$Z_c = \frac{377}{\sqrt{\epsilon_{r,eff}}} F_z, \quad v_f = \frac{c}{\sqrt{\epsilon_{r,eff}}}$$

Microstrip

The most important quasi-TEM line in practical applications is the microstrip. It belongs to the category of planar structures.



- h substrate thickness
- w strip width
- ϵ_r relative dielectric constant of substrate
- t metallic thickness

Formulas for microstrip

Quasi-static (t=0):

$$\frac{60}{\sqrt{\epsilon_{r,eff}}} \ln(8h/w + 0.25w/h) \quad w/h \leq 1$$

$Z_c =$

$$\frac{120\pi}{\sqrt{\epsilon_{r,eff}}} \left[1.393 + w/h + 0.667 \ln(w/h + 1.444) \right]^{-1} \quad w/h > 1$$

con: $\epsilon_{r,eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2\sqrt{1 + 10h/w}}, \quad v = \frac{300}{\sqrt{\epsilon_{eff}}} \text{ cm/sec}$

Finite metallic thickness:

For taking into account the finite value of t , an effective strip width (W_e) is introduced:

$$w_e/h = \begin{cases} \frac{w}{h} + \frac{1.25 \cdot t}{\pi h} \left(1 + \ln \left(\frac{4\pi w}{t} \right) \right) & w/h \leq 0.159 \\ \frac{w}{h} + \frac{1.25 \cdot t}{\pi h} \left(1 + \ln \left(\frac{2h}{t} \right) \right) & w/h > 0.159 \end{cases}$$

A simple model for introducing the frequency variation in $\epsilon_{r,eff}$ is given by:

$$\epsilon_{r,eff} (f_{GHz}) = \epsilon_r - \frac{\epsilon_r - \epsilon_{r,eff} (0)}{1 + (h_{mm}/Z_c)^{1.33} (0.43 f_{GHz}^2 - 0.009 f_{GHz}^3)}$$

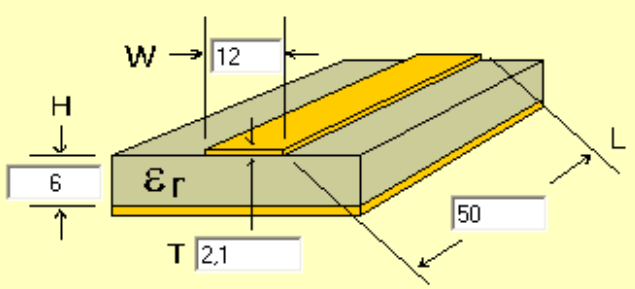
CAD tool for evaluating Z_c

AppCAD - [Microstrip]

File Calculate Select Parameters Options Help

Main Menu [F8]

Microstrip



Calculate Z0 [F4]

$Z_0 = 32,28 \Omega$

Elect Length = 0,433 λ

Elect Length = 155,7 degrees

1.0 Wavelength = 115,574 mm

$v_p = 0,386$ fraction of c

$\epsilon_{eff} = 6,728$

W/H = 2,000

Dielectric: $\epsilon_r = 9,5$

Alumina (96% Al2O3)

Frequency: 1 GHz

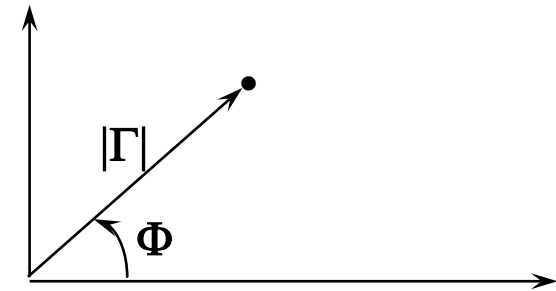
Length Units: mm

Normal

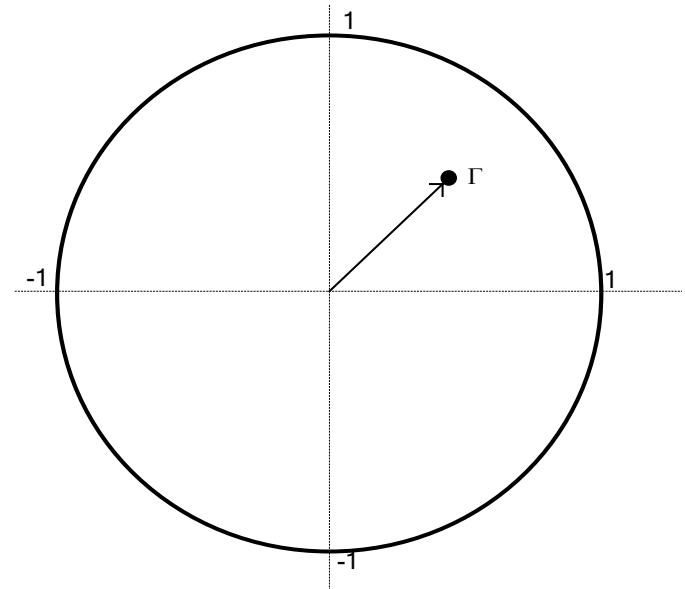
[Click for Web: APPLICATION NOTES - MODELS - DESIGN TIPS - DATA SHEETS - S-PARAMETERS](#)

Graphic representation of Γ

Γ is a complex number in the polar plane:

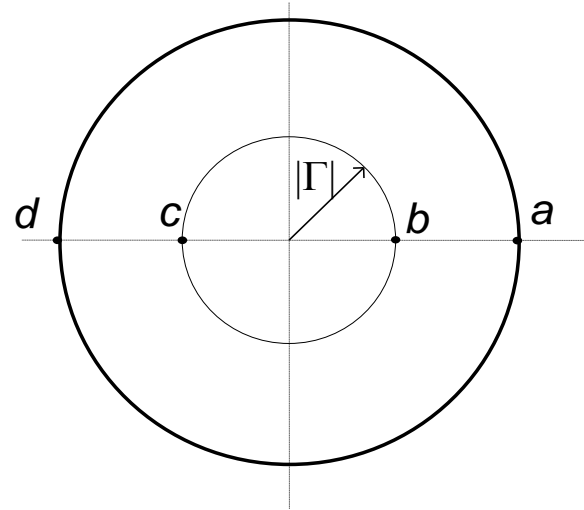


For the properties of Γ , assuming the line terminated with a passive load, the point in the polar plane must be within the circle with unit radius



Graphic representation of Γ

The curve in the polar plane representing the reflection coefficient on a lossless line terminated with a passive load is **a circle with radius equal to $|\Gamma|$** . The phase changes by **2π** with a variation of z of **$\lambda/2$**



Characteristic points:

Matched line $\Gamma = 0$ (centre of the chart)

Open circuit $\Gamma = 1$ (a)

Short circuit $\Gamma = -1$ (d)

Maximum of voltage (b) (Γ real and positive)

Minimum of voltage (c) (Γ real and negative)

Smith Chart

- On the polar plane of Γ the curves representing the locus of points where the real (or imaginary) part of $z_n = Z/Z_c$ is constant are defined by:

$$\operatorname{Re}\{Z_{in}\} = \operatorname{Re}\left\{\frac{\Gamma(z) + 1}{\Gamma(z) - 1}\right\} = \operatorname{cost}(r)$$

$$\operatorname{Im}\{Z_{in}\} = \operatorname{Im}\left\{\frac{\Gamma(z) + 1}{\Gamma(z) - 1}\right\} = \operatorname{cost}(x)$$

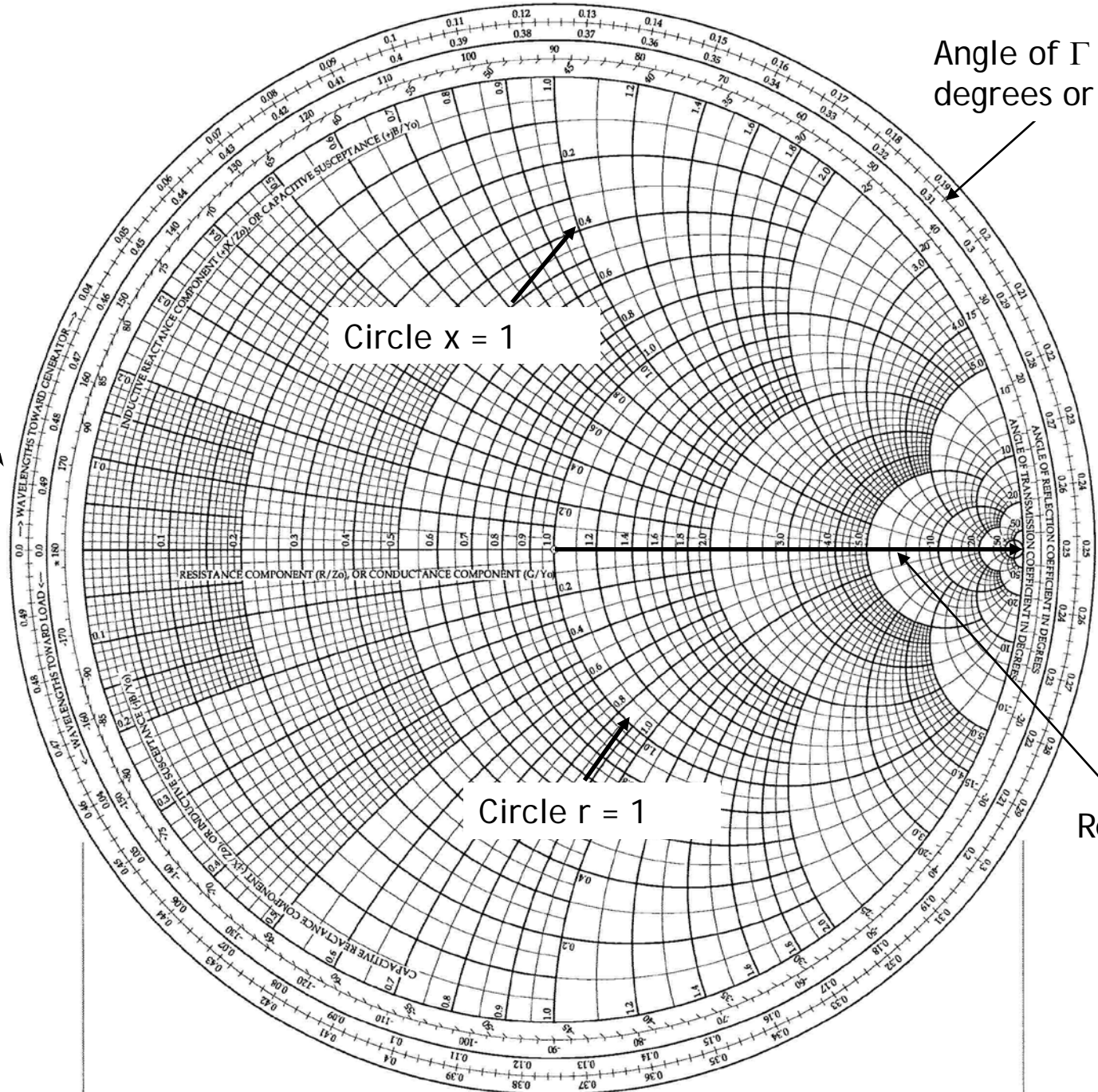
- These curves are circles, whose center and radius depend on r (or x). The Smith Chart is the graphic representation of such circles. It allows to solve graphically several problems related to the transmission lines used in microwave circuits.

Toward Source

Angle of Γ (it is measured in degrees or in L/λ_0)

Toward Load

Reference Axis



Circle $x = 1$

Circle $r = 1$

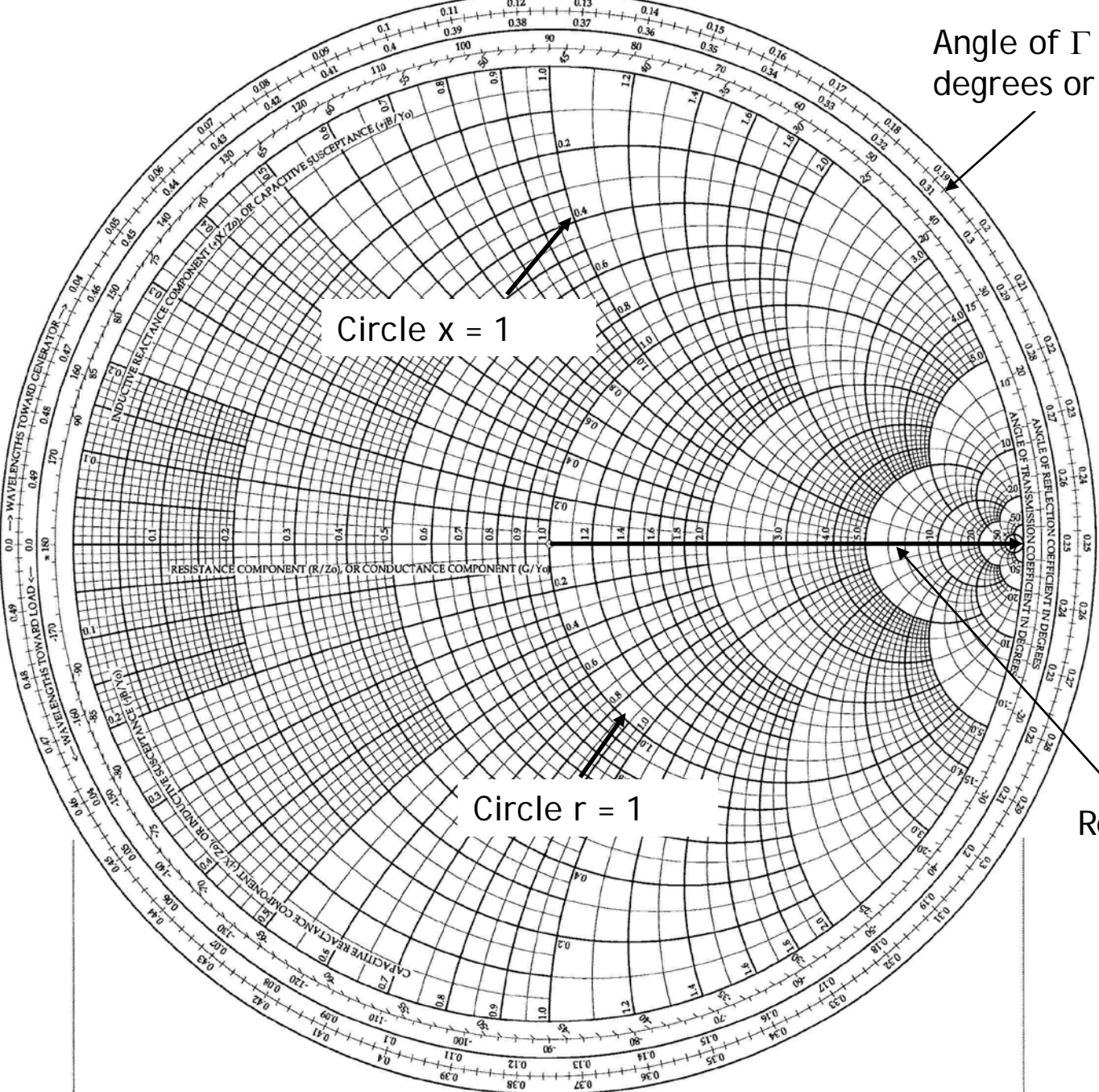
RESISTANCE COMPONENT (R/Z_0) OR CONDUCTANCE COMPONENT (G/Y_0)

INDUCTIVE REACTANCE COMPONENT ($+jB/Z_0$) OR CAPACITIVE SUSCEPTANCE ($+jB/Y_0$)

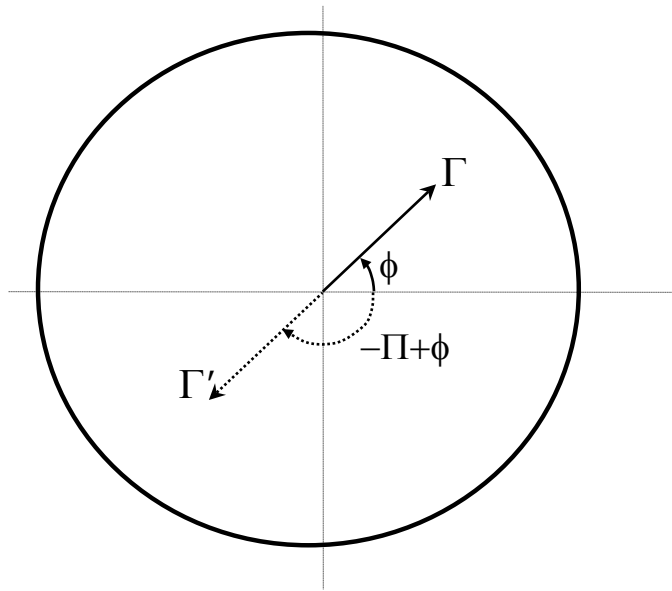
CAPACITIVE REACTANCE COMPONENT ($-jB/Z_0$) OR INDUCTIVE SUSCEPTANCE ($-jB/Y_0$)

ANGLE OF REFLECTION COEFFICIENT IN DEGREES

ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES



Admittance representation of the Smith Chart



Impedance at Γ :
$$z_n = \frac{1 + \Gamma}{1 - \Gamma}$$

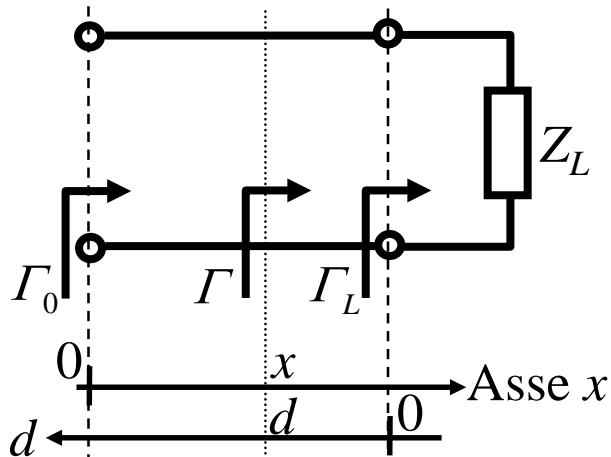
Impedance at Γ' :

$$z'_n = \frac{1 + \Gamma'}{1 - \Gamma'} = \frac{1 + |\Gamma| e^{j(-\pi + \phi)}}{1 - |\Gamma| e^{j(-\pi + \phi)}} = \frac{1 - |\Gamma| e^{j\phi}}{1 + |\Gamma| e^{j\phi}} = \frac{1 - \Gamma}{1 + \Gamma} = \frac{1}{z_n}$$

The diametrically opposite point presents the inverse impedance (i.e. the admittance) with respect the original point. As a consequence, the Smith Chart can be used for representing either impedances or admittances

Smith Chart

Moving at $\Gamma = \text{const}$



$$\begin{cases} \Gamma(x) = \Gamma_0 \cdot e^{j2\beta x} = |\Gamma_0| e^{j\angle\Gamma_0} \cdot e^{j4\pi \cdot (x/\lambda)} \\ \Gamma(d) = \Gamma_L \cdot e^{-j2\beta d} = |\Gamma_L| e^{j\angle\Gamma_L} \cdot e^{-j4\pi \cdot (d/\lambda)} \end{cases}$$

Displacement on a lossless line:



Movement on a circle with constant radius

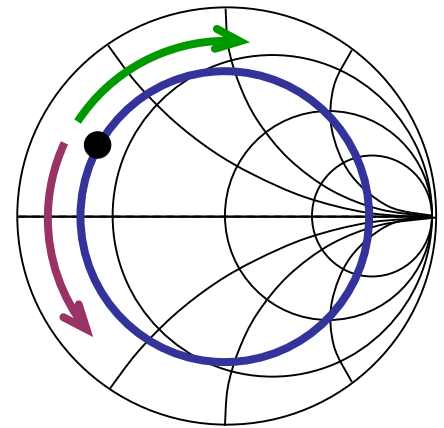
Displacement:

➤ **from source** $\rightarrow x$ increasing

$$\angle\Gamma(x) = \angle\Gamma_0 + 2\beta x \rightarrow \text{counterclockwise rotation}$$

➤ **from load** $\rightarrow d$ increasing

$$\angle\Gamma(x) = \angle\Gamma_L - 2\beta d \rightarrow \text{Clockwise rotation}$$

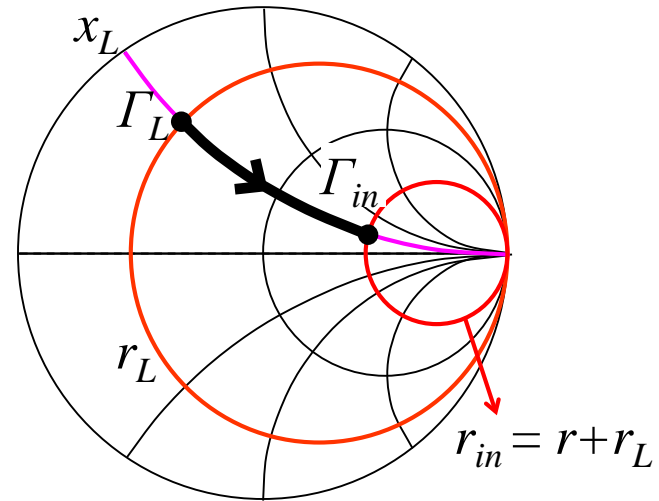
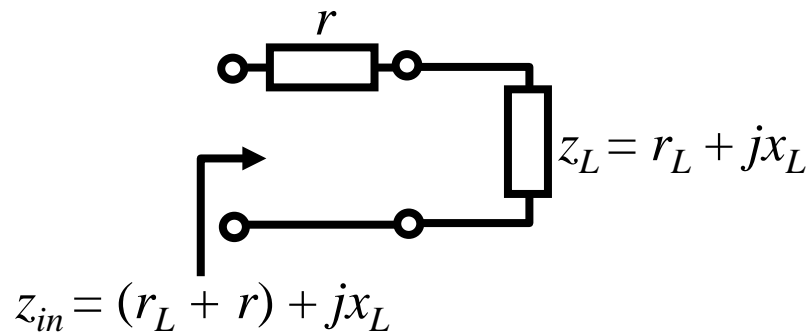


NOTE: $\Gamma(z)$ is a periodic function of $z(d)$ with period $\lambda/2$

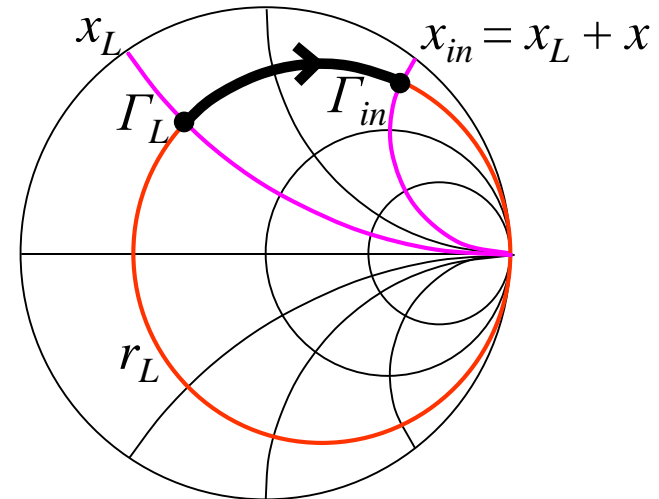
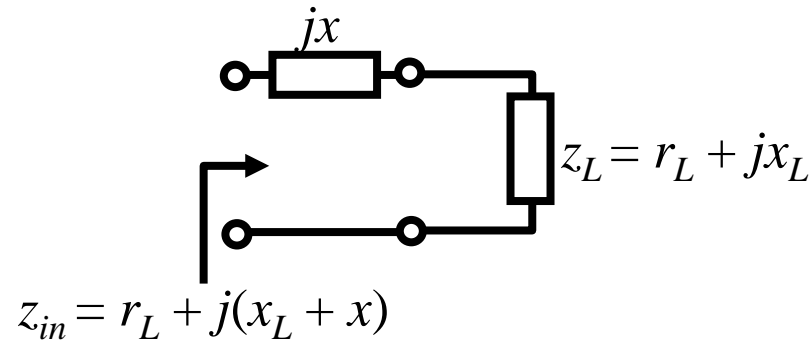
Smith Chart

Displacement at r (or x) constant

Displacement at x constant



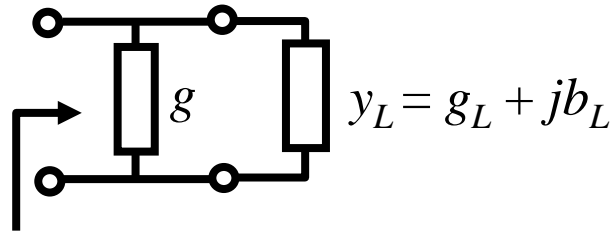
Displacement at r constant



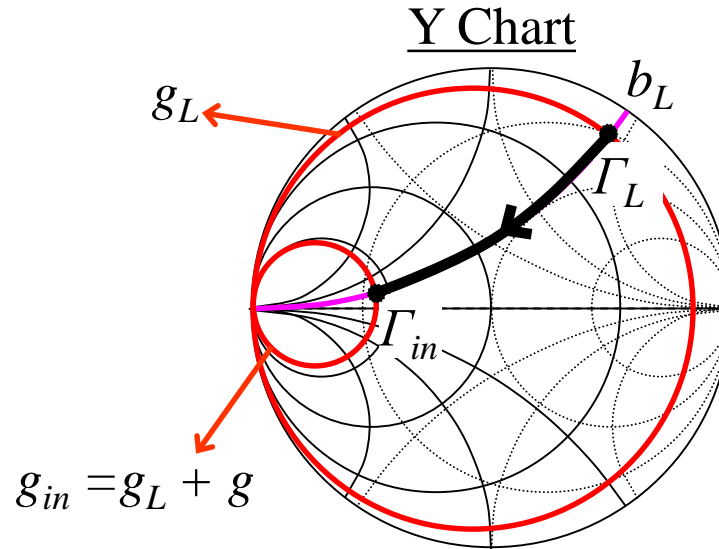
Smith Chart

Displacement at g (or b) constant

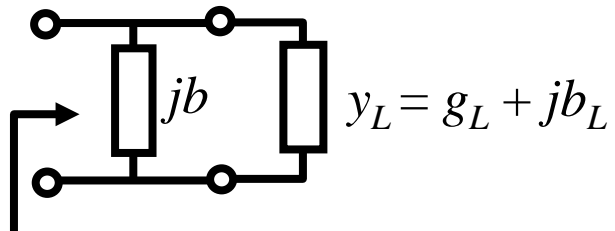
Displacement at $b=\text{const}$



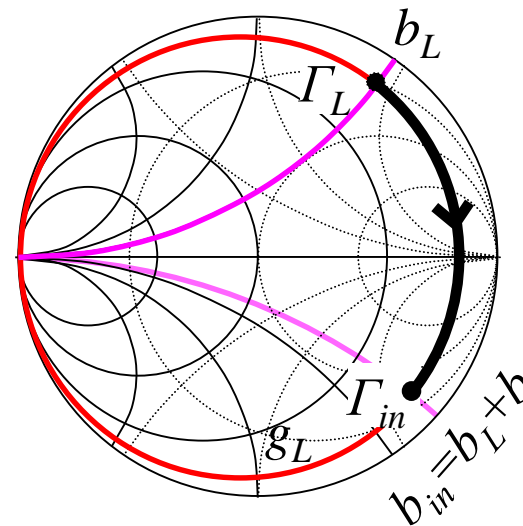
$$y_{in} = (g_L + g) + jb_L$$



Displacement at $g=\text{const}$

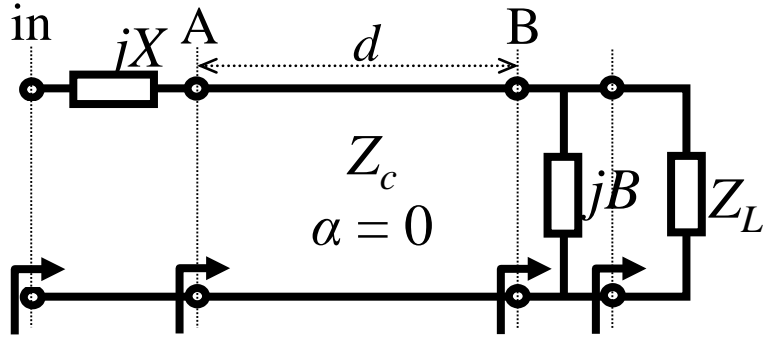


$$y_{in} = g_L + j(b_L + b)$$



Smith Chart

Example of graphical solution



$$Z_c = 50 \text{ } [\Omega];$$

$$\epsilon_r = [4];$$

$$f_0 = 3 \text{ } [\text{GHz}];$$

$$Z_L = 20 + j40 \text{ } [\Omega];$$

$$(Z_n = 0.4 + j0.8)$$

$$\beta = 7.2 \text{ } [^\circ/\text{mm}]$$

$$B = 0.05 \text{ } [\Omega^{-1}]; (b_n = 2.5)$$

$$X = -80 \text{ } [\Omega]; (x_n = -1.6)$$

$$d = 15 \text{ } [\text{mm}]; (\beta d = 108^\circ)$$

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = 0.62 \angle 97.1^\circ$$

$$\Gamma_B = 0.75 \angle -116.6^\circ$$

$$2\beta d = 40\pi d = 3.77 \text{ } [\text{rad}] = 216.0^\circ$$

$$\Gamma_A = 0.75 \angle 27.4^\circ$$

$$\Gamma_{in} = 0.51 \angle 31.1^\circ \Rightarrow Z_{in} = Z_c \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \cong 96 + j68 \text{ } [\Omega]$$

