RF SYSTEMS
Mid-course Test - November 21th, 2016

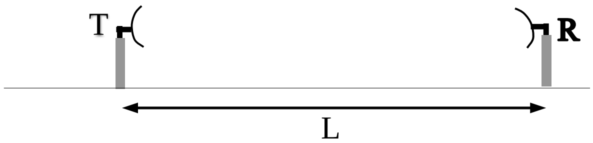
Surname & Name
Identification Number
Signature

Exercise 1

It is given the following antenna directivity function: $f(\theta,\varphi)=\dfrac{\cos^2\left(\dfrac{\pi}{2}\cos\left(\theta+\dfrac{\pi}{2}\right)\right)}{\sin^2\left(\theta+\dfrac{\pi}{2}\right)}$.

The operating frequency is 3 GHz.

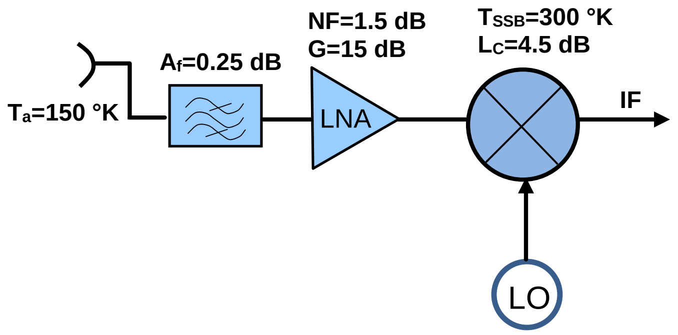
1) What is the direction θ_{MAX} for which $f=f_{max}=1$? Evaluate the 3dB beamwidth (θ_{3dB}) around this direction
2) Assuming that the emitted power is uniformly distributed on the spherical cap closing the cone of aperture $2\theta_B$ equal to θ_{3dB}, evaluate the directivity gain D_{MAX}.
3) It is known that $\dfrac{1}{r^2}\iint\limits_{\Sigma} f(\theta,\varphi)\cdot d\Sigma=1.745$ where Σ is the surface of the sphere of radius r centered on the antenna. Evaluate the exact value of D_{MAX}.
4) It is also known the radiation impedance $R_{rad}=50\ \Omega$ and the efficiency factor $\eta=0.85$. Using the antenna in a receiver, evaluate the power P_L delivered to the receiver ($Z_L=50\ \Omega$) when the incident power density S_R on the antenna is $2{\cdot}10^{-6}$ W/m^2.
5) Suppose that this antenna is used (both at T and R) in a communication system with transmitted power $P_T=30$ dBm, equivalent noise temperature of the receiver $T_{eq}=350°$, signal bandwidth B=30 MHz:



Evaluate the maximum distance L for which the SNR at the receiver R is equal to 30 dB.

Exercise 2

Consider the following block diagram referring to the RF front-end of a receiver operating in the band 6-6.5 GHz (10 channels with 50 MHz width):



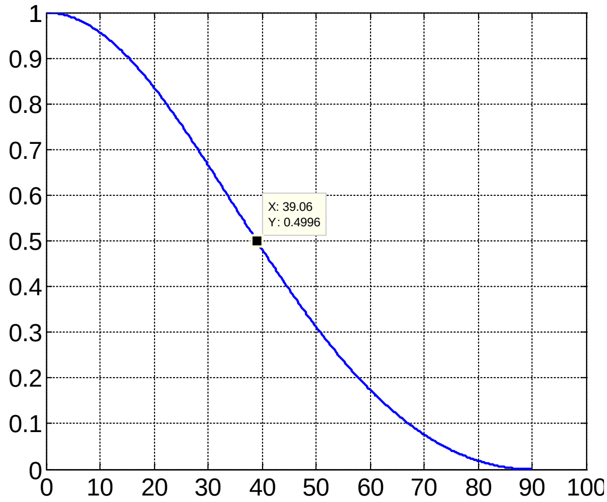
Assume the filter ideal in the stopband and equivalent to an attenuator (A_f) in the passband.

1) Assuming that the local oscillator operates at frequencies below the RF band (6-6.5 GHz), determine the range of the LO frequencies f_{OL} imposing the intermediate frequency f_{IF}=470 MHz.
2) What is the range of frequencies of the image band?
3) Evaluate the equivalent noise temperature T_{eq} of the RF front-end. Compute also the overall noise power at the mixer output (Assume the signal bandwidth B=50 MHz)
4) We know the minimum power density of the radiation incident on the antenna S_R= 10^{-8} W/m^2. Evaluate the antenna gain G_A in order the signal-to-noise-ratio of the RF front-end is at least 15 dB (refer to the channel at f_{RF}=6 GHz)
5) Assume the receiver is tuned at f_{RF}=6.425 GHz and the received power at antenna output is Pr=-80 dBm. In the adjacent channel at f_{ADJ}=6.375 GHz there is a strong interference constituted by 2-tone spaced by 40 MHz (and centered at f_{ADJ}) and mean power P_{ADJ}=-30 dBm. Note that the LNA amplify both signals because they are in the tuning band (the interference at f_{ADJ} is finally removed by the IF filter)
   a. Verify that one of the intermodulation tones produced in the LNA by the signal at f_{ADJ} falls inside the received channel (6.4-6.45 GHz)
   b. Evaluate the minimum IP3 of LNA to get the SNR=Po/P_{int}=30 dB, with Po the power at f_{RF} at the LNA output and P_{int} the power in the intermodulation line.
   c. Assuming the IIP3 of the mixer equal to 0 dBm evaluate the IIP3 of the whole RF front-end

Solution

Exercise 1

1) $\theta_{MAX} = 0°$. Drawing graphically the function f(θ) it has:



From the graph f(θ)=0.5 for θ=39.06°. Then θ_{3dB}=2*39.06=78.12°

2) $$D_{MAX} = \frac{2}{\left(1-\cos(\theta_B)\right)} = \frac{2}{\left(1-\cos\left(\frac{\theta_{3dB}}{2}\right)\right)} = 8.955$$

3) $$\left(D_{MAX}\right)_{exact} = \frac{4\pi}{\frac{1}{r^2}\iint_{\Sigma} f(\theta,\varphi)\cdot d\Sigma} = \frac{4\pi}{1.745} = 7.2$$

4) $$A_e = \frac{G\cdot\lambda^2}{4\pi} = \frac{\eta\cdot D_{\max}\cdot\lambda^2}{4\pi} = \frac{0.85\cdot 7.2\cdot(3\cdot 10^8/3\cdot 10^9)^2}{4\pi} = 0.0049\ \text{m}^2$$

$$P_r = A_e S_R = 9.8\cdot 10^{-9}\ \text{W (-50.09 dBm)}$$

Being the receiver input matched to the antenna (loss resistance can be discarded), P_L=Pr=-50.09 dBm

5) The Friis equation for the considered link is given by:

$$P_r = P_t - L_f + 10\log\left(2\eta D_{MAX}\right) = 45.73 - L_f$$

with $L_f = 20\cdot\log\left(\frac{4\pi L}{\lambda}\right)$ and λ=0.1m.

The noise power at the receiver input is $P_N=K\cdot T_{eq}\cdot B=1.45\cdot 10^{-13}$ W (-98.39 dBm). Imposing SNR=P_r/P_N=30 dB we get *Pr*=-68.39 dBm. From the above equation:

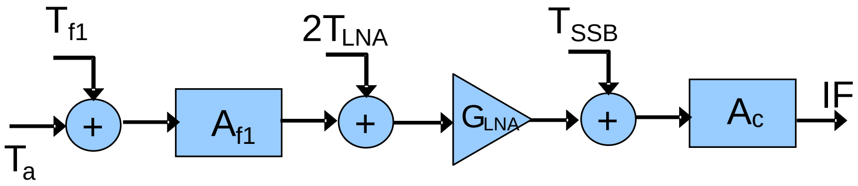
L_f=45.73-(-68.39)=114.12 dB → L=4.04 Km.

Exercise 2

1) The RF band is f_{IF} above f_{OL}, then $f_{OL}=f_{RF}-f_{IF}=5.53-6.03$ GHz

2) The image band is f_{IF} below f_{OL}, then $f_{IM}=f_{OL}-f_{IF}=5.06-5.56$ GHz

3) Consider the equivalent block diagram referring to noise temperature:



It has:

a_f=1.059, a_c=2.818 $T_f = T_0\left(10^{A_f/10}-1\right) = 17.18°K,\ g_{LNA} = 31.62$

Evaluation of T_{IF}:

$$T_{IF} = \left(T_a + T_f\right)\frac{g_{LNA}}{a_f a_c} + 2T_{LNA}\frac{g_{LNA}}{a_c} + T_{SSB}\frac{1}{a_c} = 4562.016\ °K$$

The equivalent noise temperature at input:

$$T_{eq} = T_{IF}\frac{a_f a_c}{g_{LNA}} = 430.68\ °K$$

4) The equivalent noise power at input is given by:
$P_N=K \cdot T_{eq} \cdot B=2.97 \cdot 10^{-13}$ W (-95.27 dBm).
Imposing SNR=Pr-P_N=15 dB we get Pr=-80.27 dBm ($9.4 \cdot 10^{-12}$ W).
Being $Pr=A_e \cdot S_R$ → $Ae=9.4 \cdot 10^{-4}$ m and $G=(4\pi/\lambda^2)A_e=4.7236$ (6.74 dB)
The wavelength λ is given by $3 \cdot 10^8/6 \cdot 10^9=5 \cdot 10^{-2}$m.

5) The intermodulation tone are located at $f_0 \pm 1.5\Delta=6.375\pm0.06=(6.435, 6.315)$. The first tone is inside the received channel.
The power received in the desired channel after the LNA is given by Po=-80-0.25+15= -65.25 dBm. The power Pint in the intermodulation line must be 30 dB below Po, then Pint=-95.25 dBm. The power in adjacent channel at LNA output is $P'_{ADJ}= P_{ADJ}+G_{LNA}-A_f=$ -15.25 dBm (mean power). Imposing Pint=3(P'_{ADJ} -3)-2IP3, we obtain IP3=(3(P'_{ADJ} -3)-Pint)/2=20.25 dBm.
The overall IIP3 of the RF front-end can be derived from:

$$\left(\frac{1}{iip_3}\right)^2_{tot} = \left(\frac{g_{LNA}}{a_f ip_{3,LNA}}\right)^2 + \left(\frac{g_{LNA}}{a_f iip_{3,MIX}}\right)^2$$ → $iip_{3,tot}$=0.0335 mW ($IIP_{3,tot}$=-14.75 dBm)