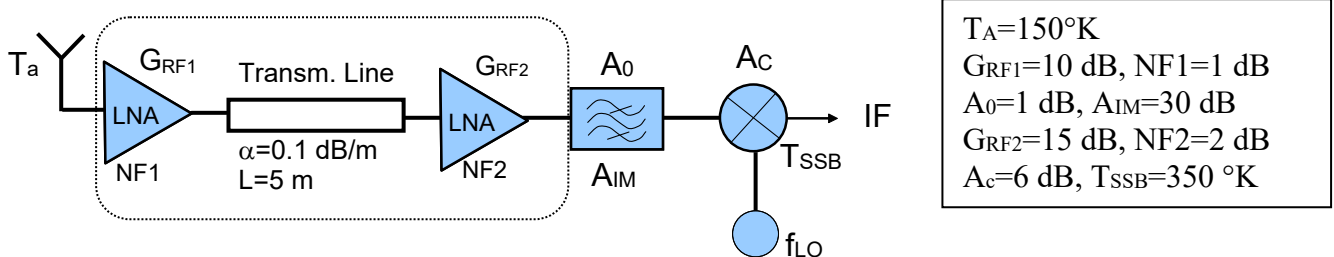


**RF SYSTEMS**  
**1<sup>th</sup> February 2017**

<b>Surname &amp; Name</b>
<b>Identification Number</b>
<b>Signature</b>

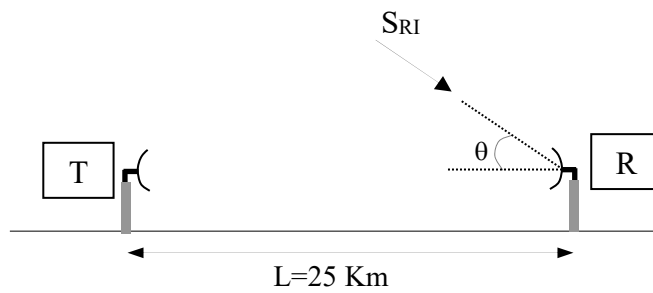
Exercise 1

The scheme in the figure represents the RF front-end of a receiver operating at 12 GHz. The two LNAs are separated by a transmission line 5 m long (attenuation  $\alpha=0.1$  dB/m). The filter is tuned at the center frequency of the receiver (12 GHz) with 10 MHz passband and 1 dB attenuation in passband; the attenuation in the stopband at the image frequency is  $A_{IM}=30$  dB. The other parameters are reported below.



- a) Assuming the IF frequency equal to 140 MHz and the local oscillator frequency ( $f_{LO}$ ) above the signal band, where is located the image band?
- b) The elements inside the dashed box are connected in cascade. They can be replaced by an equivalent amplifier with  $G_{RF,eq}$ ,  $NF_{eq}$ . Evaluate these parameters (assume  $G_a=G_T$  for all the elements. Hint: the transmission line operates as a dissipative attenuator the noise figure of which is equal to its attenuation).
- c) Draw the equivalent scheme for the evaluation of the noise temperature
- d) Evaluate the equivalent noise temperature  $T_{eq}$  at the input of the front-end

## Exercise 2

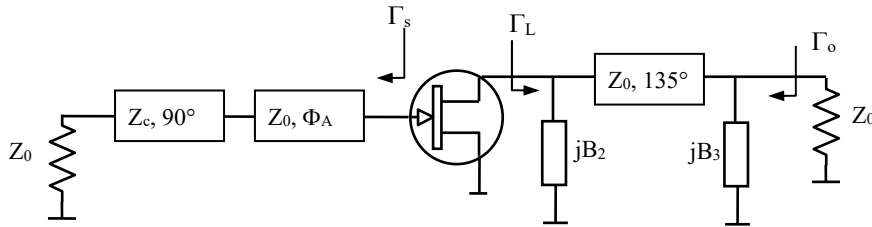


The transmitter operates at 6 GHz with a transmitted power  $P_T=10$  dBm. The receiver is located 25 Km apart from the transmitter. The two antennas are equal, optimally pointed and exhibit  $G_T=G_R=25$  dB with directivity function  $f(\theta)=\cos^4(\theta)$ . A QAM signal with band  $B=10$  MHz is employed and the receiver is characterized by  $E_B/N_0=15$  dB and  $T_{sys}=500$  °K (System Noise Temperature).

- 1) Discarding  $S_{RI}$ , evaluate the  $SNR_{sys}$  at the input of the receiver and the maximum allowed data rate  $R$ .
- 2) Consider now  $S_{RI}=5 \cdot 10^{-12}$  W/m<sup>2</sup>. It represents the power density of a wave incident at the receiving antenna with  $\theta=45^\circ$ . Assuming this signal as an interference noise with power  $P_{NI}$ , the overall noise at the receiver input becomes  $KT_{sys}B+P_{NI}$ .
  - a) Evaluate the power  $P_{NI}$ .
  - b) Evaluate the new transmitted power from  $T$  that determines the same  $SNR_{sys}$  in absence of the interfering signal.

### Exercise 3

The following scheme shows a single stage low noise amplifier operating at 12 GHz



The transistor is characterized by the following parameters ( $Z_0=50 \Omega$ ):

$$S_{11}=0.569 \angle 78.2^\circ, S_{12}=0.1 \angle -58.5^\circ, S_{21}=3.226 \angle -52.1^\circ, S_{22}=0.132 \angle 120.7^\circ$$

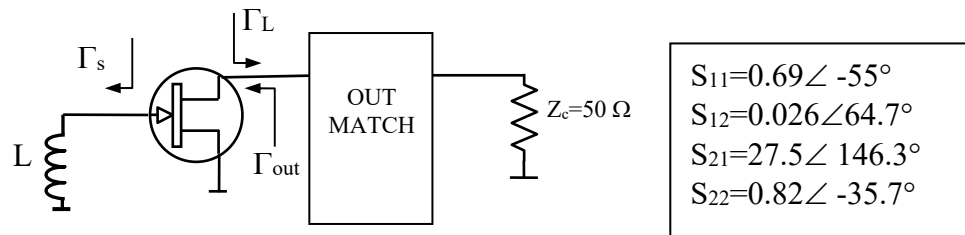
$$NF_{\min}=0.51 \text{ dB}, \Gamma_{\min}=0.358 \angle -137.2^\circ, r_n=0.12$$

The amplifier must be designed for  $NF=0.8 \text{ dB}$  (the transducer gain must be maximized compatibly with the  $NF$  imposed).

- 1) Evaluate the values of  $\Gamma_s, \Gamma_L$  in order to fit the requirements
- 2) Design the two transforming networks.
- 3) What is the output reflection coefficient ( $\Gamma_o$ ) of the amplifier?

#### Exercise 4

The following scheme refers to an oscillator working at  $f_{osc}=425$  MHz. The S parameters of the transistor are also reported on the figure.



- 1) Imposing  $|\Gamma_{out}|=1.3$  compute the values of  $\Gamma_s$  and  $\Gamma_L$  determining the start of oscillation
- 2) Evaluate the values of  $L$  producing the requested value of  $\Gamma_s$
- 3) Check the conditions for the start-up of oscillation
- 4) Chose a suitable topology and design the output network

## Solution

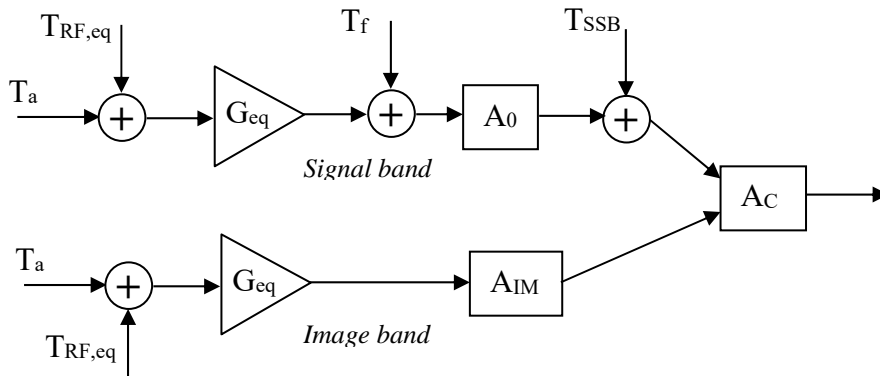
### Exercise 1

a) It has  $f_{im}-f_{ol}=f_{IF}$  and  $f_{ol}-f_{RF}=f_{IF}$ . then  $f_{im} = f_{RF}+2f_{IF} = 12.28$  GHz

b) The transmission line is equivalent to an attenuator with  $NF_L=A=0.1\cdot 5=0.5$  dB and gain  $G_L=-0.5$  dB. The overall gain (available) is then  $G_a=G_{RF1}+G_L+G_{RF2}=24.5$  dB. The transducer gain is equal to  $G_a$  being the output of LNA2 matched:  $G_{eq}=24.5$  dB. For the overall noise figure:

$$NF_{eq} = NF_1 + \frac{NF_L - 1}{G_{RF1}} + \frac{NF_2 - 1}{G_{RF1}G_L} = 1.33 \text{ (1.26 dB)}$$

c) Considering both the RF and the image channels it has:



With  $T_{RF,eq}=290(10^{NF_{eq}/10}-1)=95.7$  °K,  $T_f=290(10^{A_0/10}-1)=75.09$  °K

d)  $T_{eq}$  at the input of the receiver is evaluated as:

$$T_{eq} = T_a + T_{RF,eq} + \frac{T_f}{G_{eq}} + T_{SSB} \frac{A_0}{G_{eq}} + (T_a + T_{RF,eq}) \frac{A_0}{A_{IM}} = 247.84 \text{ °K}$$

## Exercise 2

The  $SNR_{sys}$  is defined as  $SNR_{sys} = \frac{P_r}{KT_{sys}B} = \left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right)$ , where  $P_r$  is obtained from the Friis

equation:  $P_r = P_t \left(\frac{\lambda}{4\pi L}\right)^2 G_T \cdot G_R$  with  $\lambda = 3 \cdot 10^8 / 6 \cdot 10^9 = 0.05$  m. Replacing we get  $P_r$ :

$P_r = 10 + 25 + 25 - 10 \log(3.95 \cdot 10^{13}) = -76$  dBm ( $2.5119 \cdot 10^{-11}$  W).

$SNR_{sys}$  is then obtained:  $SNR_{sys} = -76 - 30 - 10 \log(1.38 \cdot 10^{-23} \cdot 500 \cdot 10^7) = 25.6$  dB

We can compute the data rate  $R$  from the first equation:

$$R = B \frac{SNR_{sys}}{(E_b/N_0)} = 10 \cdot 10^{2.56-1.5} = 114.8 \text{ Mbit/s}$$

The power received from the interference is given by

$$P_{NI} = A_e S_{RI} f(45^\circ) = S_{RI} G_R \frac{\lambda^2}{4\pi} \cos^4(45^\circ) = 7.86 \cdot 10^{-14} \text{ W}$$

The noise power from the receiver is  $KT_{sys}B = 6.9 \cdot 10^{-14}$  W. Then, in order to maintain the  $SNR_{sys}$  unchanged we must have:

$$SNR_{sys} = \frac{P'_r}{P_{NI} + KT_{sys}B} = \frac{P_r}{KT_{sys}B} \Rightarrow \frac{P'_r}{P_r} = 1 + \frac{P_{NI}}{KT_{sys}B} = 2.139$$

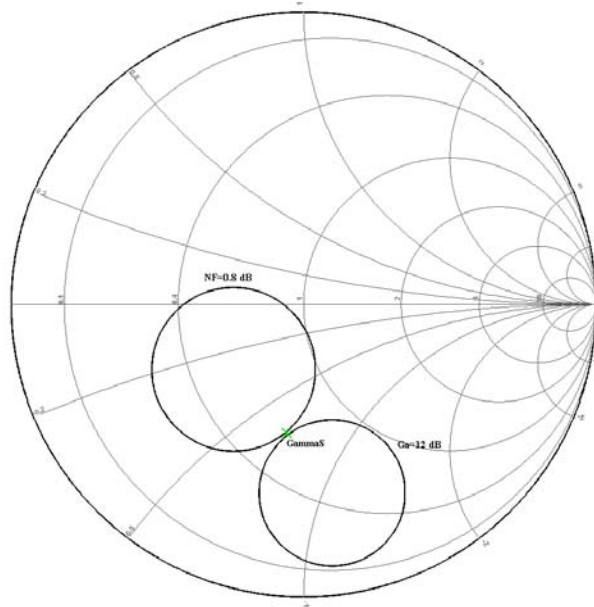
The transmitted  $P'_t$  power must increase of the same amount, then:

$$P'_t = P_r \left(1 + \frac{P_{NI}}{KT_{sys}B}\right) = 10 \cdot 2.139 = 21.39 \text{ mW (13.3 dBm)}$$

### Exercise 3

Inserting the S parameters in the e-Smith Chart we discover that the transistor is unconditionally stable with  $G_{Tmax}=12.788$ ,  $\Gamma_{S,opt}=0.74\angle-81.4$ ,  $\Gamma_{L,opt}=0.51\angle-155.2$ .

1) We draw the circle with  $NF=0.8$  dB on the S. C. Then, in order to find the value of  $\Gamma_s$  determining the maximum available gain compatible with the assigned NF, we draw some circles with  $G_a=\text{const}$  ( $< G_{Tmax}$ ) and look for the one about tangent to the NF circle:



We found  $G_a=11.96$  dB. The tangent point gives  $\Gamma_s=0.44\angle-97.9^\circ$ . In order to get  $G_a=G_T$  we impose conjugate matching at the output of the transistor getting from the S.C.:  $\Gamma_L=(\Gamma_{out})^*=0.311\angle-135^\circ$ .

2) Input network: we move on the circle with  $|\Gamma|=|\Gamma_s|$  toward the load up to the intersection with the real axis  $\rightarrow \Phi_A=48.9^\circ$ . The impedance seen in this point is  $Z=2.571\cdot 50=128.55 \Omega$ . The characteristic impedance  $Z_c$  of the  $\lambda/4$  transformer is the given by  $Z_c=\text{sqrt}(128.55\cdot 50)=80.17 \Omega$ .

Output network: draw the circle  $g=1$  rotated by  $270^\circ$  toward the source. Set the current point to  $\Gamma_L$  and store in memory. Draw the circle  $g=\text{const}$  passing for  $\Gamma_L$  and select one intersection between the two circles ( $\Gamma=0.161\angle-170.76^\circ$ ). The value of imaginary part of  $\Delta Y$  with the sign reversed gives  $b_2=0.594$ . Give an increment to the current point  $\Gamma$  by  $+270^\circ$ ; the new current point has  $y=1+jb_3 \rightarrow b_3=-0.325$ .

3) Being the transistor output in conjugate matching and the output network lossless also the amplifier output is matched, i.e.  $\Gamma_o=0$ .

#### Exercise 4

Inserting the scattering parameter into the S.C. we discover the device potentially instable and the suitable for an oscillator.

- 1) Draw the mapping circle for  $|\Gamma_{out}|=1.3$  and select one of the intersection with the unit circle:  $\Gamma_s=1\angle 132.31^\circ$ . The corresponding reactance results  $X_s=0.442\cdot 50=22.1\ \Omega$ . Evaluate  $\Gamma_{out}=1.3\angle -10.4 \rightarrow Z_{out}=-5.21-j3.53$ . The assign  $Z_L=1.7+j3.52$ . Using the S.C. we enter this value as current point and compute  $|\Gamma_{in}|=1.79$ , so the oscillation start up is guaranteed.
- 3) We have at  $f_{osc}=425\text{ MHz}$ :  $X_s = \omega_{osc} L = 22.1$ . Replacing  $\omega_{osc}=2\pi\cdot 425\text{ MHz}$  we get  $L=8.28\text{ nH}$ .
- 4) Using a single stub network:  $\Phi=58.75^\circ$ ,  $b=-2.73$ .