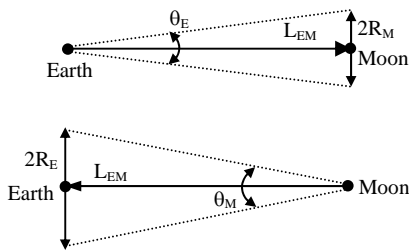


**RF SYSTEMS**  
**Written Test of February 9<sup>th</sup>, 2015**

<b>Surname &amp; Name</b>
<b>Identification Number</b>
<b>Signature</b>

Exercise 1

A bi-directional communication link must be established between Earth and Moon. Each station employs a single antenna for transmitting and receiving. The link Earth → Moon operates at  $f_{TE}=6$  GHz and the opposite (Moon → Earth) at  $f_{TM}=4$  GHz. The antennas must be dimensioned in order that their beam width correspond to the solid angle under which the moon is seen by the earth (earth antenna) and the earth is seen by the moon (moon antenna). The following picture defines the geometry and the parameters of the links.



$L_{EM}$ (Earth-Moon distance): 384400 Km
$R_E$ (Earth radius): 6371 Km
$R_M$ (Moon radius): 1378 Km
$\theta_E$ Beam width of the earth antenna
$\theta_M$ Beam width of the moon antenna

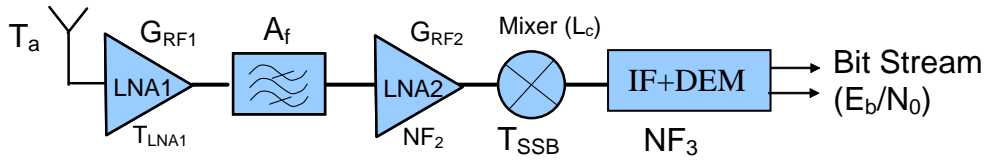
- a) Evaluate the beam width of the two antennas and the corresponding directivity gain (use the approximate formulas). Assuming that dish antennas are employed, evaluate their diameter. Dimension the antennas at the transmitting frequencies ( $f_{TE}, f_{TM}$ ); assume the aperture efficiency  $e_a=0.65$  and the efficiency factor  $\eta=0.7$  for both antennas.
- b) Assuming that the power transmitted from the Earth is 500W and that transmitted from the Moon is 50W evaluate the power of the received signals at the two stations
- c) The SNR (signal-to-noise ratio) at the input of the receivers must be at least 20 dB. The signal bandwidth is 100 MHz and the noise temperature of the antennas is 50 °K (the transmitted powers are the same as in previous point). Compute the maximum values of the equivalent noise temperature at input of the receivers in order to meet the SNR requirement.

Use  $K=-228.6$  dBW/(Hz·°K) (Boltzmann Constant)

### Exercise 2

The scheme in the figure represents the RF front-end of a receiver for a satellite earth station operating at 4 GHz with signal bandwidth 100 MHz.

The goal is to get the system SNR equal to 20 dB with the power of the received signal equal to -109.2 dBW



$T_A=25^\circ\text{K}$
$G_{RF1}=? \text{ dB}$
$T_{LNA1}=20^\circ\text{K}$
$A_f=0.1 \text{ dB}$
$G_{RF2}=20 \text{ dB}$
$NF_2=1 \text{ dB}$
$L_c=4 \text{ dB}$
$T_{SSB}=200^\circ\text{K}$
$NF_3=2 \text{ dB}$

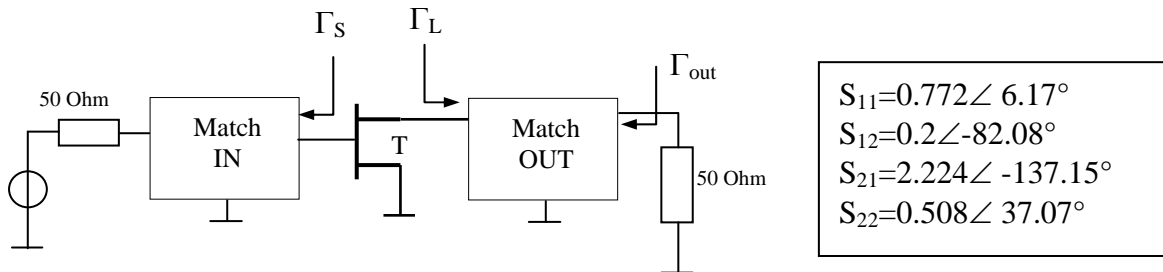
- Evaluate the gain  $G_{RF1}$  of LNA1 in order to meet the goal
- What is the maximum data rate  $R$  of the demodulated bit stream with  $(E_b/N_0) = 15 \text{ dB}$ ?

Hint: Use the expression relating the system SNR to  $E_b/N_0$

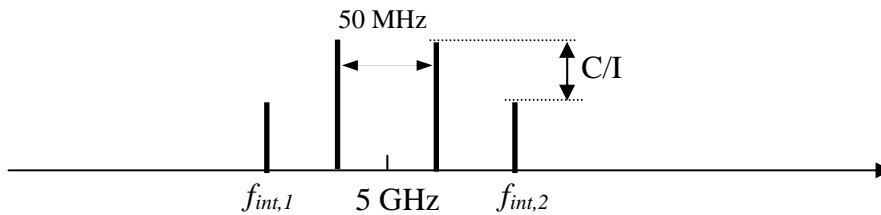
Use  $K=-228.6 \text{ dBW}/(\text{Hz}\cdot^\circ\text{K})$  (Boltzmann Constant)

Exercise 3

The following figure is the scheme of a single stage amplifier to be designed at 5 GHz. The design goal is to get the largest value for the transducer gain compatible with the stability condition. Moreover we also require that the output is matched ( $\Gamma_{out}=0$ )



- 1) Choose  $\Gamma_S$  e  $\Gamma_L$  in order to meet the requirements. Specify the corresponding transducer gain
- 2) Design the Match OUT network (single-stub configuration)
- 3) Assume a 2-tone driving signal with  $f_0=5$  GHz and  $\Delta f=50$  MHz. Assuming the 3th order intercept point of the device  $IP_3=40$  dBm, compute the output peak power for which the ratio  $C/I$  is equal to 30 dB. What is the power in each intermodulation line? What are the frequencies ( $f_{int,1}, f_{int,2}$ ) of the intermodulation lines?



## Solution

### Exercise 1

The beam width angles are immediately derived from the figures:

$$\theta_E = 2 \tan^{-1} \left( \frac{R_M}{L_{EM}} \right) = 0.411^\circ, \quad \theta_M = 2 \tan^{-1} \left( \frac{R_E}{L_{EM}} \right) = 1.9^\circ$$

The directivity gain of the antennas is obtained with the following formula:  $\cos(\theta/2) = 1 - 2/D$  :

$$D_{ET} = \frac{2}{1 - \cos(\theta_E/2)} = 3.113 \cdot 10^5 \text{ (54.93 dB)}, \quad D_{MT} = \frac{2}{1 - \cos(\theta_M/2)} = 1.4565 \cdot 10^4 \text{ (41.63 dB)}$$

The dishes radius is then obtained from:

$$\left( \frac{d}{\lambda} \right)^2 = \frac{D \cdot \eta}{\pi^2 \cdot e_a}$$

For the earth antenna  $\lambda_E = 3 \cdot 10^8 / 6 \cdot 10^9 = 0.05 \text{ m}$  while for the moon antenna  $\lambda_M = 3 \cdot 10^8 / 4 \cdot 10^9 = 0.075 \text{ m}$ .

Then:

$$d_E = \frac{\lambda_E}{\pi} \sqrt{\frac{\eta D_{ET}}{e_a}} = 9.21 \text{ m}, \quad d_M = \frac{\lambda_M}{\pi} \sqrt{\frac{\eta D_{MT}}{e_a}} = 2.99 \text{ m}$$

The gain of the antennas (transmission):

$$G_{ET} = D_{ET} + 10 \cdot \log(\eta) = 53.38 \text{ dB}, \quad G_{MT} = D_{MT} + 10 \cdot \log(\eta) = 40.08 \text{ dB}$$

The gain of antenna in reception is computed from the dish formulas:

$$G_{ER} = e_a \left( \frac{\pi d_E}{\lambda_M} \right)^2 = 49.86 \text{ dB}, \quad G_{MR} = e_a \left( \frac{\pi d_M}{\lambda_E} \right)^2 = 43.6 \text{ dB}$$

Computation of the links attenuation:

$$\text{Earth} \rightarrow \text{Moon (6Hz): } A_{EM} = 20 \cdot \log(4\pi L_{EM} / \lambda_E) = 219.7 \text{ dB}$$

$$\text{Moon} \rightarrow \text{Earth (4Hz): } A_{ME} = 20 \cdot \log(4\pi L_{EM} / \lambda_M) = 216.2 \text{ dB}$$

The received power at the two receiving stations is obtained with the Friis Equation:

$$P_{RM} = P_{TE} - A_{EM} + G_{ET} + G_{MR} = -95.72 \text{ dBW}, \quad P_{RE} = P_{TM} - A_{EM} + G_{MT} + G_{ER} = -109.24 \text{ dBW}$$

The noise at the antennas output is determined by the SNR (20 dB) and the received signals power:

$$N_E = P_{RE} - \text{SNR} = -129.24 \text{ dBW}, \quad N_M = P_{RM} - \text{SNR} = -115.72 \text{ dBW}.$$

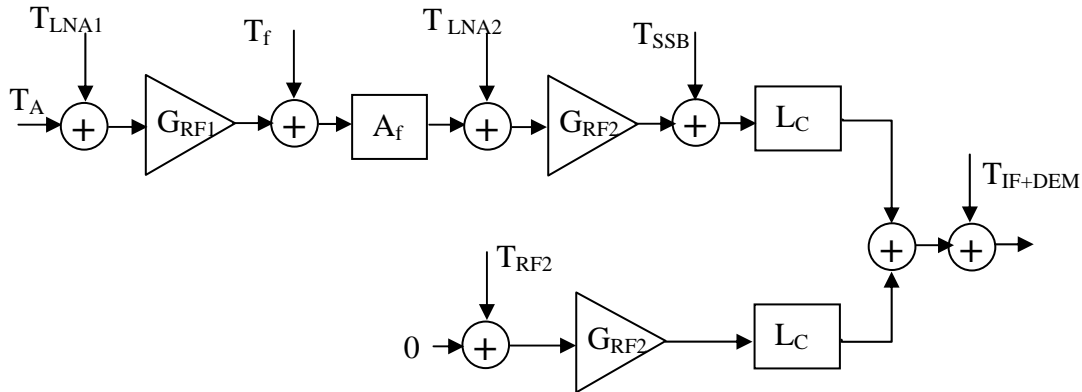
The noise power can be expressed as  $N = K T_{\text{sys}} B$ , with  $T_{\text{sys}} = T_a + T_{\text{rec}}$ . It is then obtained:

$$T_{\text{eq},E} = 86.22 \text{ }^\circ\text{K} \rightarrow T_{\text{rec},E} = 36.2 \text{ }^\circ\text{K}$$

$$T_{\text{eq},M} = 1939.7 \text{ }^\circ\text{K} \rightarrow T_{\text{rec},M} = 1889.7 \text{ }^\circ\text{K}$$

## Exercise 2

Equivalent noise scheme :



Where:

$$T_{LNA2} = 290 \left( 10^{\frac{NF_2}{10}} - 1 \right) = 75.088 \text{ °K}, \quad T_f = 290 \left( 10^{\frac{A_f}{10}} - 1 \right) T_0 = 6.755 \text{ °K}$$

$$T_{IF+DEM} = 290 \left( 10^{\frac{NF_3}{10}} - 1 \right) = 169.62 \text{ °K}$$

The equivalent noise temperature at the input of the receiver is then given by:

$$T_{sys} = T_A + T_{LNA1} + \frac{T_f}{G_{RF1}} + 2 \frac{T_{LNA2} A_f}{G_{RF1}} + \frac{T_{SSB} A_f}{G_{RF1} G_{RF2}} + \frac{T_{IF+DEM} L_C A_f}{G_{RF1} G_{RF2}}$$

$$= 45 + \frac{166.83}{G_{RF1}}$$

Imposing the SNR it results:  $N = KT_{sys}B = P_{rec} - SNR = -129.2 \text{ dBW}$ . The equivalent temperature is then given by:

$$10 \log(T_{sys}) = -129.2 - (KT_{sys})_{dB} = 19.4 \rightarrow T_{sys} = 87.1 \text{ °K}$$

Finally the gain is obtained:

$$G_{RF1} = \frac{166.83}{T_{eq} - 45} = 3.96 \text{ (6 dB)}$$

The system SNR is equal to 20 dB and is expressed as:

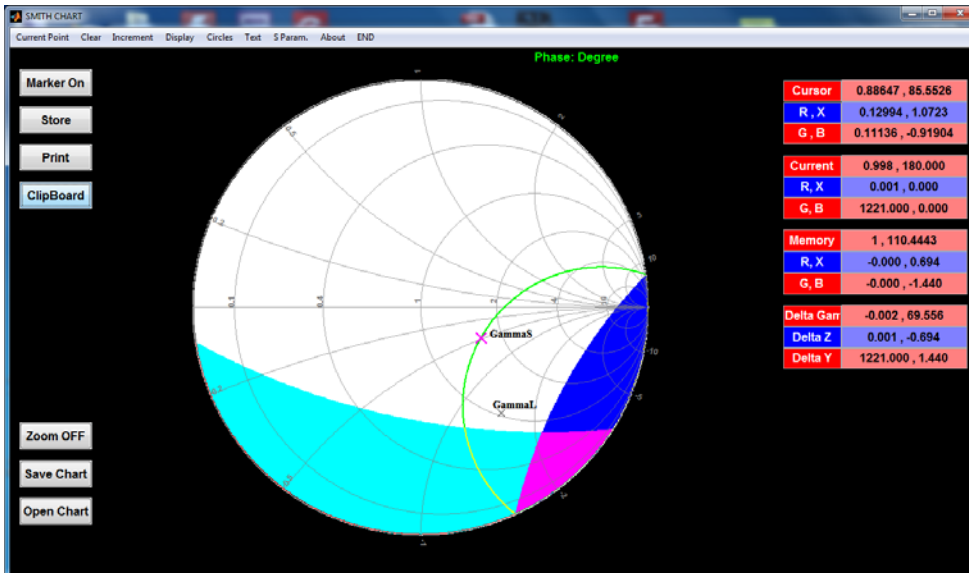
$$SNR = \frac{P}{N_0 \cdot B} = \frac{E_B}{N_0} \left( \frac{R}{B} \right) = 20 \text{ dB}$$

$$\text{then: } (R/B)_{dB} = 20 - 15 = 5 \rightarrow R = 3.16 \cdot B = 316 \text{ Mbit/sec}$$

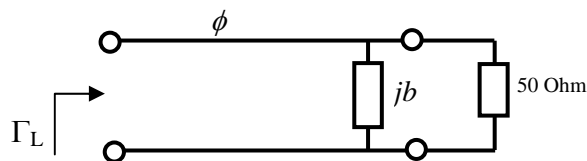
### Exercise 3

After inserting on the electronic S.C. the S parameters, we get the transistor information (potentially instable, MGS=10.46 dB). Then we draw the stability circles for source and load.

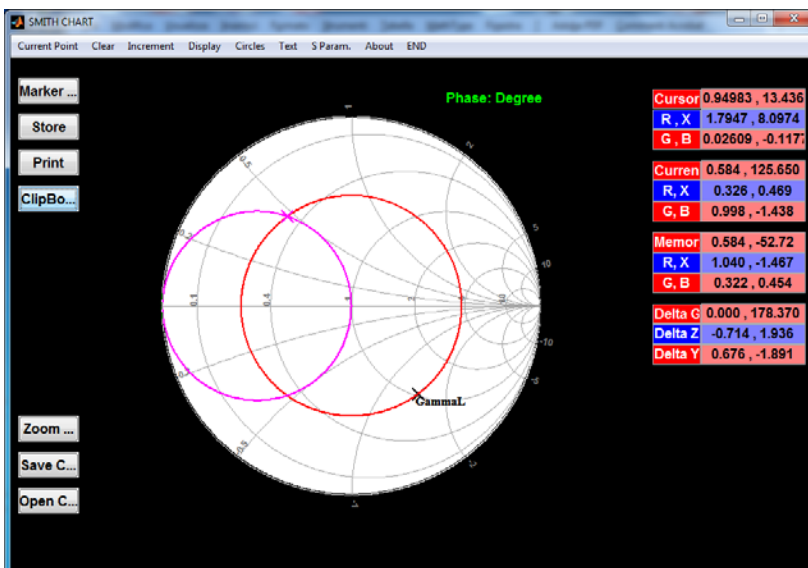
Being the output matched required, we draw the circle with available power gain equal to 10.4 and select  $\Gamma_S = 0.3 \angle -26.85^\circ$ . The conjugate match at the transistor output determine  $\Gamma_L = 0.884 \angle -52.72^\circ$  and  $G_T = G_{AV} = 10.4$  dB



The OUT network is implemented as a single stub matching network:



Using the S.C. we enter  $\Gamma_L$ , store it, and draw the circle with const  $|\Gamma|$ . Then draw the circle  $g=1$  and select one of the intersection between the two circles.



The angle of "DeltaG" tab gives  $2\phi = 178.37 \rightarrow \phi = 89.18^\circ$ .

The imaginary part of Y (current point tabs) gives  $b = -1.438$ .

The susceptance  $b$  is implemented as a short-circuited stub with:

$$\phi_s = \tan^{-1}\left(\frac{1}{1.438}\right) = 34.78^\circ$$

It has  $CI=2(IP3-P_0)$ , with  $P_0$  power per line at  $f_0$ . Then  $P_0=IP3-CI/2=25$  dBm.

Moreover  $PEP=P_0+6=31$  dBm.

The power in each intermod. line is  $P_0-CI=-5$  dBm.

The frequencies of intermod. line are  $5\pm 0.075=(4.925,5.075)$  GHz