## RF SYSTEMS

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## Identification Number

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## Exercise 1



The figure shows a terrestrial link operating at 12 GHz with bandwidth $\mathrm{B}=10 \mathrm{MHz}$. The two stations are located 15 Km away and the antennas are identical with the following features:

Efficiency $\eta=0.8$. Directivity function $f(\theta)=1$ for $0<\theta<20^{\circ},=0$ elsewhere.

1) Evaluate the gain $G$ of the antennas
2) It is known the system noise temperature Tsys $=1000{ }^{\circ} \mathrm{K}$ of the station R . What it the power to be transmitted from T in order to get the system SNR equal to 30 dB ? Assume the antennas pointed for the maximum gain.

Consider the following scheme for the receiving station R:

3) Evaluate the equivalent noise temperature of the receiver $\left(T_{R}\right)$ compatible with the imposed system SNR .
4) Assuming $\mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0}=20 \mathrm{~dB}$ at the receiver output, what is the maximum data rate R ?

Consider now another transmitting station located as in the following figure, transmitting a power $\mathrm{P}=10 \mathrm{~W}$ at the same frequency of $\mathrm{T}(12 \mathrm{GHz})$. The antenna of this station is characterized by $\mathrm{G}=10$ dB and $g(\theta)=\cos \left(5.7^{\cdot} \theta\right)$. The signal received by R represents in this case an interference and should be treated as noise which is added to the thermal noise of the receiving station.

5) Evaluate the minimum distance $L$ so that the overall system $S N R$ of $R$ is not lower than 20 dB.

## Exercise 2

A base station transmitter generates a signal constituted by 4 channels 5 MHz spaced, each with bandwidth 1 MHz . The first channel starts at 1.8 GHz . This signal is produced by 4 amplifiers (PA) followed by a selective combiner (producing a loss of 1 dB for each channel).
Assume that the signal in each channel can be represented by 2 tones 1 MHz apart. If the mean power per channel must be 26 dBm (at the output of the combiner), find the $\mathrm{P}_{1 \mathrm{~dB}}$ of the amplifiers which determine the minimum carrier-to-intermodulation (CI) equal to 30 dB (see the figure).


Consider now the use of a single PA (MCPA). Assume that the signal in each channel is represented by a single tone with the assigned mean power ( 26 dBm ). We ask to determine the $\mathrm{P}_{1 \mathrm{~dB}}$ of the MCPA which still determine $\mathrm{CI}=30 \mathrm{~dB}$. For performing this computation, assume that the 4 -tone signal at the output of the MCPA is equivalent (for the IM3 generation) to a 2-tone signal with the same mean power:


The gain of the MCPA is not sufficient to get the requested power at the output. A driver is then used before the MCPA. Knowing that $\mathrm{G}_{\mathrm{MCPA}}=10 \mathrm{~dB}, \mathrm{P}_{\mathrm{in}, \mathrm{m}}=6 \mathrm{dBm}$, find the gain $\mathrm{G}_{\mathrm{drv}}$ and $\mathrm{P}_{1 \mathrm{~dB}}$ of the driver in order to get the requested output power and the overall $P_{1 d B}$ reduced by 1 dB with respect that of the MCPA.

In what way is it possible to reduce the requested $\mathrm{P}_{1 \mathrm{~dB}}$ of the MCPA still maintaining the same CI (and not resorting to a linearizer)?

## Exercise 3

We want to design the oscillator in the figure operating at 2 GHz :


The S parameters of the active device at 2 GHz are given in the following table as function of the bias current:

| Ibias | S11 | S12 | S21 | S22 |
| :--- | :--- | :--- | :--- | :--- |
| 10 mA | $0.745 \angle-162.9^{\circ}$ | $0.063 \angle-7.1^{\circ}$ | $1.875 \angle 25.1^{\circ}$ | $0.602 \angle-119.6^{\circ}$ |
| 20 mA | $0.76 \angle-145^{\circ}$ | $0.06 \angle-1.2^{\circ}$ | $1.92 \angle 43.6^{\circ}$ | $0.603 \angle-105.3^{\circ}$ |
| 30 mA | $0.864 \angle-93.4^{\circ}$ | $0.064 \angle 27.4^{\circ}$ | $2.545 \angle 93.8^{\circ}$ | $0.627 \angle-64.2^{\circ}$ |

1) Select the bias current (imposing the necessary oscillation condition)
2) Assign a suitable value to the resonant frequency $f_{\text {res }}$
3) Assuming the relative dielectric constant of the lines $\varepsilon_{\mathrm{r}}=2.2$, evaluate the length $L_{i n}$ of the input line
4) Evaluate the reflection coefficient $\Gamma_{L}$ to be presented at the transistor output and design the output network (i.e. evaluate the lengths $L_{\text {out }}$ and $L_{\text {stub }}$ )

## Exercise 1

The antenna gain is obtained from the formula:
$G=\eta 4 \pi\left[\int_{0}^{2 \pi} d \varphi \int_{0}^{\pi} g(\theta) \sin \theta d \theta\right]^{-1}=2 \eta \int_{0}^{20^{\circ}} \sin \theta d \theta=\frac{2 \eta}{1-\cos \left(20^{\circ}\right)}=26.53(14.24 \mathrm{~dB})$
The system SNR is defined as:
$S N R_{s y s}=\frac{P_{\text {rec }}}{K T_{\text {sys }} B}=30 \mathrm{~dB}$
Then the received power must be $\mathrm{Pr}=30+\left.\mathrm{KT}\right|_{\mathrm{dBm}}+10 \log (\mathrm{~B})=30-168.6+70=-68.6 \mathrm{dBm}$
The Friis equation for the given link is:
$P_{r e c}=P_{t r}+2 G-L_{f}$
with: $L_{f}=20 \cdot \log \left(\frac{4 \pi R}{\lambda}\right)=137.55 \mathrm{~dB},(\lambda=3 \mathrm{e} 8 / 12 \mathrm{e} 9=0.025 \mathrm{~m})$
Then the transmitted power results:
$P_{t r}=P_{\text {rec }}-2 G+L_{f}=-68.6-2 G+L_{f}=40.45 \mathrm{dBm}$
From the scheme of the receiving system it has:
$T_{\text {sys }}=T_{a}+T_{f}+290 \cdot\left(10^{A_{f} / 10}-1\right)+T_{L N A} \cdot 10^{A_{f} / 10}+T_{R} \cdot 10^{\left(A_{f}-G_{\text {LNA }}\right) / 10}=1000^{\circ} \mathrm{K}$
from which: $\mathrm{T}_{\mathrm{R}}=1875.2^{\circ} \mathrm{K}$
The $\mathrm{SNR}_{\text {sys }}$ is related to $\mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0}$ as follows:
$S N R_{s y s}=\frac{E_{b}}{N_{0}} \frac{R}{B}$
The value of R is then given by: $\mathrm{R}=\mathrm{B} \cdot 10^{(30-20) / 10}=100 \mathrm{Mbit} / \mathrm{sec}$
From the Friis equation for the second link the received power at R is expressed as:
$\mathrm{P}^{\prime}{ }_{\text {rec }}=\mathrm{P}^{\prime} \mathrm{t}+\mathrm{G}_{\mathrm{T}}+\mathrm{G}_{\mathrm{R}}+f\left(15^{\circ}\right)+g\left(15^{\circ}\right)-\mathrm{Lf}{ }^{\prime}=40+10+14.24+0+10 \log \left(\cos \left(5.7 \cdot 15^{\circ}\right)\right)-\mathrm{Lf} \mathrm{f}^{\prime}=53.19 \mathrm{dBm}-\mathrm{L}_{\mathrm{f}}$
The received power is imposed by the minimum system $S N R=20 \mathrm{~dB}$ (100):
$S N R_{s y s}^{\prime}=\frac{P_{r e c}}{K T_{s y s} B+P_{r e c}^{\prime}}=\frac{1}{\frac{K T_{s y s} B}{P_{r e c}}+\frac{P_{r e c}^{\prime}}{P_{r e c}}}=\frac{1}{\frac{1}{S N R_{s y s}}+\frac{P_{r e c}^{\prime}}{P_{r e c}}}=100$
It has then:
$\frac{P_{r e c}^{\prime}}{P_{r e c}}=\frac{1}{S N R_{s y s}^{\prime}}-\frac{1}{S N R_{\text {sys }}}=0.01-0.001=0.009$,
$\left.P_{\text {rec }}^{\prime}\right|_{d B m}=\left.P_{\text {rec }}\right|_{d B m}+10 \cdot \log (0.009)=-68.6-20.46=-89.06$
The link attenuation can be obtained as follows:
$\mathrm{L}_{\mathrm{f}}=53.18+89.06=142.24$
The minimum distance $L$ is then given by:
$L=10^{142.24 / 20} \cdot \frac{0.025}{4 \pi}=25.75 \mathrm{Km}$

## Exercise 2

With the mean power per channel at output equal to 26 dBm , each amplifier must exhibit an output power in each tone given by $\mathrm{P}_{0}=26+1-3=24 \mathrm{dBm}$. It has then:
$\mathrm{CI}=2\left(\mathrm{IP} 3-\mathrm{P}_{0}\right) \rightarrow$ IP3 $=\mathrm{CI} / 2+\mathrm{P}_{0}=15+24=39 \mathrm{dBm} \rightarrow \mathrm{P}_{1 \mathrm{~dB}}=39-10=29 \mathrm{dBm}$
In the case of using a MCPA, the mean power of the equivalent 2-tone signal is given by $P^{\prime}$ mean $=4 \mathrm{Pm}=26+6=32 \mathrm{dBm}$. The power per tone is $P^{\prime} m=32-3=29 \mathrm{dBm}$, so the new IP3 is given by:
IP3' $=\mathrm{CI} / 2+\mathrm{P}^{\prime}{ }_{\mathrm{m}}=15+29=44 \mathrm{dBm} \rightarrow \mathrm{P}^{\prime}{ }_{1 \mathrm{~dB}}=44-10=34 \mathrm{dBm}$
With Pin $=6 \mathrm{dBm}$ and $\mathrm{G}_{\mathrm{MCPA}}=10 \mathrm{~dB}$, in order to get Pout=32 dB we need $\mathrm{G}_{\mathrm{drv}}=16 \mathrm{~dB}$.
The $\mathrm{P}_{1 \mathrm{~dB}}$ of the driver is obtained from this equation:
$\frac{1}{I P 3_{\text {tot }}^{2}}=\frac{1}{I P 3_{M C P A}^{2}}+\frac{1}{G_{M C P A}^{2} \cdot I P 3_{d r v}^{2}} \rightarrow I P 3_{d r v}=\frac{1}{G_{M C P A}} \sqrt{\frac{1}{\left(\frac{1}{10^{0.2 \cdot I 3_{30 t}}}-\frac{1}{10^{0.2 \cdot P P_{3 C P A}}}\right)}}=3284.4(35.16 \mathrm{~dB})$
$P_{1 \mathrm{~dB}, \mathrm{drv}}=35.16-10=25.16 \mathrm{dBm}$
The $\mathrm{P}_{1 \mathrm{~dB}}$ of the MCPA can be reduced by 3 dB without affecting the CI by using a balanced configuration (two identical amplifiers + two $90^{\circ}$ hybrids).

## Exercise 3

Using the electronic Smith Chart it can be observed that the active device is potentially instable $(k<1)$ only with Ibias $=30 \mathrm{~mA}$.
The resonant frequency of the resonator is assigned equal to the oscillation frequency. The input line is then an open stub with $b_{s}=\tan \left(\beta \cdot \mathrm{L}_{\mathrm{in}}\right)$. For choosing bs the mapping circle of the source is drawn with $\left|\Gamma_{\text {out }}\right|=1.2$. The chosen point must be also on the outer circle (two choices); we have selected $b s=-1.39$. The electrical length of the input stub is then given by:
$\left(\beta \mathrm{L}_{\text {in }}\right)=\tan ^{-1}(-1.2)=129.8^{\circ}$
It has:
$\mathrm{Z}_{\text {out }}=-0.27-\mathrm{j} 1.386 \rightarrow \mathrm{Z}_{\mathrm{L}}=0.09+\mathrm{j} 1.386$.
The single-stub matching network transforms $\mathrm{Z}_{\mathrm{L}}$ into 50 Ohm . We get:
$\left(\beta \mathrm{L}_{\text {out }}\right)=44.4^{\circ}, \mathrm{b}_{\text {stub }}=-5.55 \rightarrow\left(\beta \mathrm{~L}_{\text {stub }}\right)=\tan ^{-1}(1 / 5.55)=10.21^{\circ}$
Lengths computations:
$\lambda=\frac{300}{f_{0} \sqrt{\varepsilon_{r}}}=101.13 \mathrm{~mm}, \quad \beta=\frac{360}{\lambda}=3.56 \% \mathrm{~mm}$
$\mathrm{L}_{\text {in }}=129.8 / \beta=36.46 \mathrm{~mm}, \mathrm{~L}_{\text {out }}=44.4 / \beta=12.47 \mathrm{~mm}, \mathrm{~L}_{\text {stub }}=10.21 / \beta=2.87 \mathrm{~mm}$

