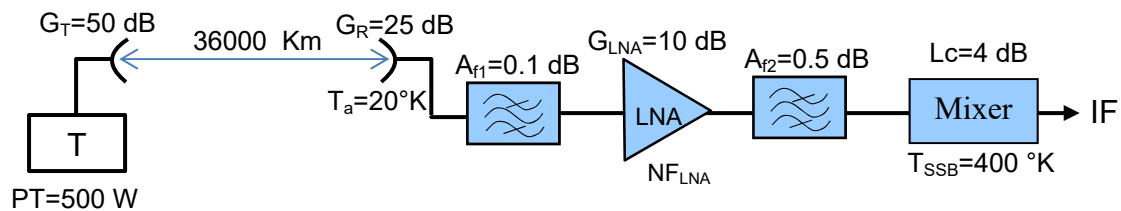


**RF SYSTEMS**  
**Mid-course Test - November 19<sup>th</sup>, 2015**

<b>Surname &amp; Name</b>
<b>Identification Number</b>
<b>Signature</b>

Exercise 1



Consider the scheme in the figure, which refers to the up-link of a satellite communication system operating at 6.4 GHz with a signal having  $B = 32 \text{ MHz}$  bandwidth. All the relevant system parameters are reported in the scheme (except  $NF_{LNA}$  to be assessed).

- 1) Evaluate the received power  $P_r$  of the signal at the output of the satellite receiving antenna
- 2) Imposing  $SNR = 30 \text{ dB}$  at the receiving antenna, find the equivalent input temperature  $T_{eq}$  of the receiver
- 3) Evaluate  $NF_{LNA}$  in order to get the requested  $T_{eq}$
- 4) If the LNA and the second filter are removed, what is the new value of  $T_{SSB}$  for maintaining  $T_{eq}$  unchanged?
- 5) Determine the minimum value of  $P_{1dB}$  of the transmitter in order to obtain the mean power of intermodulation at the receiver input equal to the system noise power  $K \cdot T_{eq} \cdot B$  (use a 2-tone signal with 500 W mean power). What is the back-off of the transmitter?

## Exercise 2

It is known the directivity function of an antenna operating at 1 GHz:  $f(\theta, \varphi) = \sin(\theta) \cdot \sin^2(\varphi/2)$ .

- 1) Evaluate the directivity gain  $D_M$
- 2) Find the direction  $(\theta_{MAX}, \varphi_{MAX})$  where  $D(\theta, \varphi) = D_M$ .
- 3) Evaluate the exact value of the half-power beamwidth in the elevation plane ( $\theta_{3dB}$ ). Note that the reference axis is along  $\theta_{MAX}$ .
- 4) Assuming the radiation impedance  $Z_{rad} = 50 \Omega$  and the loss resistance  $R_p = 12.5 \Omega$ , evaluate the antenna efficiency factor  $\eta$
- 5) Compute the intensity of the electric field  $|E|$  at 30 m from the antenna in the direction  $(\theta = 135^\circ, \varphi = 180^\circ)$ . Assume the transmitted power  $P_T = 1500$  W.

Note: 
$$\int \sin^2(x) dx = \frac{1}{2} \left[ x - \frac{\sin(2x)}{2} \right]$$

## Solution

### Exercise 1

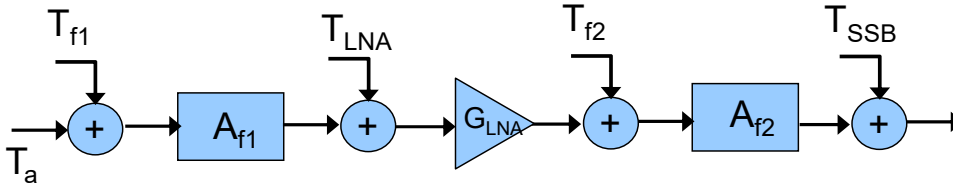
Link equation (in dBm):

$$P_r = P_t + 20 \log \left( \frac{\lambda}{4\pi R} \right) + G_T + G_R = 57 + 50 + 25 - 199.7 = -67.7 \text{ dBm}$$

Evaluation of  $T_{eq}$ :

$$SNR = P_r - 10 \log(KT_{eq}B) = 30 \Rightarrow T_{eq} = 384.41 \text{ } ^\circ\text{K}$$

$NF_{LNA}$ :



$$A_{f1}=1.023, A_{f2}=1.122 \quad T_{f1}=T_0(10^{A_{f1}/10}-1)=6.75^\circ\text{K}, \quad T_{f2}=T_0(10^{A_{f2}/10}-1)=35.38^\circ\text{K} \quad g_{LNA}=10$$

$$T_{eq} = T_a + T_{f1} + A_{f1}T_{LNA} + \frac{A_{f1}[T_{SSB}A_{f2} + T_{f2}]}{G_{LNA}} = 384.41^\circ\text{K}$$

$$T_{LNA} = \frac{1}{A_{f1}} \left\{ 384.41 - \left[ T_a + T_{f1} + \frac{A_{f1}[T_{SSB}A_{f2} + T_{f2}]}{G_{LNA}} \right] \right\} = 301.1^\circ\text{K}$$

$$NF_{LNA} = 10 \log \left( 1 + \frac{T_{LNA}}{290} \right) = 3.09 \text{ dB}$$

Removing the LNA and second filter:

$$T_{eq} = T_a + T_{f1} + A_{f1}T_{SSB} = 384.41^\circ\text{K}$$

$$T_{SSB} = \frac{1}{A_{f1}} \left\{ 384.41 - [T_a + T_{f1}] \right\} = 349.52^\circ\text{K}$$

As the noise power is imposed equal to the power of intermodulation, the CI (carrier-to-intermodulation) is equal to SNR (30 dB). Moreover, CI value is conserved moving from the receiver input to the transmitter output. It has then:  $CI = 2(IP_3 - P_m) + 6 = 30$  with  $P_m = 57 \text{ dBm}$ .

We get  $IP_3$  and  $P_{1dB}$ :

$$IP_3 = \frac{CI}{2} + P_m - 3 = 69 \Rightarrow P_{1dB} = IP_3 - 10 = 59 \text{ dBm}$$

The back-off is given by:  $BO = P_{1dB} - P_m = 2 \text{ dB}$  (with  $\Delta_p \approx 10 \text{ dB}$ )

## Exercise 2

$$1) D_M = 4\pi \int_0^\pi \int_0^{2\pi} f(\vartheta, \varphi) \sin(\theta) d\theta d\varphi = 4\pi \int_0^\pi \int_0^{2\pi} \sin^2(\theta) \sin^2\left(\frac{\varphi}{2}\right) d\theta d\varphi = 4\pi \left[ \frac{2}{\pi^2} \right] = \frac{8}{\pi}$$

$$2) \theta_{MAX} = 90^\circ, \quad \varphi_{MAX} = 180^\circ$$

3) Imposing  $f(\theta_B, \varphi) = f(\theta_{MAX}, \varphi) / 2 = 0.5 \sin^2(\varphi/2)$  we get  $\sin(\theta_B) = 0.5$  and  $\theta_B = 30^\circ$ . Taking into account that the reference direction is  $\theta = 90^\circ$ , it has:  $\theta_{3dB} = 2 \cdot (90 - 30) = 120^\circ$ .

$$4) \eta = \frac{R_{rad}}{R_{rad} + R_p} = 0.8$$

5) We know that:

$$|E| = \frac{1}{R} \sqrt{\frac{Z_0 \cdot P_{ERP}}{2\pi}} \quad \text{with } R=30\text{m}, Z_0=377\Omega, P_{ERP}=\eta D_M P_T f(135^\circ, 180^\circ)=2160.1$$

Then  $|E|=12 \text{ V/m}$