# Electrical Quantities

Given an electrical signal s(t) in the time domain we can introduce the following quantities:

- Instantaneous value s(t)=v(t) (voltage)
- Average value: $\overline{s} = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{+T} s(t)\,dt$
- Instantaneous power: $P(t) = \frac{1}{R} s^2(t)$
- Mean Power: $\overline{P} = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{+T} \frac{s^2(t)}{R}\,dt$
- Mean Energy $\overline{E} = \lim_{T\to\infty} \int_{-T}^{+T} \frac{s^2(t)}{R}\,dt$

# Periodic signal

- Average value: $\overline{s} = \frac{1}{T} \int_{-T/2}^{+T/2} s(t)\,dt$
- Mean Power: $\overline{P} = \frac{1}{T} \int_{-T/2}^{+T/2} \frac{s^2(t)}{R}\,dt$
- Effective value $s_e = \sqrt{\frac{1}{T} \int_{-T/2}^{+T/2} s^2(t)\,dt}$
- Mean Energy $\overline{E}_a = \int_{-T/2}^{+T/2} \frac{s^2(t)}{R}\,dt$

# Ratio between quantities

- The ratio may represent gain (no dimensions) or immittance (V/I or I/V)
- In case of gain are often used logarithmic units:

$$R_{dB} = 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

The quantities are Powers

Taking into account that $P = k \cdot s^2$, it has also:

$$R_{dB} = 10 \cdot \log_{10}\left(\frac{k \cdot s_{out}^2}{k \cdot s_{in}^2}\right) = 20 \log_{10}\left(\frac{s_{out}}{s_{in}}\right)$$

The quantities are voltages or currents

***$R_{dB}$* remains the same in both cases**

# Definition of Neper

- Is a logarithmic unit used sometime instead of dB:

$$R_{Np} = \log_e\left(\frac{S_{out}}{S_{in}}\right)$$

The quantities are voltages or currents

$$R_{Np} = \frac{1}{2}\cdot\log_e\left(\frac{P_{out}}{P_{in}}\right)$$

The quantities are Powers

It has:

$$1\ \mathrm{Np} = 20\log_{10}(e) \cong 8.686\ \mathrm{dB}$$

# Absolute Quantities expressed in logarithmic units

- Logarithmic units can be used for representing absolute quantities (typically power) by assigning a reference value.

It is quite usual to assign as reference power 1 mW, then:

$$P_{dBm} = 10 \cdot \log_{10}\left(\frac{P}{1\ \mathrm{mW}}\right)$$

Power in dBm

# Sum of quantities in logarithmic units

- The sum of two powers expressed in dBm is NOT the sum of the dBm values!

$$P_{tot,dBm} = 10 \cdot \log_{10}\left(10^{\frac{P_{1,dBm}}{10}} + 10^{\frac{P_{2,dBm}}{10}}\right)$$

# Sum of two signals

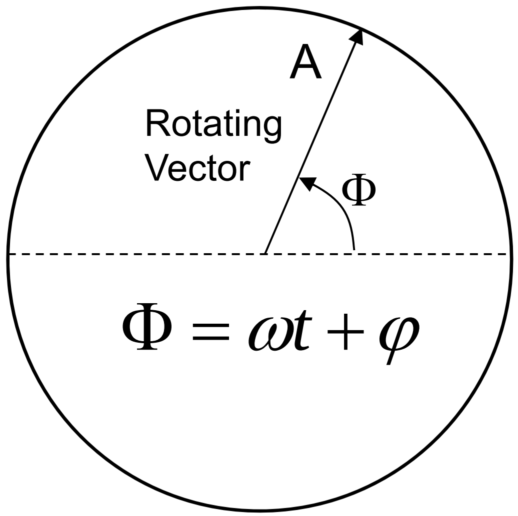
- The sum can be made in power or in voltage (current)
- If there is no correlation between the two signals the results is the same (i.e. the power of the resulting signal is the same in both cases
- In general, for deterministic signals $s_1$(t) and $s_2$(t), the result is the same if:

$$\lim_{T \to \infty} \frac{1}{T} \int_{-T}^{+T} s_1(t) \cdot s_2(t) dt = 0$$

- For sinusoidal signals: if the two signals are equal the sum in voltage gives a power twice with respect the sum of the powers. If the frequencies are different the sum in power and voltage is the same (the signals are uncorrelated)

# Sinusoidal signals: phasor representation

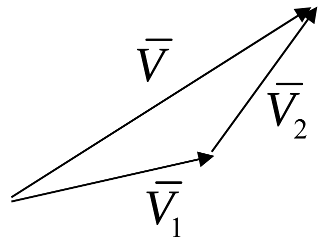
$$v(t) = A\cos(\omega t + \varphi)$$




Phasor Notation: $\overline{V} = Ae^{j\varphi}$

Power: $\overline{P} = \frac{1}{T}\int_{-T}^{+T} \frac{s^2(t)}{R} dt = \frac{1}{2}\frac{A^2}{R}$

Sum of two phasors:




$$\overline{V} = \overline{V}_1 + \overline{V}_2 = A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2}$$

$$P = \frac{|A|^2}{2} = \frac{A_1^2}{2} + \frac{A_2^2}{2} + A_1 A_2 \cos(\varphi_1 - \varphi_2) = P_1 + P_2 + P_{cross}$$

For $A_1=A_2=A_0$ → $P=2P_0$ (as already observed)

# Phasors and RF signals

- We have seen than a RF signal can be represented with the complex envelope notation:

$$s_{RF}(t) = V_M(t)\cos\left(\omega_0 t + \Phi(t)\right) \quad \Rightarrow \quad V_M(t)\exp\left(j\Phi(t)\right)$$

- It is evident the similitude of complex envelope with phasor notation. Actually the complex envelope vector is a phasor whose amplitude and phase are variable in time
- In most cases all operations performed on phasors can be also applied to RF vectors. Some caution must be exercised when the power must be computed (the time average of the squared envelope is involved).

# Friis equation computations

$$P_{r,dBm} = P_{t,dBm} - 20 \cdot \log\left(\frac{4\pi R}{\lambda}\right) + G_{t,dB} + G_{r,dB} + g\left(\bar{\vartheta}_r, \bar{\varphi}_r\right)\Big|_{dB}$$

$$+ f\left(\bar{\vartheta}_t, \bar{\varphi}_t\right)\Big|_{dB}$$

$$L_f = 20 \cdot \log\left(\frac{4\pi R}{\lambda}\right) \equiv \text{Free Space Attenuation}$$

For $\lambda$=10cm (f=3 GHz):

| R | $L_f$ |
|---|---|
| 100m | 81.98 dB |
| 10Km | 121.98 dB |
| 36000 Km | 193.11 dB |
| 384400Km | 213.68 dB |

Doubling the frequency, Lf increases of 6 dB.
Considering fixed area antennas (like disk antennas), the gain is proportional to the frequency squared :

$$G = A_e \frac{4\pi}{\lambda^2} = A_e \frac{4\pi}{c^2} f^2$$

So, doubling the frequency, also the antenna gain increases of 6 dB and the received power remain the same.
It must be however remarked that the beamwidth depends on 1/G, so increasing the frequency the radiated power is more concentrated in the desired direction