RF SYSTEMS
Written Test of 28 September 2015

## Surname \& Name

Identification Number

## Signature

## Exercise 1

Consider the following receiver operating at 15 GHz (signal bandwidth $\mathrm{B}=70 \mathrm{MHz}$ ), with the specified parameters of the filter, mixer and antenna:


$$
\begin{aligned}
& \mathrm{A}_{0}=0.5 \mathrm{~dB} \\
& \mathrm{~A}_{\mathrm{c}}=4 \mathrm{~dB} \\
& \mathrm{~T}_{\mathrm{SSB}}=400{ }^{\circ} \mathrm{K} \\
& \mathrm{~T}_{\mathrm{A}}=40^{\circ} \mathrm{K}
\end{aligned}
$$

It is now requested that:

- the minimum power of the received signal at the antenna port is -43 dBm
- the minimum power level of the signal at IF ( $\mathrm{P}_{\mathrm{IF}}$ ) must be -30 dBm
- the $\mathrm{S} / \mathrm{N}$ (power) ratio at IF must be at least $55 \mathrm{~dB}\left(\mathrm{~S} / \mathrm{N}=\mathrm{P}_{\mathrm{IF}} / \mathrm{N}_{\mathrm{IF}}\right)$

Compute the gain $\mathrm{G}_{\mathrm{RF}}$ and the noise figure NF of the LNA so that the above requirements are satisfied (assume that the filter eliminates completely the image band)

Hint: The noise power at IF can be expressed as $\mathrm{N}_{\mathrm{IF}}=\mathrm{K}_{\mathrm{IF}} \cdot \mathrm{B}$, where K is the Boltzmann Constant $\left(1.38 \cdot 10^{-23} \mathrm{~W} /{ }^{\circ} \mathrm{K} \cdot \mathrm{Hz}\right) ; \mathrm{T}_{\text {IF }}$ is the equivalent temperature at IF; B is the signal bandwidth.

## Exercise 2

A satellite earth station operates at 6 GHz with a geostationary satellite (distance: 36000 Km ). The parameters of the employed antennas are the following:

## Earth station:

Gain $G_{T}=55 \mathrm{~dB}$, directivity function $f_{T}=\cos \left(110 \cdot \theta_{T}\right)$
Satellite:
Gain $G_{s}$ to compute, directivity function $f_{S}=\cos \left(\theta_{S}\right)$
The transmitted power from the earth station is 5.5 kW .
a) Assuming initially the two antennas perfectly aligned $\left(\theta_{T, S}=0\right)$, evaluate the gain $G_{s}$ of the satellite antenna in order the received power $\left(\mathrm{P}_{\mathrm{R}}\right)$ is $\geq-43 \mathrm{dBm}$.
b) Let assume now that the earth antenna is misaligned ( $\theta_{T} \neq 0$ ). Evaluate the maximum value of $\theta_{T}$ that determines the received power to decrease by 3 dB (note that the satellite antenna remains optimally directed, so $f_{s}=1$ )

## Exercise 3

The following figure represents the general configuration of a microwave oscillator. Using the reported scattering parameters of the active device, evaluate the reflection coefficients $\Gamma_{1}$ and $\Gamma_{2}$ which ensure the start of oscillation (the magnitude of $\Gamma_{1}$ must be imposed equal to 1 ).

Hint: draw the mapping circle of the source with $\left|\Gamma_{\text {out }}\right|=1.2$ for determining $\Gamma_{1}$. For evaluating $\Gamma_{2}$ determine the value of $\mathrm{Z}_{\text {out }}$ corresponding to the selected $\Gamma_{1}$ and assign $\mathrm{Z}_{2}=\left|\mathrm{R}_{\text {out }} / 3\right|-\mathrm{j} \mathrm{X}_{\text {out }}$.


Then design the network B , using the scheme in the following figure (assume $\mathrm{Z}_{\mathrm{c}}=50 \mathrm{Ohm}$ and evaluate the electrical length $\phi_{0}$ and the susceptance B):


## Solution

## Exercise 1

From the imposed signal power at IF the gain $\mathrm{G}_{\mathrm{RF}}$ is obtained:
$P_{I F}=P_{i n} \frac{G_{R F}}{A_{0} A_{C}} \Rightarrow G_{R F}=\frac{P_{I F}}{P_{i n}} A_{0} A_{C}=17.5 \mathrm{~dB}$
The equivalent noise temperature at IF is obtained from the imposed $\mathrm{S} / \mathrm{N}$ :

$$
N_{I F}=\frac{P_{I F}}{S / N}=K \cdot T_{I F} \cdot B=3.16 \cdot 10^{-12} \mathrm{~W} \Rightarrow T_{I F}=\frac{3.16 \cdot 10^{-12}}{K \cdot B}=3273.6^{\circ} \mathrm{K}
$$

$\mathrm{T}_{\text {IF }}$ results from the following equivalent representation:

$T_{I F}=\frac{\left[\frac{\left[\left(T_{R F}+T_{A}\right) G_{R F}+T_{f}\right]}{A_{0}}+T_{S S B}\right]}{A_{C}}$
where the parameter $T_{f}$ is given by $T_{f}=T_{0}\left(A_{0}-1\right)$.
The unknown is represented by $\mathrm{T}_{\mathrm{RF}}$, which results:
$T_{R F}=\frac{\left[\left(A_{C} T_{I F}-T_{S S B}\right) A_{0}-T_{f}\right]}{G_{R F}}-T_{A}=115.46{ }^{\circ} \mathrm{K}$
Finally the noise figure of the amplifier is given by:
$N F=10 \cdot \log _{10}\left(1+\frac{T_{R F}}{T_{0}}\right)=1.45 \mathrm{~dB}$

## Esercizio 2

Link budget equation (in dB ) with aligned antennas:

$$
P_{S, d B m}=P_{T, d B m}+20 \cdot \log (\lambda)-20 \cdot \log (4 \pi R)+G_{T, d B}+G_{S, d B}
$$

Substituting: $\mathrm{P}_{T, d B m}=67.4 \mathrm{dBm}, \mathrm{P}_{\mathrm{S}, \mathrm{dBm}}=-43 \mathrm{dBm}, \lambda=50 \mathrm{~mm}, \mathrm{R}=36000 \mathrm{Km}, \mathrm{G}_{\mathrm{T}}=55 \mathrm{~dB}$ we get:

$$
G_{S, d B m}=-43-67.4-20 \cdot \log (0.05)+20 \cdot \log \left(4 \pi \cdot 36 \cdot 10^{6}\right)-55=33.73 \mathrm{~dB}
$$

Evaluation of the misalignment angle of the earth antenna that determine 3 dB reduction of the received power:

$$
P_{S, d B m}-3=P_{T, d B m}+20 \cdot \log (\lambda)-20 \cdot \log (4 \pi R)+G_{T, d B}+G_{S, d B}+10 \cdot \log (\cos (110 \cdot \theta))
$$

Then:

$$
\begin{gathered}
10 \cdot \log (\cos (110 \cdot \theta))=-3 \\
\theta=\frac{\operatorname{acos}\left(10^{-0.3}\right)}{110}=0.0095 \mathrm{rad}\left(0.545^{\circ}\right)
\end{gathered}
$$

## Exercise 3

The assigned transistor is potentially instable ( $\mathrm{k}=0.53$ ), so it can be used for realizing an oscillator. Using the electronic Smith Chart, the mapping circle with $\left|\Gamma_{\text {out }}\right|=1.2$ is drawn. The two intersections with the outer circle are: $\Gamma_{1 \mathrm{a}}=1 \angle 23^{\circ}$ and $\Gamma_{1 \mathrm{~b}}=1 \angle-161.5^{\circ}$
Selecting "S Param." $\rightarrow$ "Gamma OUT" the reflection coefficient at port 2 is obtained:
$\Gamma_{\text {out, }, ~}=1.2 \angle-168^{\circ}\left(\Gamma_{\text {out,a }}=1.2 \angle-28.7^{\circ}\right)$ The S. chart reports also the normalized impedance $\mathrm{Z}_{\text {out }, \mathrm{a}}=-0.092-\mathrm{j} 0.103$ ( $\mathrm{Z}_{\text {out }, \mathrm{b}}=-1.31-\mathrm{j} 3.44$ ).
Imposing the condition suggested in the text, the values of $Z_{2}$ and $\Gamma_{2}$ are then obtained:
$\mathrm{Z}_{2 \mathrm{a}}=0.03+\mathrm{j} 0.103 \rightarrow \Gamma_{2 \mathrm{a}}=0.94 \angle 168.2^{\circ}\left(\mathrm{Z}_{2 \mathrm{~b}}=0.44+\mathrm{j} 3.44 \rightarrow \Gamma_{2 \mathrm{~b}}=0.935 \angle 32^{\circ}\right)$
The matching network is designed according the well-known procedure:


$\phi_{0}=15.85^{\circ}, b=5.5$

$\phi_{0}=63.7^{\circ}, \mathrm{b}=-5.3$

