RF SYSTEMS -
Written Test of February 3th, 2016

Surname & Name
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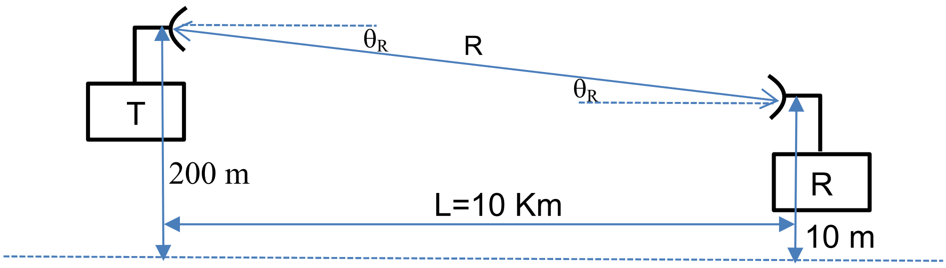
Exercise 1



A terrestrial link operating at 6 GHz is illustrated in the above figure. The antennas of the receiver and transmitter are identical and are characterized by the following directivity function:

$$f(\theta)=\left[1-\frac{\sin(\theta)}{\sin(\theta_0)}\right] \qquad 0<\theta<\theta_0 \qquad \theta_0=\pi/18$$

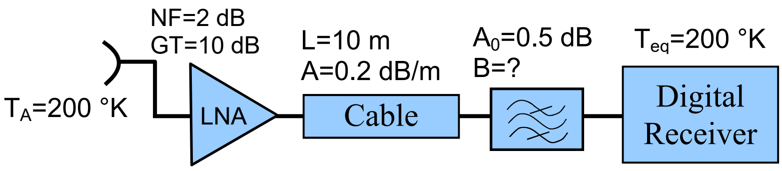
$$f(\theta)=0 \qquad \theta_0<\theta<\pi$$

The radiation impedance Z_R and the loss resistance R_p of the antennas are also given: $Z_R=50\Omega$, $R_p=12.64\ \Omega$.

1) Evaluate the gain of the antennas [Hint: $\int \sin^2(x)dx=\frac{1}{2}(x-\sin(x)\cos(x))$]

The transmitted signal is a 64-QAM modulated carrier with the following parameters: roll-off factor $\alpha=0.3$, data rate R=155 Mbit/sec, transmitted power $P_T=10$ dBm.

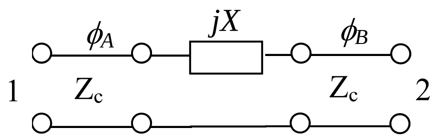
The RF front-end of R is schematized in the following figure:



1) Evaluate the filter bandwidth
2) Evaluate the system SNR_{sys} and the (E_b/N_0) ratio requested by the digital receiver
3) Assume the P_{1dB} of the transmitter equal to 20 dBm. Evaluate the transmitted mean power (2-tone signal) determining the intermodulation mean power equal to the system noise power at the input of R

Exercise 2

Consider the following 2-port circuit:

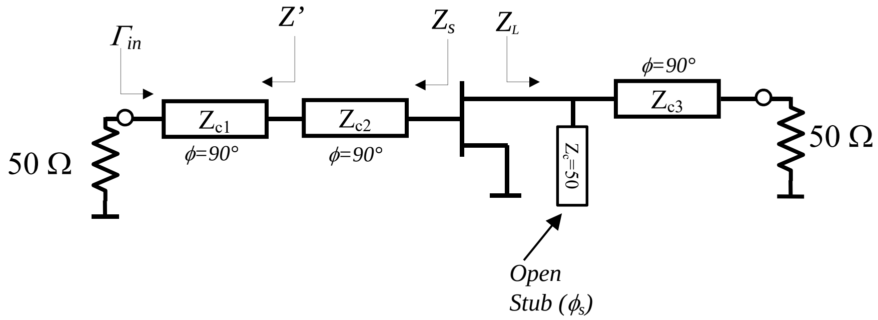


Z_c=50 Ω
ϕ_1=45°
ϕ_2=100°
X=100 Ω

Evaluate the scattering parameters S_{11}, S_{22}, S_{12}. Hint: use the eigenvalues method for the inner part (reactance X) and then move the reference sections of the two ports.

Exercise 3

The low noise amplifier at 2 GHz with the scheme in the figure must be designed.



$S_{11}=0.519\angle{-133.27°}$
$S_{12}=0.095\angle 17.7°$
$S_{21}=4.071\angle 41.2°$
$S_{22}=0.392\angle{-90.39°}$

$NF_{min}=0.35$ dB
$\Gamma_{s,opt}=0.52\angle 94.6$
$r_n=0.081$

The goal of the design is to obtain the transducer gain G_T>14 dB and the noise figure NF=1 dB.

A) Evaluate Z_S and Z_L in order to fit this requirements.
B) Evaluate Z_{c1}, Z_{c2}, Z_{c3} and ϕ_s.
C) Evaluate the input reflection coefficient Γ_{in}

The input network is constituted by two sections of transmission lines λ/4 long. The chosen value of Zs must be compatible with this network topology (for computing Z_{c1} and Z_{c2} select a suitable value for Z').

The stub length ϕ_s and the characteristic impedance Z_{c3} of the line must be evaluated for the output network (note that the line reports a real admittance in parallel to the stub).

Solution

Exercise 1

Efficiency: $\eta = \frac{Z_R}{Z_R + R_p} = 0.7982$

Antenna Gain:

$$G = \frac{4\pi\eta}{\int_0^{2\pi} d\varphi \int_0^{\theta_0} \left(1 - \sin(\theta)/\sin(\theta_0)\right)\sin(\theta)\, d\theta} = \frac{2\eta}{1 - \cos(\theta_0) - \left[\frac{\theta_0 - \sin(\theta_0)\cos(\theta_0)}{2\sin(\theta_0)}\right]} = 316.23 \text{ (25 dB)}$$

Link Equation:

$$P_r = P_t - 20 \cdot \log\left(\frac{4\pi R}{\lambda}\right) + 2G_A + 20 \cdot \log\left(f(\theta_R)\right)$$

$$R = \sqrt{L^2 + 0.19^2} = 10.002 Km, \quad \theta_R = \tan^{-1}\left(\frac{0.19}{10}\right) = 1.0885°$$

$$f(\theta_R) = 1 - \frac{\sin(1.0885°)}{\sin(10°)} = 0.8906$$

Substituting:

$$P_r = 10 - 20 \cdot \log\left(\frac{4\pi 10^4}{5 \cdot 10^{-2}}\right) + 2 \cdot 25 + 20 \cdot \log(0.8906) \cong -69 \text{ dBm}$$

Expression of SNR_{sys}:

$$SNR_{sys} = \frac{P_r}{KT_{sys}B}, \quad B = \frac{R(1+\alpha)}{\log_2 64} = 33.58 \text{ MHz}, \ K = 1.38 \cdot 10^{-23}.$$

Evaluation of T_{sys}:

$$T_{sys} = T_A + T_{LNA} + \frac{T_1}{G_T} + T_{eq}\frac{A_1}{G_T}$$

$$T_{LNA} = 290\left(10^{0.2} - 1\right) = 169.62, \quad G_T = 10$$

$$A_1 = A \cdot 10 + A_0 = 2.5 \text{ dB}, \quad T_1 = 290\left(10^{0.25} - 1\right) = 225.7$$

$$T_{sys} = 427.75 \text{ °K}$$

Evaluation of SNR_{sys}:

$$SNR_{sys} = \frac{P_r}{KT_{sys}B} = \frac{1.259 \cdot 10^{-10}}{1.98 \cdot 10^{-13}} = 635.82 \text{ (28 dB)}$$

$$\frac{E_b}{N_0} = \frac{SNR_{sys}}{(R/B)} = \frac{635.82}{4.61} = 138 \quad \text{(21.4 dB)}$$

Received intermodulation power ($P_{int,rec}$):

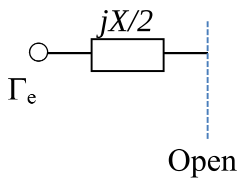
$$P_{int,rec} = P_{int} - 20\cdot\log\left(\frac{4\pi R}{\lambda}\right) + 2G_A + 20\cdot\log\left(f\left(\theta_R\right)\right) = P_{int} - 79 = P_{noise} = -97 \text{ dBm}$$

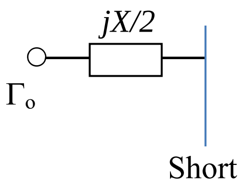
$$P_{int} = 79 - 97 = -18 \text{ dBm} = 3\left(P_m - 3\right) - 2IP_3 + 3 = 3P_m - 2P_{1dB} - 26$$

$$P_m = \frac{P_{int} + 2P_{1dB} + 26}{3} = 16 \text{ dBm}$$

Exercise 2

Let consider the inner part. The eigen-networks and the corresponding eigenvalues result:





$$\Gamma_e = 1 \text{ , } \Gamma_o = \frac{jX/2 - 50}{jX/2 + 50}$$

Then:

$$S'_{11} = \frac{1}{2}\left(\Gamma_e + \Gamma_o\right) = \frac{j100}{100 + j100} = \frac{j}{1+j} = \frac{1}{\sqrt{2}}e^{j\frac{\pi}{4}}$$

$$S'_{12} = \frac{1}{2}\left(\Gamma_e - \Gamma_o\right) = \frac{100}{100 + j100} = \frac{1}{1+j} = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{4}}$$

Now, moving outwards the reference sections, we get the scattering parameters of the complete 2-port:

$$S_{11} = S'_{11}e^{-2j\phi_A} = \frac{1}{\sqrt{2}}e^{j\left(-\frac{\pi}{2}+\frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{4}}$$

$$S_{22} = S'_{11}e^{-2j\phi_B} = \frac{1}{\sqrt{2}}e^{j\left(-\frac{10\pi}{9}+\frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}}e^{-j\frac{31\pi}{36}}$$

$$S_{12} = S'_{12}e^{-j\left(\phi_A+\phi_B\right)} = \frac{1}{\sqrt{2}}e^{j\left(-\frac{29\pi}{36}-\frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}}e^{-j\frac{19\pi}{18}}$$

Exercise 3

The value of Zs must be on the circle with NF=1 dB and inner to the circle Gav=14 dB. Moreover, due to the topology of the input network Zs must be real. Using the electronic Smith chart we find that there is only one point on the Γs plane satisfying the above conditions:

$$\Gamma_s = 0.501\angle 180° \Rightarrow \quad z_s = 0.332 \Rightarrow \quad Z_s = 50\cdot z_s = 16.6\ \Omega$$

The two λ/4 transformers operate as impedance inverters with K=Zc. Imposing Z'=30 Ω (about in the middle of the range Zs→50) we get:

$$Z_{c1} = \sqrt{50 \cdot Z'} = 38.73\ \Omega, \qquad Z_{c2} = \sqrt{Z_s \cdot Z'} = 22.32\ \Omega$$

Using the Smith chart, selecting “optimum gamma, load” we get:

$$\Gamma_L = 0.615\angle 96.92°, \qquad G_T = 14.482\ \text{dB}, \qquad NF = 1\ \text{dB}$$

For designing the output network we observe that the impedance presented by this network at the transistor output is constituted by the parallel of the susceptance of the open circuited stub with the resistance obtained from the λ/4 transformer. The obtaining YL from the Smith chart:

$$y_L = 0.506 - j0.993 \Rightarrow \quad Y_L = \frac{y_L}{50} = 0.01012 - j0.01986$$

Then:

$$b_s = \tan(\phi_s) = -0.993 \Rightarrow \phi_s = 135.2°$$

$$\frac{Z_{c3}^2}{50} = \frac{1}{0.01012} \Rightarrow Z_{c3} = 70.29\ \Omega$$

For computing Gin we must evaluate first gamma at the input of the transistor ($\Gamma_{in,t}$) from the smit chart (“S Param., Gamma IN”):

$$\Gamma_{in,t} = 0.696\angle -158.035° \quad (Z_{in,t} = 9.3 - j9.4)$$

Then we evaluate the impedance Z_1 at the input of the second λ/4 transformer (see the figure):

$$Z_1 = \frac{Z_{c2}^2}{Z_{in,t}} = 26.5 + j26.8$$

The input impedance is obtained from the first λ/4 transformer:

$$Z_{in} = \frac{Z_{c1}^2}{Z_1} = 28 - j28.3 \quad (z_{in} = 0.56 - j0.57)$$

$$\Gamma_{in} = 0.432\angle -107.91° \ \text{(from the Smith chart)}$$